A TREATISE

ON

PLANE SURVEYING.

BY

DANIEL CARHART, C.E.,

PROFESSOR OF CIVIL ENGINEERING IN THE WESTERN
UNIVERSITY OF PENNSYLVANIA.

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PREFACE.

THIS work, as its name indicates, extends over the field of plane surveying. It illustrates and describes the instruments employed, their adjustments and uses; it exemplifies the best methods of solving the common problems occurring in practice, and furnishes solutions for many special cases which not unfrequently present themselves. An experience of twenty years in the field and in technical schools confirms the opinion that a work of this kind should be eminently practical; that the student who desires to become a reliable surveyor needs frequently to manipulate the various surveying instruments in the field, to solve many examples in the class-room, and to exercise good judgment in all these operations. With this in view, therefore, the different methods of surveying are treated, directions for using the instruments are given, and these are supplemented by numerous examples to be solved, by various field exercises to be performed, and by many queries to be answered.

Chapter I. is devoted to Chain Surveying, in which directions are given for measuring and ranging out lines, and methods of overcoming obstacles, recording field notes, obtaining areas, and plotting a chain survey.

Chapter II. treats of Compass and Transit Surveying, or when, in addition to the chain, an instrument for measuring angles is employed. In this chapter the compass and transit, the solar attachment, the adjustments of these, and auxiliaries of the transit, such as the stadia wires, gradienter, etc., are fully illustrated and described, and their uses shown. Here the various methods of obtaining the data requisite to deter-

mine the area, as well as the different methods employed in calculating the contents of land, are exhibited. Tests of the accuracy of a survey are indicated, numerous methods of overcoming obstacles, supplying omissions, of ascertaining heights and distances, of keeping the field notes, and of plotting a survey are given, while the uses of the solar attachment in determining the latitude of a station and its geographic meridian are exemplified.

The student now having been taught how to survey land, using a needle instrument, should become acquainted with the declination of the magnetic needle, or variation of the compass, as it is frequently called. This subject is accordingly discussed in Chapter III. Some of the tables and much of the matter is taken from the Reports of the United States Coast and Geodetic Survey. The student will do well to give this chapter a careful inspection, examining the tables and formulas and the directions for determining the true meridian, thus being prepared with facts, figures, and methods, which will enable him intelligently to undertake the retracing of old lines, as well as to establish with considerable precision his geographic meridian, and thereby obtain the declination of the needle.

Chapter IV. is devoted to Laying Out and Dividing Up Land. This subject is of more importance than some suppose, especially to practitioners in the older States of the Union, and is here treated very fully. The principal cases are exemplified, and general directions and suggestions given, so that, it is believed, with a thorough knowledge of this chapter, the student will be enabled, without embarrassment, to meet the requirements of an extensive practice.

The description, adjustment, and use of the Plane Table form the subject of Chapter V. This instrument is being employed more frequently than formerly in park surveys, in determining positions in harbors, along the lines of proposed highways, in "filling in" large surveys, and generally in locating points where extreme accuracy is not required.

In Chapter VI. the system employed by the government in the Survey of the Public Lands is set forth. The description and adjustment of the Solar Compass, which is used quite extensively in these surveys, precede an account of the origin of the system, and the leading points in the "Instructions to Surveyors-General" from the commissioner of the land office. A form of recording the notes extracted from the "Instructions" is also given, the chapter closing with formulas and a table for determining the inclination of meridians and deviation of parallels.

Chapter VII., on City Surveying, is from the pen of my friend and former colleague, Frederic H. Robinson, C.E., City Engineer of Wilmington, Del. This subject has received but little notice from writers on surveying, although the need of some systematic and practical treatment of it has long been recognized. It therefore affords me much pleasure to acknowledge my indebtedness to Professor Robinson for supplying this want, and so enhancing the value of this publication as a text-book. Experience in teaching, and ten years' practice in city surveys and improvements, eminently qualify him to speak on this important subject with authority and in a manner readily understood by students.

The special instruments needed in this branch of surveying are illustrated and described; the adjustment of the Y-level and directions how to level and to record the notes are given; more refined means of measuring lines are discussed; temperature, pull, sag, wind, etc., are considered, and corrections indicated; best directions and width of streets, together with the subject of grades, sewers, the establishment of permanent reference points, and adjusting property lines, are fully set forth.

To my college classmate and esteemed friend, F. Z. Schellenberg, C.E., Superintendent of Westmoreland Coal Co., Irwin, Pennsylvania, I am indebted for Chapter VIII., on Mine Surveying. This chapter, though in general explanatory of what is applicable and peculiar to this branch of surveying,

includes directions for running contours and sketching topog raphy. It is replete with suggestions that will be value when, by the aid of the study of mine workings themselves an their ground, illustrations will be afforded which otherwise, a drawings alone, cannot readily be understood.

The Judicial Functions of Surveyors, as given by Chief Jus tice Cooley, are set forth in an Appendix.

Those who are familiar with the elegant tables of logarithm of numbers and of trigonometrical functions prepared by Pro fessor Wentworth, will likely recognize the use of his electro plates, from which I have been permitted to print Tables I. III., IV., and VII. To him my personal acknowledgment are due. The plates from which Tables II., V., VI., VIII., an IX. are printed were prepared expressly for this work. It i thought that the *four-place* tables of the natural trigonometri cal functions will be found very useful in connection with sur veying and engineering operations. They are believed to b correct, having been very carefully compared with others whos accuracy is unquestioned.

In addition to acknowledgments made elsewhere, I tak pleasure in expressing here my thanks to Messrs. W. and L. E Gurley, of Troy, New York, for the use which I have been per mitted to make of their valuable catalogue, in the descriptio of certain instruments, and for the loan of several plates fo the engraving of instruments; also to Messrs. Fauth and Co. Washington, D.C., and to Messrs. Heller and Brightly, an Messrs. Young and Sons, Philadelphia, Pa., for plates whic they kindly furnished for the illustration of the subject.

D. C.

WESTERN UNIVERSITY OF PENNSYLVANIA,
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SURVEYING.

DEFINITIONS, AND DIVISION OF THE SUBJECT.

1. Surveying is the art of determining and delineating the relative position of points upon the surface of the earth. It consists principally in measuring, laying out, and dividing land; in establishing lost positions; in the measurement of heights and distances; and in the graphical representation of the peculiarities of any part of the earth's surface.

2. It may be divided into two parts: PLANE SURVEYING and GEODETIC SURVEYING.

In Plane Surveying the spherical form of the earth is neglected; in other words, the portion of the earth included in the survey is regarded as a horizontal plane. This may be done without sensible error where, as in ordinary land surveying, the operations are limited to surfaces of small extent.

In Geodetic Surveying the shape of the earth is regarded, since the surfaces under consideration are so extensive, as in the United States Coast and Geodetic Surveys, sensible errors would otherwise arise.

REMARK. The spherical excess of a spherical triangle, each of whose sides is one mile, is less than six-thousandths of a second. The excess amounts to only one second for an area of 75.5 square miles, each side of the equilateral triangle being then about 13 miles.

3. In the following pages Plane Surveying only will be considered, and the subject treated under the following heads:

CHAIN SURVEYING.
COMPASS AND TRANSIT SURVEYING.
PLANE TABLE SURVEYING.
GOVERNMENT SURVEYING.
CITY SURVEYING.
MINE SURVEYING.

In Plane Surveying there are usually three operations:

1. *The Field Work.*
2. *The Graphical Representation, or Plot.*
3. *The Computation.*

CHAPTER I.

CHAIN SURVEYING.

SECTION I.

INSTRUMENTS.

4. **Chain Surveying** has chiefly for its object the determination of areas from data obtained by direct measurement of distances between points. The instruments needed are therefore simply those for measuring *lines*.

5. **Gunter's Chain,** so called from its inventor, is generally used for this purpose. It is made of iron or steel wire, is 66 feet in length, and divided into 100 links, so that each link, with half the rings connecting it with the adjoining links, is seven and ninety-two hundredths inches (7.92), or one-hundredth of a chain. Swivels are inserted to keep it from twisting, and every tenth link has a metallic mark attached, so that the number of tens from either end is readily ascertained. Its advantages in surveying farms or fields are apparent: there being 4840 square yards in an acre, and the chain 22 yards long, a square chain will contain one-tenth of an acre; or, there being 10,000 square links in a square chain, which is one-tenth of an acre, 100,000 square links are equivalent to an acre. Hence, if the area of a field is calculated in links, the area is at once shown in acres, by cutting off the last five figures. If the area is found in chains, then since there are ten square chains in an acre, the area is given in acres by cutting off the last figure.

6. **A Two-Pole,** or **Half-Chain** is sometimes used instead of Gunter's Chain. It is quite convenient for measuring lines where the ground is rough and hilly.

7. The Engineer's Chain is used in surveying railroads and canals, and generally where extensive line surveys are being conducted; hence not unfrequently it is employed in connection with these surveys, as well as otherwise, in determining areas. It is 100 feet in length, and is divided into 100 links, every tenth link being marked by a piece of brass, as in the four-pole chain.

8. The Tape Measure is very convenient for taking offsets in a survey, for measuring the boundaries of city lots, cross-sectioning in railroad work, etc. Tapes are "metallic," or steel, and made of various lengths,* — 50 feet or 100 feet are commonly used, — and divided into feet and inches, or feet and tenths of a foot. The latter graduation is preferable for the railroad engineer, and the former for the city engineer.

9. Eleven Marking-Pins, 12 or 14 inches long, one of which is made of brass, the others of No. 4 iron wire or No. 6 steel, all pointed at one end and formed into a ring at the other, are used in chaining.

10. Straight Poles about 8 feet long, shod at the bottom with a conical shoe, point down, and painted alternately red and white in foot-width bands, are used to indicate the direction of the line which is being measured, or the position of points to be located.†

SECTION II.

A. CHAINING.

11. Two men are required, a "leader" and a "follower," or *head* and *hind* chainman. The chain is first thrown out in the general direction of the line which it is desired to measure, and

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* Steel tapes 1000 feet in length have been frequently used for special purposes. See Mine Surveying, p. 380.

† See Article 383.

examined carefully to see if there are any kinks in it, or bends in the links; the leader having the marking-pins in one hand takes hold of the forward end of the chain with the other, and moves on as nearly as he may judge in the direction of the line; the follower places the rear end of the chain at the station whence the line is to be measured, directs the leader by signals as he approaches the chain's length to get in *line*, and then calls, "halt"; then the chain must be drawn taut and straight, and the follower having his end of the chain precisely at the starting-point, calls out, "down"; the leader then thrusts one of the iron marking-pins into the ground exactly at the end of the chain and calls out, "down," which is the signal to the follower to advance: proceeding as before until the second length of chain is measured, which is indicated by the follower coming to the pin set in the ground by the leader, when the follower cries, "halt," and after placing his end of the chain at the pin, the chain having been drawn taut and straight as before, calls, "down"; the leader, as before, leaving a pin to mark the end of the chain, repeats, "down"; the follower then takes up the pin first placed by the leader, and moves on; thus the party proceeds until the end of the line is reached, the leader placing the pins at his end of the chain, and the follower picking them up at his end.

If the line ends with less than the length of the chain, the leader places his end at the point which marks the extremity of the line, calls out, "down"; the follower then reads off the number of links between the last pin and the end of the line. The number of whole chain's length of the line is shown by the pins in the hands of the follower, and the number of links counted off added thereto will give the total length in chains and links.

12. Tally. If the line exceeds eleven chains in length, a transfer of pins from the hind chainman to the head chainman is necessary; this is called *tallying*, and is performed in the following manner: At the end of the eleventh chain, the *brass*

pin — the last pin left in the hands of the leader — is placed, when he call out "tally"; at this signal the follower drops his end of the chain, advances to the leader, counts over with him the ten iron pins which he has gathered up, and transfers them to the leader, who then withdraws the brass pin, sets an iron one in its place, and the measuring is continued as before.* Each tally should be recorded, especially when chaining very long distances, to avoid error in the final count.† It is obvious that the total length of the line will be equal to the chains and links as indicated above, plus the number of tens shown by the tallies.

13. The surveyor should guard against error in chaining, by frequently testing his chain, to see that it is of the proper length, — if it has been stretched, make a file mark showing its true length, — and when in use, see that it is drawn straight, that the forward chainman sticks the pin in line *exactly* at the end of the chain, or at the mark indicating its true length, and as nearly vertical as possible; ‡ and when obtaining the number of links at the end of the line, see that they are not counted

* Some surveyors use only ten marking-pins, and tally by marking the end of the eleventh chain with a pencil, the finger, or a scratch on the ground, and when the ten pins are transferred to the leader, one of them is thrust in the place thus indicated, and the work is continued as before.

† In chaining long distances where there are several tallies, the leader and follower may, at each tally, change places, and thereby lessen the liability to error in the final count. See Articles 352, 353.

‡ "It has been found by many trials with as good men as can generally be obtained, that with two sets of chainmen instructed alike in the proper manner of keeping their chain level and straight on the line, and of setting the tally pins plumb, as well as holding the ends of the chain to them, a difference has sometimes been made of 36 links, and an average difference of 15 or 16 links to a mile in common timbered land." —*Burt*, "Government Surveying," p. 35.

The surveyor should have laid down by means of a standard steel tape or otherwise, in a convenient place, and between permanent marks in the ground or on the floor of a large hall, the exact length of a standard chain by which he could test his chain from time to time.

from the wrong end of the chain, nor the wrong way from the brass mark.

The pull on the chain, when in use, has a tendency to increase its length; and moreover, since there are a great number of wearing surfaces, if each of these be worn by an extremely small amount, the chain will be considerably elongated.

In either the surveyor's or engineer's chain there are two small links which connect with the two pieces of wire which form the principal part of what is called the *link* of the chain, thus giving six wearing surfaces to every link; therefore, if each of these surfaces wears only .005 of an inch, the chain will be increased in length *three inches*, so that in measuring only a quarter of a mile with a four-pole chain, the error from this cause alone would be *five feet*,* making an error in area of about 4.9 acres in a tract one mile square. This stretching of the chain is partially compensated by the difficulty, and often impracticability, of drawing the chain *precisely straight;* and so long as the chain is not elongated beyond one-tenth or one-twelfth of one per cent of its length, it may be relied on for accurate work.†

The true length of a line which has been measured by a chain stretched beyond the standard length may be found from the proportion:

The length of standard chain : the length of chain used
: : the distance measured : the true distance.

* This error, it is perceived, increases directly with the number of applications of the chain: it is called *cumulative*. The error arising from erroneous setting of the pin is termed *compensative*, that is, it is as likely to be additive as subtractive, and it is shown by the Method of Least Squares, that for this class of errors the square root of the number of errors are probably not compensated. If the error in setting is one inch, in chaining a mile with a Gunter's chain, the probable error would be $\sqrt{80}$ = about 9 inches.

† To remove the difficulty of drawing the chain perfectly straight, the instructions issued from the United States Land Office, 1880, to Government Surveyors-General, states that the 66 feet chain must be 66.06 feet. See p. 301.

For example, if, with a chain stretched one link over the standard, a line be measured for 2000 feet, we should have

$$100 : 101 = 2000 : 2020, \text{ the true distance.}$$

In like manner, for the area of a tract measured with a stretched chain:

The square of the length of the standard chain
: the square of the length of the chain used
: : the computed area
: the true area.

If the chain was stretched one link, as in the above example, and the area computed therefrom 20 acres, we should have

$$100^2 : 101^2 = 200 \text{ sq. chs.} : 204.02 \text{ sq. chs. for the true area}$$
$$= \tfrac{102}{100} \text{ of the computed area, nearly.}$$

In general, if A = true area, A_1 = computed area, L = length of chain, and dL = error in its length (always small). Then $A : A_1 = (L \pm dL)^2 : L^2$.

Reducing and rejecting d^2 as inconsiderable, there results $A = (1 \pm 2d)A_1$; or, the correction to be applied to obtain the true area $= 2dA_1$.

This correction is additive when the chain is too long, which is the usual case, and subtractive when the chain is too short.

14. The surfaces to be measured are in general uneven and broken, not plane; but however great the inequalities, the area of a tract is considered to be that part of the horizontal plane which is intercepted by vertical planes through its boundaries.* The horizontal distance is therefore required; hence, when the

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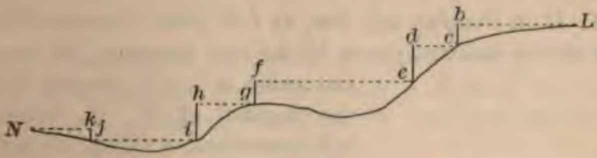
* A vertical line is a line directed to the centre of the earth, or it is a line having a plummet freely suspended to it, and at a state of rest; *a plumb line*.

A vertical plane is a plane embracing a vertical line.

A horizontal line is a line perpendicular to a vertical line.

A horizontal plane is a plane perpendicular to a vertical line.

ground slopes, it is necessary to raise the down-hill end of the chain. If the slope is considerable, only a part of the chain should be used. For example, to measure from *L* down to *N*, the follower holds one end of the chain at *L*, while the leader, stretching the other towards *N*, takes as much of it as he can raise to a horizontal position *b*, and, holding a plummet there, fixes the point *c*; the follower, who is now signalled to come forward, places at *c* that point in the chain whence the plummet was suspended to fix *c*, while the leader advances and, using as much of the chain as possible, locates *e*, and so on: when the end of the chain is reached, a pin should be transferred



from the leader to the follower. Where great accuracy is not required, a marking-pin or pebble may be dropped to indicate the points *c*, *e*, etc.* To measure up hill from *N* to *L* is less accurate, on account of the difficulty experienced by the follower in holding his end of the chain at the points *h*, *f*, *d*, etc., over their counterparts, *i*, *g*, *e*, etc.

When chaining steep hills, especially if through a wood or over rough, rocky ground, the work may be greatly facilitated by an extra chainman. He may assist in getting line, straightening the chain, noting the points *c*, *e*, etc., marked by the plumb bob, and other duties.†

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* If in connection with the chain a survey is being made with an instrument for measuring angles, — vertical and horizontal, — the inclination of a slope may be observed, and the length of it measured; then the horizontal distance required will be equal to the measured distance multiplied by the natural cosine of the angle of inclination.

† For extreme accuracy in measuring lines, see Chapter VII. Article 389.

EXERCISES.

1. Set two marks on gently undulating ground and about 1000 feet apart, and measure forward and back between these points several times; the same party once at least each way.

2. The same between points on hilly and, if possible, bush land.

3. Chain down a steep hill, and chain up between the same points.

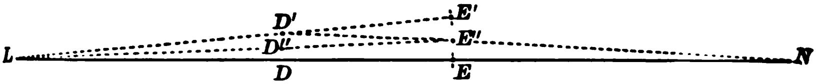
B. RANGING OUT LINES.

15. If in chaining any line, as *LN*, from *L* toward *N*, a rod at *N* can be constantly seen by the rear chainman, he can keep the leader in line by ranging him with the flagstaff at *N*. If, however, a hill intervenes, a valley, or brush or woodland interferring with the alignment, then the line must be first *ranged out* or points determined in it before the chaining can be performed.

L ———————————— N

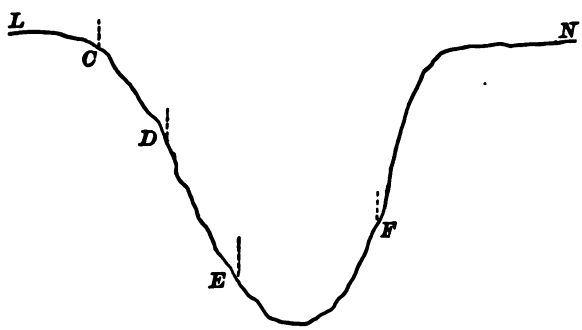
16. **Ranging out a Line.** To range out a line requires three persons, each having a rod eight or ten feet long, and a plummet to indicate when his rod is vertical. Calling these men *A*, *B*, and *C*, and supposing *A* and *B* in the line, *C* goes forward, and sighting back to *A* and *B*, puts his rod in line; *A* then advances beyond *C* and sets his rod in line with *C* and *B*; next *B* advances and places his rod in line with *C* and *A*, and so on the line may be extended any desired length. If, as frequently is the case, one of the party has had more experience or is naturally better qualified for *sighting a line*, the best results would be obtained by such an one setting all the rods; for example, *C* would place his rod in line, then call up *A*, to whom he would turn over the rod just set, and go forward to line the next; after which call up *B*, exchange rods with him, and so on.

17. Over a Hill. To fix points in a line over a hill, both ends of which are visible from points near the summit, proceed as follows:



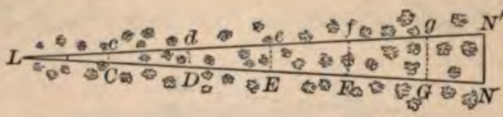
Place a flagstaff at L, another at N. A man at E' signals one at D' in line with L; D' then directs E' to E'' in line with N; and so on alternately, until the men are at D and E in the line LN.

18. Across a Valley. To locate points in a line, the ends of which may be seen from each other, but which are separated by a wide, deep valley.



Fix a point C in line with LN; then a man holding a plumb line at C, and sighting N can direct the setting of the stakes D, E, F, and others.

19. Through a Wood. In chaining through a wood or thick brush land, where the ends cannot be seen from each other, a line * is measured as nearly as may be in the direction of the desired line, and stakes driven every two or three chains, or oftener if necessary. When the end of the line is reached, the distance to the corner is measured, and, by proportion, the amount to move each stake to bring it into line is determined.



For example, let LN be the true line, and LN' the measured line; c, d, e, etc., points three chains apart. Now, if the length LN' equals 17.40 chains, and NN' measured at right angles to $LN' = 35$ links, LN † will equal

$$\sqrt{\overline{LN'}^2 + \overline{NN'}^2},$$

and

$$LN'(1740 \text{ links}) : NN'(35 \text{ links}) = Lg \ (1500 \text{ links}) : gG \ (30 \text{ links});$$

or 30 links from g at right angles to LN' will indicate the position of G, a point in the true line LN.

$$1740 : 35 = 1200 : 24, \text{ the distance } fF,$$
$$1740 : 35 = 900 : 18, \text{ the distance } eE;$$

and so on.

Or, after finding the first distance to set off, either gG or cC, the others are readily obtained by taking a proportional part of this distance, shown by the several divisions of the line thus: gG represents the fifth division, fF the fourth, eE the third, and so on; hence, if gG is 30 links, fF will be $\frac{4}{5}$ of 30, or 24,

* Called a *random line* or *trial line*.

† If the distance NN' is a small per cent of the total length of the line, the shortest distance between the ends of the lines may be taken for NN', and the length of the measured line for that of the true line. See Article 177.

links; eE, $\frac{3}{5}$ of 30, or 18; dD, $\frac{2}{5}$ of 30, or 12; and cC, $\frac{1}{5}$ of 30, or 6 links.

EXERCISES.

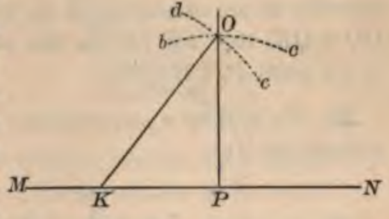
1. Let each student range out a line of several hundred feet, setting all the poles forward, and back again to the starting-point, and on different kinds of ground, undulating, hilly, and bushy.

2. Measure a line through a wood or where the ends are not visible from each other. Set stakes, as indicated in Article 19, in the true line 200 feet apart. See how near these stakes are placed in line by ranging.

C. SETTING OFF PERPENDICULARS.

20. *To erect a perpendicular at a given point in a line.*

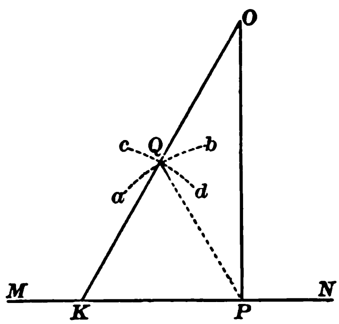
Let MN be the given line, and P the point at which it is desired to erect a perpendicular. Since a triangle formed of the sides 3, 4, and 5, or any multiple of these, will contain a right angle, we may take parts of a chain representing these distances or multiples, having the angle made by the shorter sides at P, and set off a perpendicular to a given line, thus: Fasten one end of the chain at K, 30 links from P, the end of the ninetieth link at P; then when both parts of the chain are drawn straight by a pull at the fiftieth link, the end of that link will indicate the point O which if connected with P will give the perpendicular required.



21. *If the perpendicular is to be of considerable length*, then a greater length than $PO = 40$ links should be used, and the following method would be better: Fasten one end of the chain at P, and with the eightieth link describe an arc bc; measure

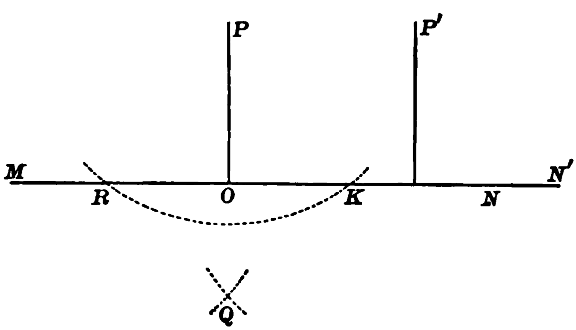
$PK = 60$ links, and with K as a centre, and with a radius $= 100$ links, the whole length of the chain, describe another arc de; the intersection of these arcs will give the point O required.

22. *Another Method.* With the whole length of the chain as a radius, and P as a centre, describe an arc ab; locate K a chain from P, and with the same radius, and with a centre K,



describe an arc cd cutting ab in Q; extend KQ to O, so that $OQ = QK$, then will OP be the perpendicular to the line MN at the point P. Why?

23. *To let drop a perpendicular on a line* from a given point without the li ve.



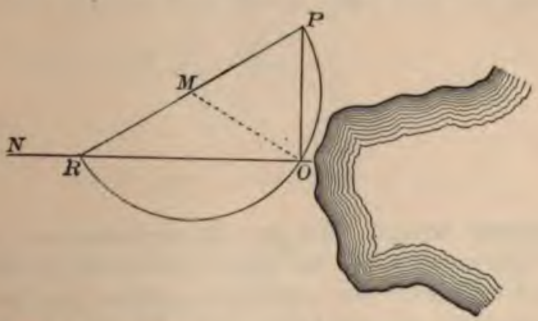
First, *When the point is accessible.*

Let MN represent the line, and P the point. With a length

of chain somewhat greater than PO, describe an arc cutting MN in the points R and K. With centres R and K, and any radius greater than the half of RK, describe arcs intersecting in Q. A line drawn from P to O in the direction of Q will be the perpendicular required.

If the point is at P' at or nearly opposite one end of the line, extend the line if it be possible to N' until a sufficient distance is obtained to describe the arcs required.

24. Or if it is impracticable to prolong the line, as in the figure, where a pond of water prevents, proceed as follows:



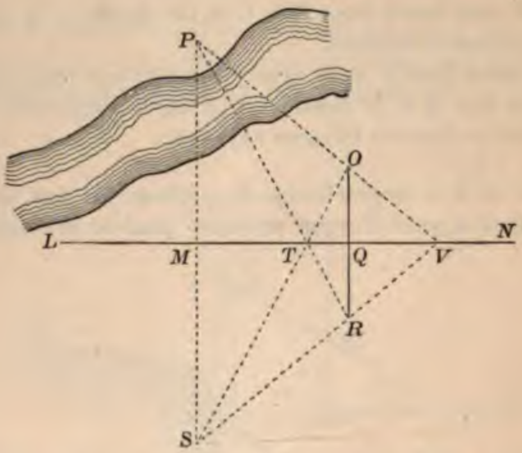
Extend the chain or any convenient portion of it from P to any point R in the line NO. Fix the middle point of RP, as M, and with this as a centre, and a radius MP, or its equal MR, describe an arc cutting the given line in O. Join PO for the perpendicular required.*

25. Second, *When the point is inaccessible.*

Let P be the given point, and LN the line. At any convenient point Q in the line LN erect the perpendiculars QO and QR of equal length. Locate V in the line PO and T in the line RP; then if a point S be found at the intersection of the

* The angle ROP is measured by one-half a semi-circumference, and is therefore a right angle.

prolongation VR and OT, and a point M be located in LN and SP, a line joining M and P will be the perpendicular sought. Why?



26. Optical Square. To set off perpendiculars from a line, an instrument called the *optical square* may be used. It is a small cylindrical box containing a mirror, from the upper half of which the silvering is removed. The glass is placed so as to make half a right angle with the line of sight, hence two objects seen in it, the one by direct vision, and the other by reflection, subtend at the point of observation a right angle.

Or the *surveyor's cross*, which is simply two pairs of sights set at right angles to each other, and supported upon a staff.*

D. RUNNING PARALLELS.

27. *Through a given point to run a parallel to a given line, the point and line both being accessible.*

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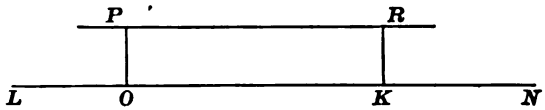
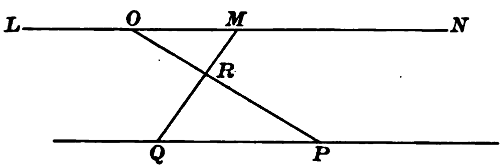
* While these instruments may be employed in chain surveying, neither of them is used in the ordinary practice of a surveyor, as perpendiculars are expeditiously set off by means of the compass or transit.

Let LN represent the line, and P the point. Let drop a perpendicular PO, and at some other point K; erect a perpen-



dicular $KR = PO$. A line drawn through P and R will be the parallel required.

28. *Otherwise.* From any point O in LN run an oblique line to the point P. Through any point R in PO measure a

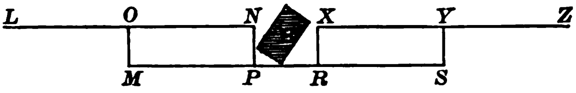


line MQ, so that $RQ = \frac{MR \cdot RP}{RO}$. A line passing through PQ will be the parallel required.

If R be taken at the middle point of OP, and QR be made equal to MR, the direction of the parallel PQ would be shown at once.

B. OBSTACLES TO ALIGNMENT.

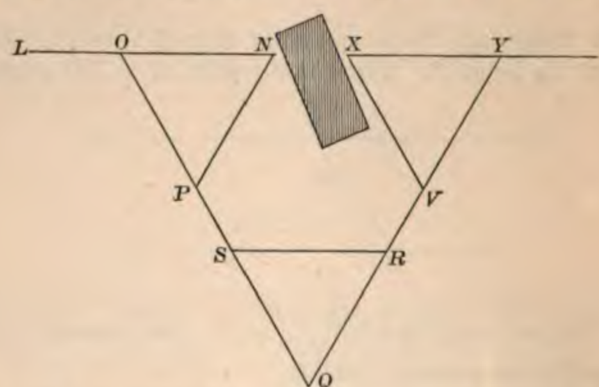
29. *To prolong a line when an obstacle, as a tree or building, prevents direct sighting*, we may proceed as follows:



By Perpendiculars. Let LN be the line which it is desired to prolong past a building B. At two points O and N in the

line, set off equal perpendiculars NP and OM, of such length that a line MP through these may be extended past the obstacle to some point S. At R and S set off perpendiculars to X and Y, of the same length as before, at O and N, and join XY; it will be the prolongation of LN.

30. *Otherwise: by Equilateral Triangles.* On LN, the line to be prolonged, take a distance ON as a base, and construct on it an equilateral triangle NOP; extend the side OP to some



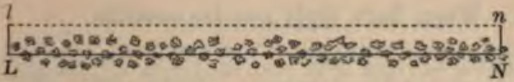
point Q. Describe an equilateral triangle QRS, and prolong the side QR to Y, making $QY = QO$; finally the construction of the equilateral triangle VXY will give XY the direction sought.

F. OBSTACLES TO MEASUREMENT.

31. *a.* **When Both Ends of the Line are Accessible.**

By Perpendiculars. For example, if it is desired to measure one side of a field or farm where a fence, hedge, or bushes prevent chaining on the line, set off perpendiculars, and measure the parallel line.

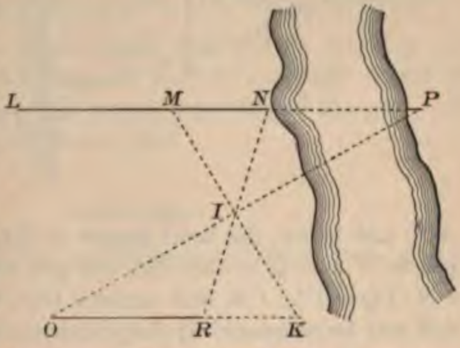
Let LN represent a line which, on account of fence and brush, it is impracticable to make the measurement exactly on the line.



Erect at L and N perpendiculars Ll and Nn, of equal and sufficient length so that a line connecting l and n will clear the obstruction. Measure ln; it will be the length of the required line.

32. *b.* **When One End is Inaccessible.**

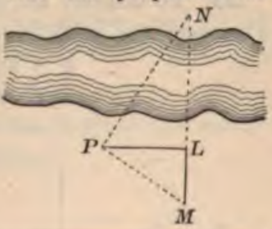
By Symmetrical Triangles. Suppose LP the line, P the inaccessible end, visible, but on the opposite bank of a river. Measure from any point N near the river, in a direction diverging from its bank to R, making $NI = IR$. Through any other point M, in the line LN, measure through I to K, so $MI = IK$. If now a point O be found in the prolongation of RK, and in line with I and P, RO may be measured and taken for their distance NP.*



33. *Otherwise.* Measure from the line the perpendicular LP; erect at P a perpendicular to PN, and extend it to a point M in the prolongation of LN. Measure LM; then the proportion

$$ML : LP = LP : LN$$

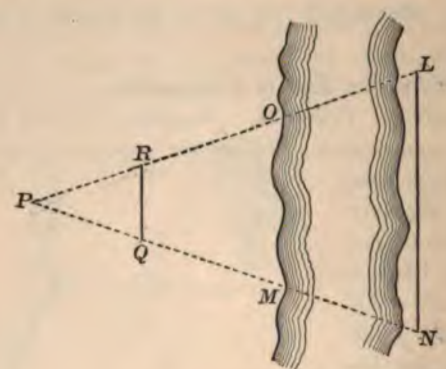
gives $$LN = \frac{PL^2}{ML}.$$



* The student will show that ROI and NIP are symmetrical triangles, and NP and RO are homologous.

34. *c.* **When Both Ends are Inaccessible.**

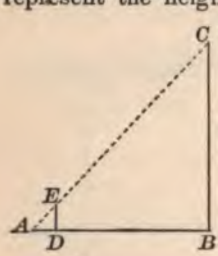
By Symmetrical Triangles. Let LN be the line, the length of which it is required to determine. Take any point P, measure PO and PM, and find by one of the preceding methods OL,



MN, and hence, the total length of PL and PN. Now take points R and Q in the lines PL and PN respectively, so that $PR : PQ = PL : PN$, and measure RQ; then the required line LN may be calculated by the proportion $PQ : PN = RQ : LN$.

G. MEASUREMENT OF HEIGHTS.

35. To measure the height of a tree or a flag-staff. Let BC represent the height required. At a point D set up a staff of a known height so that, with the eye at A, C and E will be in line of sight; measure AD and DB; then the similar triangles ADE and ABC give the proportion



$$AD : DE = AB : BC.$$

Whence $$BC = \frac{DE \times AB}{AD}.$$

EXAMPLES.

1. If the height of a staff is 4 feet, and the distance from it to a tree = 80 feet, AD being $4\frac{1}{3}$ feet, what is the height of the tree? *Ans.* $77\frac{1}{13}$ feet.

QUERIES. If the height of the staff is equal to AD, the length of neither being known, simply the distance AB given, could the height of the tree be ascertained?

If the ratio of the height of the staff to AD is known, but not the absolute length, could the required height be found by simply measuring AB?

Is this method applicable on other than horizontal ground?

2. A liberty pole, whose height was 90 feet, standing on a horizontal plane, was broken off, and the extremity of the top struck the ground 28 feet from the bottom of the pole. Required the length of the broken part.

EXERCISES.

1. Set a stake 40 feet perpendicularly distant from a given point in a given line.

2. Through a given point 50 feet from a given line run a parallel 120 feet in length.

3. Prolong a line beyond a house or other obstacle.

4. Measure the width of a stream or pond without crossing it.

5. Run a line to the bank of a stream or lake, and let fall a perpendicular on the line near its extremity from a given point without it.

6. Measure the height of a tree, flagstaff, or church spire.

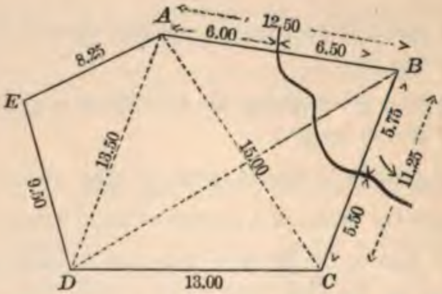
SECTION III.

RECORDING THE FIELD NOTES.

36. The Field Notes should be kept in a neat, concise, and intelligible manner, exhibiting a complete record of the work done, and the method of doing it, so that a surveyor unacquainted with the work, and having the record before him, could make a plot of the tract, or go on the field and readily ascertain the position of any point indicated in the notes.

Either of two methods may be employed, or a combination of them.

37. Sketch. One is to make a sketch of the tract as the survey progresses, writing the length of each line and indicating the intersection of fences, roads, streams, etc., as shown below.

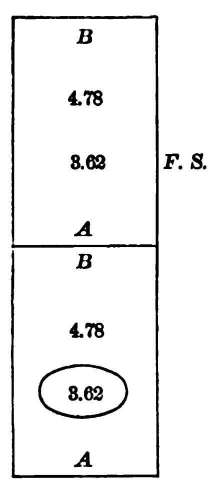


For surveying a field or small tract of land, this is a good method, but if the tract is large, many sided, and numerous points to be noted in and near the side-lines and diagonals, it would be difficult if not impossible to decipher the sketch on a page of the ordinary field-book, and to make an intelligible record of the work would require a book or sheet inconveniently large to carry about the field.

38. Columns. A method which will answer as well for complex as for simple surveys consists in drawing two parallel lines, about an inch apart, extending from top to bottom of the note-book, and near the middle of the left-hand page. Between the lines the distances and stations are to be recorded, commencing at the bottom of the page and proceeding upwards. Roads, fences, streams, etc., should be represented on either or both sides of the column as they naturally appear. The record of the measurements on any line being referred to the beginning of the line.

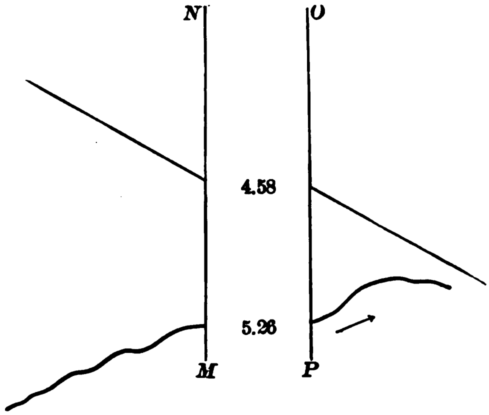
The right-hand page may be used for sketching any part of the survey to further elucidate, where necessary, the work done.

A station is indicated by a triangle (△) or a circle (⊙). If the station is at the end of a line it is usual to name it by the letter or number, designating that corner as station *A* or station 1, and the line extending from *A* to *B* is called the line *AB*, from 4 to 5, the line 4, 5 ; or a line may be designated by its length; a line that is 3 chains and 52 links long would be referred to as the line 352.

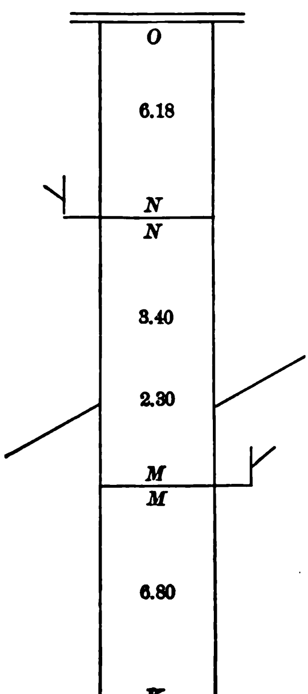


A false station is a point in a line whence other measurements are to be made either to the right or left, and are designated by enclosing in a curve its distance from the end of the measured line, or by writing *F. S.* opposite that distance, as per margin, which shows that there is a false station at a distance of 3.62 chains from *A* on the line *AB*.

A fence, brook, road, etc., intersecting the measured line, should be drawn so as to indicate, as nearly as possible, its inclination thereto, but not as a continuous line; the ends on each side being directly opposite, as at 4.58 and 5.26, so that if the vertical column



were to vanish by the two lines *MN* and *OP* coinciding, the fence or creek would be shown as continuous.

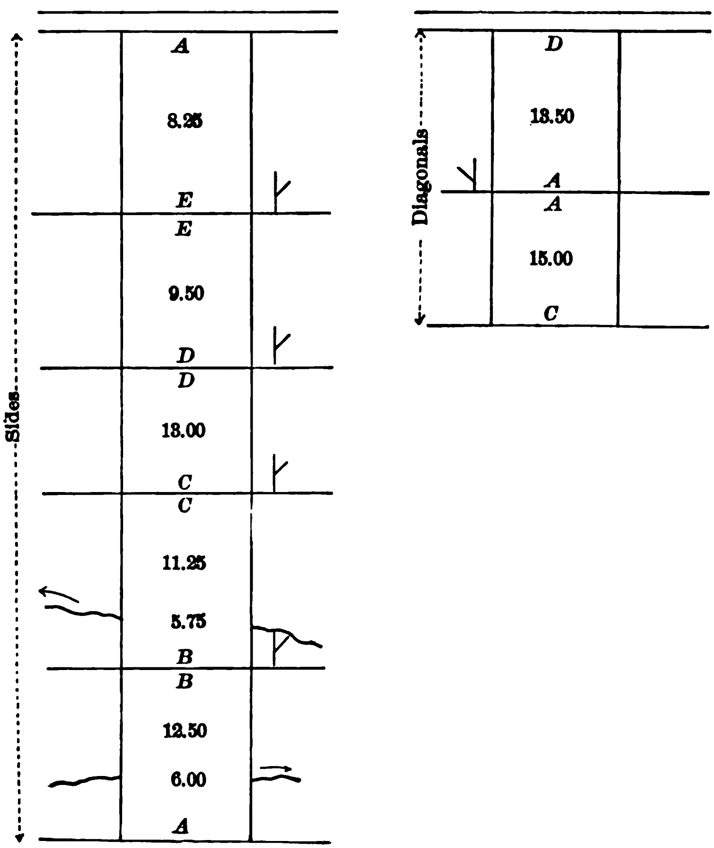


When the record of a line, as *MN*, is complete, and the measurement is continued from *N*, a horizontal line is drawn across the column as shown in the figure. But if the survey closes at the end of a line, as at *O*, or if for any reason the work is to proceed from some other point, *two* lines are drawn across the column.

A mark (ɼ) or (Γ) placed at the beginning of a line indicates by shape, as well as position, that the line along which it stands bears to the right of the preceding; the reverse position of the angle (ɺ or ˥) indicates a turn to the left.

In the figure, *MN* bears to the right of *KM*, and *NO* to the left of *MN*.

The record of the survey sketched in Article 37 would be represented by the column method as follows:



SECTION IV.

MAPPING AND PLOTTING.

39. **A Map** of a survey is a correct representation or copy of the tract surveyed, exhibiting not only its boundaries, roads, streams, etc., in relative dimensions and positions, but also the irregularities and appearances of its surface.

A Plot (or Plat) is an outline map, in which, in general, only the boundaries, roads, streams, and important lines are delineated, but no attempt is made to indicate the *topography* of the tract. The surveyor usually makes a *plot* of a field or farm survey. The civil engineer makes a map of a proposed railroad.

Instruments Useful for making a Plot of a Chain Survey.

40. Drawing-Board, T-Square, Triangles, Dividers, Scale, Drawing Pen and Pencil.*

A Drawing-Board is a rectangular, smooth board to which the paper that is to contain the drawing is fastened. There are two patterns: one consists of a frame of walnut, or other hard wood, with a detachable centre of soft white pine. The paper, which should be somewhat larger than the detachable centre, being moistened and laid on it, becomes well stretched when the parts of the board are buttoned together and the paper dries. The other is simply a rectangular white pine board made of several pieces of wood laid in different directions to prevent warping. Both patterns are made of various dimensions.

41. **A T-Square**, as its name indicates, is a *square* or ruler with a cross-piece or head at one end, giving it the appearance

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* Other instruments used in drawing are described in Chapter II. Section VIII.

of a letter T. There are two patterns of these, one with a head fixed at right angles to the ruler or *blade;* the other, in addition to the permanent head, has another head attached to it with a clamp screw, so that by properly setting the movable head, lines of any desired inclination may be drawn. The blade, being long and thin, should be tested occasionally by means of a metallic straight edge or another T-square to see whether or not it is perfectly straight. The correctness of the angles should also be tested; this may be done as indicated in the next article.

42. Triangles are made of hard wood, rubber, or metal, and are either solid or have an open centre. The angles are usually 30, 60, and 90 degrees, or 45, 45, and 90 degrees, and the longest side rarely exceeds 12 inches.

The T-square and triangles are frequently employed together to draw parallels, perpendiculars, and many of the oblique lines of a plot.*

The sides of triangles should be tested occasionally, to see if they are straight, by placing them against the edge of a metallic straight edge.

The right angle may be tested by placing one of its sides against a straight edge; mark the direction of the other side, reverse the triangle, but bring the same side against the straight edge, and having the right angle at the same point as before, mark the side again. If the two marks coincide, the angle is right; otherwise, it is not.

When correct, the right angle of the triangle may be used to test the correctness of the right angle of the T-square.

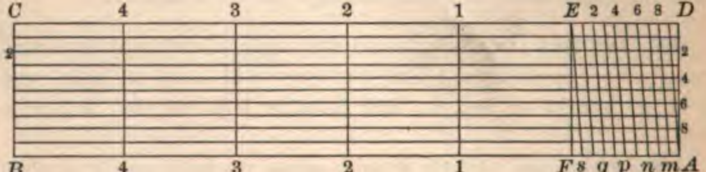
43. Dividers (or Compasses) are made of different sizes and numerous appendages. The surveyor will need at least one with a detachable leg, so that another leg, carrying a pen or

* The results are tolerably accurate within the limits usually required in a farm survey. It may be well, however, to caution the student not to rely too much upon the accuracy of a point located by means of and near the extremity of a thirty-inch T-square.

pencil point, may be inserted when necessary. These, it need hardly be said, are used for laying off lines, describing arcs, circles, etc.

44. Lead-Pencil. Fine quality, hard, used in outlining the work; and a *Drawing-Pen*, medium size, for inking in the drawing.

45. Scales are made of box-wood, metal, ivory, or paper, and are of various kinds. *Triangular* and *diagonal* are generally used for plotting chain surveys. *The triangular scale* for engineers and surveyors is usually 12 inches long, and made of good box-wood, each of the six bevelled faces being graduated with a single scale, viz.: one face contains 10 divisions to the inch, one 20, another 30, another 40, one 50, and one 60 divisions; and generally one inch on each face is subdivided so that an extremely small fraction of an inch may be set off or read. This is a very convenient scale; not only can very small divisions be readily transferred from it to a drawing, but by simply placing the instrument properly on a line of a drawing, the scale of which is known, its length may be directly determined.

The Diagonal Scale is usually six inches long, thin and flat, divided transversely into 6 equal parts of one inch each, and longitudinally into ten equal parts. At one end, as AD, one inch is divided by 10 oblique lines, as 8 m, 6 n, etc., into 10 equal parts and numbered as shown in the figure.

Now Fs being .1, the next division between the perpendicular FE and the oblique line sE is .09, the next .08, and the last

division, or one nearest F, is .01. Hence the scale may be used to measure .01 of an inch, or one hundredth of any division taken as the unit. For example, to lay off 3.4, place one foot of the dividers at 3 on the line EC and extend the other foot to 4 between DE. To lay off 3.42, place one foot at the intersection of 3, 3, and 2, 2, and the other on the same line 2, 2, at its intersection with 4 p.

The diagonal scale usually found with a box of drawing instruments contains various graduations. The simplest are divided to inches, and halves, quarters, tenths, and twelfths of an inch; each quarter and half subdivided diagonally into tenths, so that a tenth of a quarter can be taken off at once; and even tenths of these are indicated on the scale — besides other divisions of more or less utility.

Paper scales are frequently employed, and regarding hygrometric changes are better than the others, for the scale and the paper containing the drawing expand and contract more nearly alike. Generally, however, they are not divided with the same degree of accuracy.

46. Drawing to a Scale consists in drawing lines whose length shall be some fraction of the length of the line measured. Suppose, for example, a line is 13 chains long, and it is desired to draw it to a scale of 5 chains to an inch; then $2\frac{6}{10}$ inches will evidently be the distance to transfer from the scale to the paper to represent the length of the line.

A line 10 chains and 50 links in length drawn to a scale of 3 chains to an inch will be represented by a line $3\frac{1}{2}$ inches long, and so on. The length of the line divided by the number of units — chains, yards, feet, etc. — to the inch, always giving the distance to be taken off the scale. Obviously the converse of this is true; that is, the real length of a line may be ascertained when the scale is known, by multiplying the units in the length of the line in the drawing by the number of chains or feet which each unit represents. In the last example the length of the line being found $3\frac{1}{2}$ inches, and the scale 3 chains to an

inch, the true length $= 3.5 \times 3 = 10.50$ chains. The scale should always be given on the drawing. It may be stated thus: Scale, 3 chains to an inch, 1000 feet to an inch, 2 miles to an inch, or fractionally, and thereby indicating the relative length of the lines in the drawing to those which they represent; as, 1 : 500, 1 : 2000, etc.

47. Size of Drawing or Scale to Adopt. In farm surveys of small extent, 1 or 2 chains to an inch may be used; for medium tracts 3 chains to an inch (1 : 2376) is perhaps the best. The shape of the farm, the length of the shortest and longest sides, as well as the object of the drawing, will, however, influence the surveyor in his decision of the scale.

48. Scale Unknown. If the area of a tract of land is known but the scale not given, it may be found by measuring the lines of the drawing referred to any convenient scale and computing the area from these determined lengths. Then, since the areas of similar figures are to each other as the squares of their homologous sides, the true scale may be obtained by the proportion,

$$\frac{\text{computed area}}{\text{known area}} = \frac{\text{square of assumed scale}}{\text{square of true scale}}.*$$

SECTION V.

On Areas, and Illustrative Examples.

A. AREAS.

49. The following are geometrical truths with which the student is supposed to have an acquaintance, but are given here for convenience of reference.

* The protractor and other drawing-instruments used in connection with compass and transit surveying are described in Chapter II.

The Area of a Triangle is equal to one-half the product of its base and altitude.

In Terms of the Three Sides the area is equal to the square root of the continued product of one-half the sum of the sides, and the half-sum minus each side severally, or in symbols, where A = area, a, b, c, the three sides, and s their sum,

$$A=\sqrt{\tfrac{1}{2}s(\tfrac{1}{2}s-a)(\tfrac{1}{2}s-b)(\tfrac{1}{2}s-c)}.$$

If the triangle is equilateral and s = length of a side,

$$A=\frac{s^2}{4}\sqrt{3}.$$

50. The Area of a Rectangle is equal to the product of its length and breadth, or $A=bl$ where b = breadth and l = length.

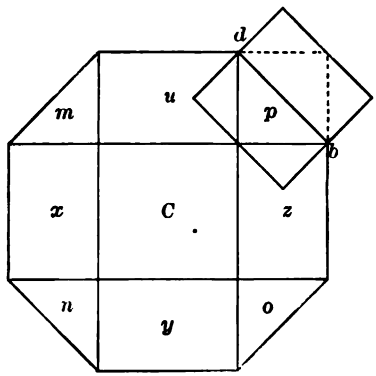
51. The Area of a Parallelogram is equal to the product of its base and altitude, or $A=bh$ where b=breadth and h=height.

52. The Area of a Trapezoid is equal to the product of one-half the sum of its parallel sides and the perpendicular distance between them, or $A=\frac{p}{2}(m+n)$ where m and n are the parallel sides, and p the perpendicular distance between them.

53. The Area of a Regular Hexagon, where s denotes the length of one of its sides, is $A=\frac{3}{2}s^2\sqrt{3}$, or it is equal to six equal equilateral triangles, having for each side the length of one side of the hexagon.

54. The Area of a Regular Octagon, each of its sides being unity, may be calculated by the rules of geometry, thus: Let the figure represent the octagon. It is evident that the area of the central square = 1. The sum of the areas of the four triangles m, n, o, $p=1$, since their sum equals the square described on db.* Now, the dimensions of each of the four

* The square described on the diagonal of a square is double the given square.



remaining figures (rectangles) x, y, z, and u, are 1, and $\frac{1}{2}\sqrt{2}$; hence the sum of the areas of these four rectangles

$$= 4 \times \tfrac{1}{2}\sqrt{2} = 2\sqrt{2};$$

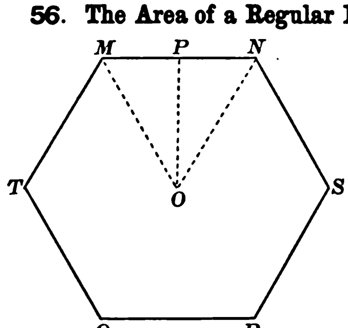
adding all the parts, there results

$$1 + 1 + 2\sqrt{2} = 2 + 2\sqrt{2}$$

for the area of the octagon.

55. The Area of a Regular Polygon in terms of the *perimeter* and *apothem*, or radius of inscribed circle, is equal to one-half the product of the perimeter and apothem, or $A = \frac{pr}{2}$; p denoting the perimeter, and r the radius of inscribed circle or apothem.

56. The Area of a Regular Polygon in terms of the *number* of *sides* and *length* of *one side* may be determined as follows: Let $r = OP$ be the radius of the inscribed circle or apothem, l the length of each side, and n the number of sides, A the area, as before; then



$$r = \frac{l}{2} \cot \frac{180^\circ}{n},$$

and $$A = \frac{nl}{2} \times \frac{l}{2} \cot \frac{180°}{n} = \frac{nl^2}{4} \cot \frac{180°}{n}.$$

If $l=1$, and $n=8$, the area of the polygon (octagon) becomes $2 \cot 22° \, 30' = 2 + 2\sqrt{2}$, as before found.

57. By the application of the formulas just found, the following table may be constructed, showing the apothems and areas of some of the regular polygons, each of whose sides is unity.

Names.	Sides.	Apothems.	Areas.
Triangle	3	0.2886732	0.4330127
Square	4	0.5000000	1.0000000
Pentagon	5	0.6881910	1.7204774
Hexagon	6	0.8660254	2.5980762
Heptagon . . .	7	1.0382601	3.6339124
Octagon	8	1.2071069	4.8284271
Nonagon	9	1.3737385	6.1818242
Decagon	10	1.5388418	7.6942088
Hendecagon . .	11	1.7028439	9.3656399
Dodecagon . . .	12	1.8660252	11.1961524

Now, since the areas of similar polygons are proportional to the squares on their homologous sides, this table may be used to find the area of any regular polygon named in the table, whatever may be the length of its side. Using the notation above, the principle just enunciated will be expressed as follows:

$$1^2 : \text{area in table} = l^2 : A, \text{ or } A = l^2 \times \text{area in table}.$$

That is, the area of a regular polygon is equal to the square of its side multiplied by the area of a similar polygon each of whose sides is 1.

Example. The area of a regular pentagon, each side being 30,
$$= 30^2 \times 1.7204774 = 1548.43.$$

58. **The Area of a Circle** is equal to π multiplied by the square of the radius, or one-half the product of the circumfer-

ence and radius. Let R represent the radius, C the circumference, and A the area; then

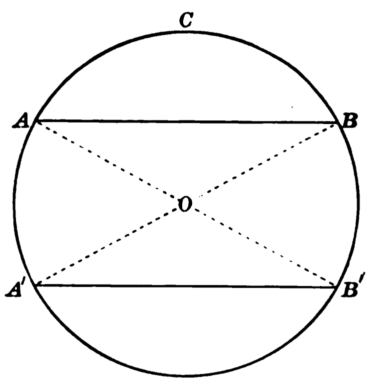
$$A = \pi R^2 = \frac{RC}{2}.$$

The area of a **Quadrant** $= \frac{\pi R^2}{4}.$

59. The Area of a Sextant $= \frac{\pi R^2}{6}$, and in general, the area of any *sector* of a circle $= \frac{n}{360} \times \pi R^2$, in which n denotes the number of degrees in the sector, or $A = \frac{Rl}{2}$, in which l denotes the length of the arc.

60. The Area of a Circular Ring is evidently the difference of the areas of the outer and inner circles; or, in symbols, if R and r equal the outer and inner radii, $A = \pi(R^2 - r^2)$.

61. The Area of a Segment of a circle, as ABC, is evidently equal to the area of the sector $AOBC$, minus the area of the



triangle AOB; or, in symbols, since the area of the triangle $= \frac{R^2 \sin n}{2}$, and the area of the sector as given above,

$$A = \frac{n\pi R^2}{360} - \frac{R^2 \sin n}{2}.$$

If n is greater than 180°, as in the segment $A'B'BCA$, $\sin n$ becomes negative, thereby making the second term of the right-hand member positive, as it should; since in this case the segment is greater than the sector, and the triangle $A'OB'$ is additive.

If the lengths of arc and chord are given, denote by $2c$ the length of chord, the other notation as above; then

$$A = \frac{Rl}{2} \mp c\sqrt{R^2 - c^2};$$

the *minus* sign to be used when the segment is less than a semicircle, and the *plus* sign when the segment is greater than a semicircle.

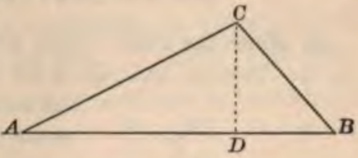
62. The Area of an Ellipse is equal to πAB, in which A and B denote the semi-axes.

B. ILLUSTRATIVE EXAMPLES.

EXHIBITING VARIOUS METHODS EMPLOYED TO SURVEY LAND, TO PLOT THE SURVEY, AND TO CALCULATE THE AREA.

TRIANGLES.

63. *First Method.* Measure the perpendicular CD, and the segments AD and DB, into which it divides the base; then



$$A = \frac{AB \times DC}{2}.$$

To Make the Plot. Draw AB according to any convenient scale, and locate D; with the same scale erect at D a perpendicular $= DC$. Join CA and CB, and the triangle ABC will result.

EXAMPLES.

1. Required the area and plot of a triangular field, the perpendicular of which measures 4.86 chains, and divides the side on which it falls into segments measuring 5.80 chains and 3.16 chains, or a total length of 8.96 chains.

Calculation. $A=\frac{8.96\times4.86}{2}=21.7728$ square chains. Dividing by 10, since there are 10 square chains in an acre, their results $\frac{21.7728}{10}=2.177+$ acres.* (The student will make the plot.)

QUERIES. Could a correct plot of the tract be made if there were given simply the base and altitude?

Would there be, usually, any choice of side to take as the base?

2. A triangular field measures 12.18 chains on one side, and the perpendicular erected at a point 5.10 chains from one end measures 7.54 chains. Calculate the area and make the plot.

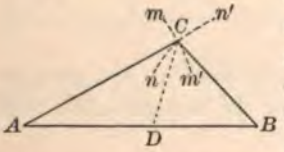
64. *Second Method.*† Measure all the sides, and calculate the area by the formula given above for that case.

EXAMPLES.

1. The lengths of the sides of a triangle are as follows: $AB=40$ chains, $AC=30$ chains, and $BC=20$ chains. Required the area and plot.

$$A=\sqrt{45\times5\times15\times25}=29.047 \text{ acres.}$$

To Make the Plot. Take 40 chains to any convenient scale in the dividers, and lay it off for the base AB; then, with A as a centre and 30 chains to the same scale in the dividers, describe an arc mm'; also, with B as a centre and 20 chains for radius, describe the arc nn'. The point C connected with A and B will give the triangle ABC required.



REMARK. It is customary when making a chain survey, to

* The area is usually expressed in acres and hundredths or thousandths of an acre.

† Other methods are given in Chapter II. Section IX.

measure a *proof* * line such as CD, and this should always be constructed to test the accuracy of the work.

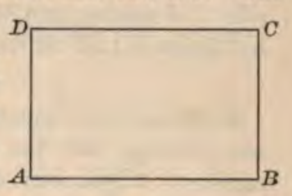
2. The three sides of a triangle measure 49, 50.25, and 25.69 chains. Find the area. *Ans.* 61.498 acres.

3. The sides of a triangular field are 24, 18, and 15 chains. A proof line, 12 chains in length, intersects the longest side or base at a point 10.25 chains from the angle formed by the two longest sides of the field. Required the area and plot. Test accuracy of latter by constructing proof line.

Rectangles.

65. Measure any two adjacent sides, as AB and BC. The area $= A = AB \cdot BC$.

To Plot. Lay off AB to any desired scale, and erect a perpendicular with the same scale at the extremities $= AD$ and BC; connect D and C, and the required figure will be formed.



EXAMPLES.

1. The length and breadth of a rectangle are 12.32 and 7.16 chains respectively. Required the area. *Ans.* 8.82 acres.

2. The length of a rectangle is 1250 feet, and its breadth 840 feet. What is its area? *Ans.* 24.1 acres.

3. A road running across a farm is $\frac{3}{8}$ of a mile long and 3 rods wide. How much land does it occupy? *Ans.* $2\frac{1}{4}$ acres.

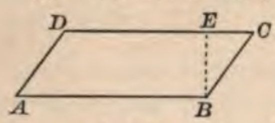
4. The length of a road on a hillside inclined to the horizon at an angle of $20°$ is 2310 feet, and its width $2\frac{3}{4}$ rods. At the rate of \$84 per acre, what must be paid to the owner across whose land the road runs? *Ans.* \$189.93.

---

* A line to check the measurement.

PARALLELOGRAMS.

66. Measure a side, as AB, the perpendicular distance, as BE, to the opposite side DC, and the distance CE. Then $A = AB \times BE$.



To Plot. Lay off the base AB, and at the extremity B erect a perpendicular equal BE. Through E draw DC equal to and parallel to AB, making EC its proper length. Join DA and CB, and the parallelogram $ABCD$ will be formed.

EXAMPLES.

1. The base of a parallelogram measures 10.54 chains. A perpendicular from one extremity of the base to the opposite side 5.16 chains, and the distance corresponding to EC in the last figure is 1.82 chains. Required the area and plot.

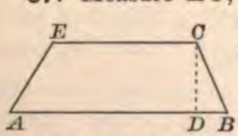
Ans. 5.439 acres.

2. A surveyor employed to determine the area of a rhombus, and knowing that the obtuse angles were double the acute, measured the shorter diagonal only, and found it 100 feet. Was the measurement sufficient? If so, give the area.

QUERIES. Can the area of a rhombus be ascertained if the lengths only of the two diagonals be given? If either diagonal and a side be given?

TRAPEZOIDS.

67. Measure EC, the perpendicular CD, and BA; note where the perpendicular CD meets the base AB.



$$A = \tfrac{1}{2}(AB + CE)\ CD.$$

To Plot. Lay off the base AB to the desired scale, and at D erect a perpendicular thereto equal to DC. Through C draw CE of the required length and parallel to AB. Join EA and CB. The figure resulting will be the trapezoid required.

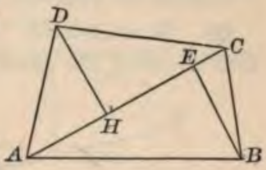
EXAMPLES.

1. The base of a trapezoid measures 12.62 chains, the parallel side 8.14 chains, and the perpendicular 7.44 chains. The distance corresponding to DB in the last figure is 1.12 chains. Required the area and plot. Area = 7.723 acres.

2. A railroad embankment extends 3240 feet perpendicularly across a farm intersecting parallel sides. At one end its base is 96 feet wide, and at the other 60 feet. Supposing the property line is 10 feet from the embankment on each side, how much of the farm is taken for railroad purposes?

TRAPEZIUMS.

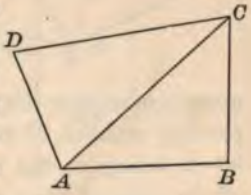
68. *First Method.* Measure either diagonal, and the perpendiculars thereto from the opposite angles, noting the distances AH and EC.



$$A = \tfrac{1}{2} AC(DH + EB)$$

To Plot. Draw the diagonal AC to the desired scale, and fix the points H and E. At these points erect perpendiculars corresponding to the scale and measurements. Joining DA and DC, and BA and BC, will complete the plot required.

Second Method. Measure all the sides and a diagonal as shown in the figure, thereby dividing the trapezium into two triangles, all the sides of which are known; whence the area may be computed by the formula for the area of a triangle in terms of the three sides.



To Plot. Lay off the diagonal AC, and locate the points B and D by methods heretofore given. Connect the points $ABCDA$ for the plot required.

EXAMPLES.

1. The diagonal of a trapezium measures 120 rods, and the two perpendiculars 30 and 40 rods; what is the area?

Ans. $26\frac{1}{4}$ acres.

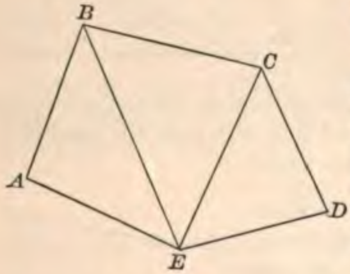
2. The sides of a trapezium taken in regular order are $AB = 5$, $BC = 9$, $CD = 11$, and $DA = 13$ chains, and the diagonal $AC = 12$ chains. Required the area and plot.

3. The sides of a trapezium are 18.10, 22.14, 28.16, and 34.62 chains, and the diagonal from the first to the third corner is 30.76 chains. Determine the area.

POLYGONS.

Regular or irregular, five or more sides.

69. *First Method.* Measure all the sides and the diagonals, thus dividing the tract into three or more triangles. The area will equal the sum of the areas of the triangles thus formed.

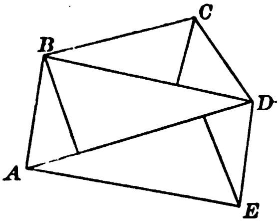


To Plot. Draw a line representing the diagonal BE, and construct the triangle ABE on it; on the other side of BE construct BCE; if a pentagon, the plot will be completed by adding CDE.

If a hexagon, there must be measured another diagonal giving four triangles, and generally, for any number of sides n, there will be $n-3$ diagonals and $n-2$ triangles, the area of the tract being equal to the sum of the areas of the $n-2$ triangles.

If the tract be a regular polygon, the measurement of one side by the aid of the table in (57) will be sufficient to determine the area.

70. *Second Method.** Measure one or more diagonals, and perpendiculars from these to the oppòsite angles, or corners, thereby dividing the tract into right triangles, or right triangles and trapezoids. The sum of the areas of these figures will equal the area of the polygon.



EXAMPLES.

1. The sides of a pentagon taken in regular order are, 6.80, 4.20, 5.30, 8.90, and 9.62 chains. The diagonals from the fifth corner to the second and third are each 10 chains. Find the area,† and make a plot.

2. A side of a regular heptagon measures 4.25 chains. What is the area?

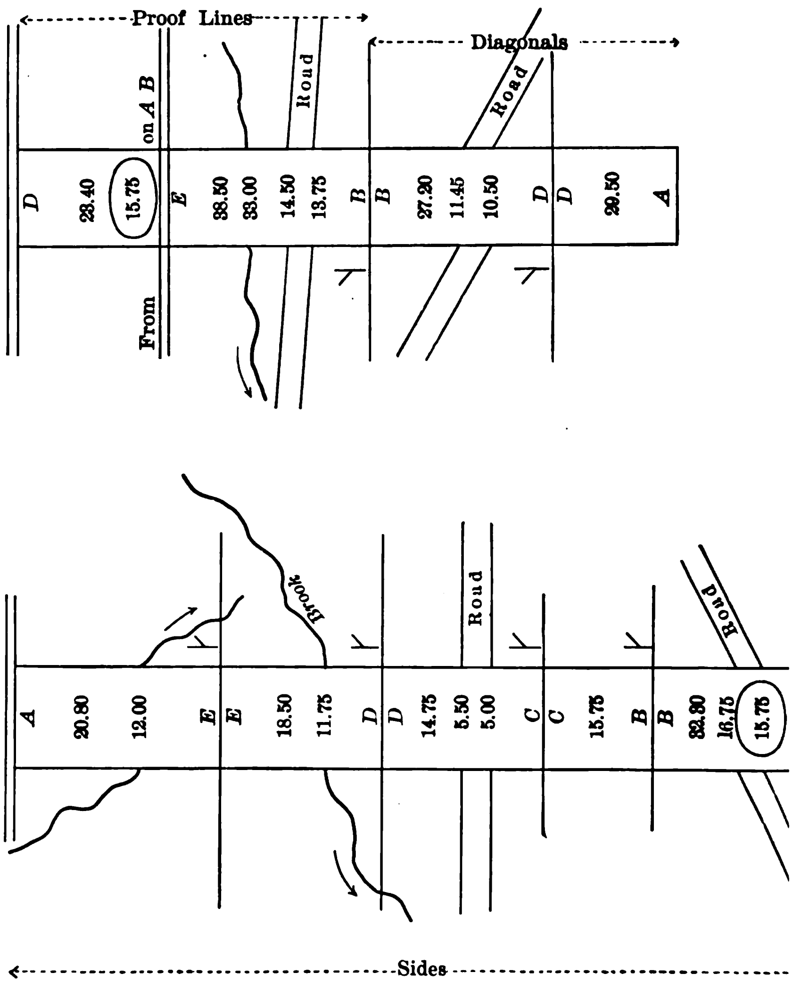
Given the following field notes to calculate the areas and make the plots. The distances are in chains.

	D	
	16.75	
C 4.50	13.50	
	12.90	4.50 *E*
	9.00	3.80 *F*
B 4.50	4.80	
	3.20	6.25 *G*
	A	

	D	
	12.50	
	7.80	4.60 *E*
C 2.80	5.90	
B 5.50	3.20	
	2.60	3.00 *F*
	A	

* Other methods are given in Chapter II.

† The work may be abridged by using logarithms.


Proof Lines
Diagonals
From on A B
D
28.40
15.75
E
38.50
33.00
14.50
13.75
B
B
27.20
11.45
10.50
D
D
29.50
A
Road
Road
A
20.80
12.00
E
E
18.50
11.75
D
D
14.75
5.50
5.00
C
C
15.75
B
B
32.30
16.75
15.75
Brook
Road
Road
Sides

Circles and Circular Rings.

71. Measure the radius or diameter of a circle, and the radii or diameters of a circular ring.

$$\text{The area of the former} = \pi R^2 = \frac{\pi D^2}{4}.$$

$$\text{The area of the latter} \; = \pi(R^2 - r^2).$$

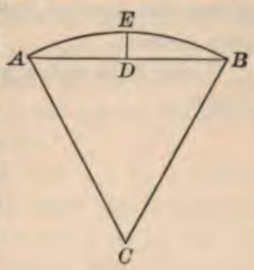
EXAMPLES.

1. The diameter of a circle is 10.16 chains. What is the area?

2. What is the area of a circular ring, the outer and inner radii measuring respectively 20 and 12 rods?

Sectors and Segments.

72. Measure the chord AB, and the perpendicular distance or height of arc DE, from the centre of AB to the arc AEB. From these data the radius and the angle at the centre may be found; and hence the area obtained. See (59) and (61). Otherwise, measure the radius BC, and by short chords the arc AEB; whence the area may be computed. (The student will supply the details for both cases.)



EXAMPLES.

1. If the length of the arc of a sector is 500 feet and the radius 1000 feet, how many acres does it contain? *Ans.* 5.739.

2. If the chord AB (last figure) $= 40$ feet, and the height of arc $DE = 10$ feet, what is the area of the segment?

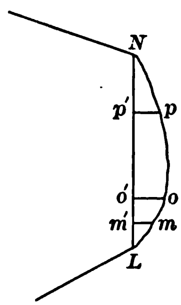
Ans. 279.558 square feet.

3. Given the radius, which is bisected by the chord, $= 100$ feet. Required the area of sector and segment.

SECTION VI.

OFFSETS AND TIE-LINES.

73. When any portion of the boundary of a tract of land is irregular, as, for example, when it is a stream or crooked road, the survey along such sides is best effected by measuring a straight line, as LN, and setting off short perpendiculars $m'm$, $o'o$, and $p'p$ at points m', o', and p', and extending them to the boundary line. Such short perpendiculars are called offsets, and they should be so chosen that the part of the curve Lm, mo, op, etc., intercepted between any two consecutive ones may be considered straight; whence the area of the part lying between the straight and curved lines may be obtained by adding together the area of the triangles and trapezoids into which it is thus divided.



If the field notes corresponding to the above figure are as below:

N	
9.60	0
7.00	1.30 *p*
2.50	1.40 *o*
1.20	1.00 *m*
L	0

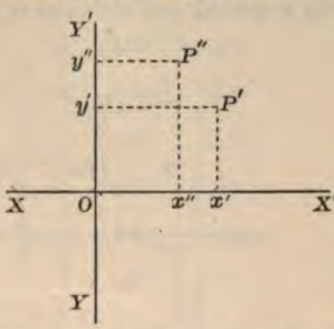
The area between straight line and boundary

$$
= \begin{cases} \text{Area triangle} & Lmm', & 6000 & \text{square links.} \\ \text{Area trapezoid} & mm'oo', & 15600 & \text{"} \quad \text{"} \\ \text{Area trapezoid} & oo'pp', & 60750 & \text{"} \quad \text{"} \\ \text{Area triangle} & p'pN, & \underline{16900} & \text{"} \quad \text{"} \end{cases}
$$

$$\text{Their sum} = 99250 \quad \text{"} \quad \text{"}$$

$$\text{or,} \quad .9925 \text{ of an acre.}$$

74. Rectangular Co-ordinates. Let XX' and YY' be two straight lines intersecting each other at right angles at O, and $P'P''$, points in their plane. Then if perpendiculars be drawn through these points to the lines XX' and YY', the distances cut off on the former are called *abscissas*, and those on the latter *ordinates*. The abscissa and ordinate referring to one point, as P', are termed the co-ordinates of that point.



The lines to which the measurements are referred are called the *axes; XX'* being called the axis of abscissas or axis of X, and YY' the axis of ordinates or axis of Y.

The axes being at right angles, the system is called the *rectangular system of co-ordinates*. O is the *origin*.* Designating the ordinates measured from the axis of X upward, and the abscissas measured to the right of the axis of Y, as *plus*, and those downward from the X-axis and to the left of the Y-axis, as *minus*, it is evident that a point can be located in either quadrant very readily by this method.

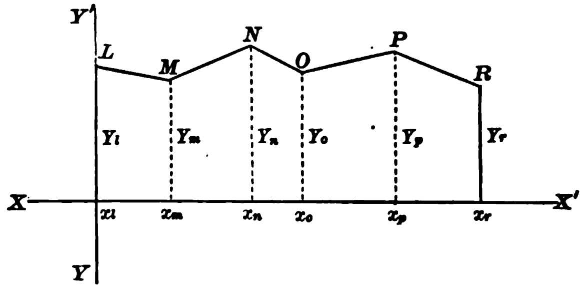
If the co-ordinates of P' are $x = 6$ and $y = 4$, it means simply that $Ox' = 6$ and $Oy' = 4$, and the point may be located by drawing the lines as indicated. If $x = -5$ and $y = 3$, the point is five units to the left of the Y-axis, and three units above the X-axis, etc.

75. *Application of Rectangular Co-ordinates to the Computation of Areas.*

Suppose it is required to find the area of any number of trapezoids formed by a broken line, and perpendiculars from its angles upon a straight line as indicated in the figure. XX', the

* Axes inclined to each other are called *oblique*.

straight line, may be taken as the axis of X, and YY' the axis of Y. Let x_l, x_m, x_n, etc., y_l, y_m, y_n, etc., denote respectively the abscissas and ordinates of the points L, M, N, etc.



The area required

$$= \tfrac{1}{2}[x_m(y_l + y_m) + (x_n - x_m)(y_m + y_n) + (x_o - x_n)(y_n + y_o) + (x_p - x_o)(y_o + y_p) + (x_r - x_p)(y_p + y_r)].^*$$

By expanding and simplifying there results

$$\tfrac{1}{2}[x_m(y_l - y_n) + x_n(y_m - y_o) + x_o(y_n - y_p) + x_p(y_o - y_r) + x_r(y_p + y_r)].$$

Whence for calculating the area of a tract of land included between a straight line and a broken line, whose angles are given by their co-ordinates upon the straight line as base, we have the following

RULE.

Multiply the difference between each ordinate and the second succeeding one by the abscissa of the intervening ordinate.

Multiply also the sum of the last two ordinates by the last abscissa.

The half of the algebraic sum of these several products will be the area.

EXAMPLES.

Calculate the areas, and make the plots from the following field notes; the distances are in chains.

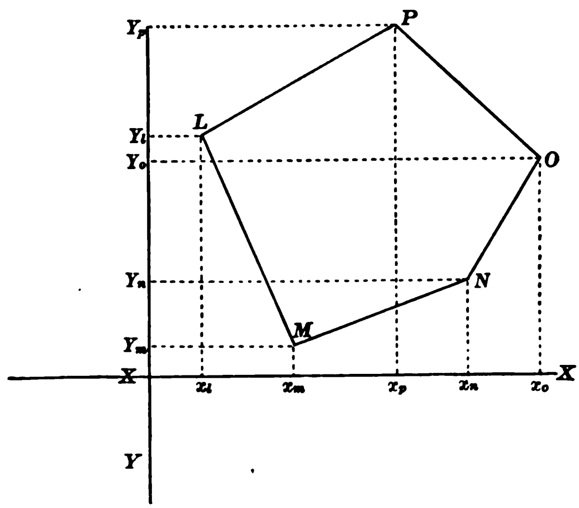
---

* A similar expression could evidently be found for any number of trapezoids.

1.20	.90
2.60	1.50
4.00	2.10
3.00	1.60
1.80	1.00
1.00	.60

2.20	1.60
4.30	2.00
5.00	2.40
3.20	1.50
2.50	1.00
1.70	.20
0	0

76. A slight modification of the rule just given will make it applicable to the case where a broken line encloses a tract or forms the boundary of a polygon.



Let the tract enclosed be represented by the figures, then the area

$$A = \tfrac{1}{2}\left[(y_n - y_m)(x_m + x_n) + (y_o - y_n)(x_n + x_o) \right.$$
$$+ (y_p - y_o)(x_o + x_p) - (y_p - y_l)(x_p + x_l)$$
$$\left. - (x_m + x_l)(y_l - y_m)\right].$$

By expanding, cancelling, and factoring, we may obtain either of the following expressions:

$$A = \tfrac{1}{2}\left[x_l(y_m - y_p) + x_m(y_n - y_l) + x_n(y_o - y_m) \right.$$
$$\left. + x_o(y_p - y_n) + x_p(y_l - y_o)\right]; \quad (1)$$

or, $$A = -\tfrac{1}{2}\left[y_l(x_m - x_p) + y_m(x_n - x_l) + y_n(x_o - x_m) \right.$$
$$\left. + y_o(x_p - x_n) + y_p(x_l - x_o)\right]. \quad (2)$$

Whence, for the area of a polygon whose corners are given by their co-ordinates, we have the following

RULE.

Take one-half the sum of the products of each $\left\{ \begin{matrix} \textit{abscissa} \\ \textit{ordinate} \end{matrix} \right\}$ *and the difference of its adjacent* $\left\{ \begin{matrix} \textit{ordinates} \\ \textit{abscissas} \end{matrix} \right\}$, *always making the subtraction in the same direction round the plot.**

EXAMPLES.

1. Given the abscissas of the several corners of a field, *L*, *M*, *N*, *O*, *P*, respectively:

2.00, 5.50, 12.00, 15.00, and 8.60 chains.

The corresponding ordinates:

10.20, 1.80, 4.00, 9.40, and 14.00 chains;

to compute the area.

---

* The work of computation may be abridged when the abscissas are greater than the ordinates, by making the differences of the abscissas the factors with the ordinates; and when the ordinates are greater than the abscissas, taking the differences of the ordinates with the abscissas. If the axis of ordinates pass through *L*, the abscissa of that point would vanish. Regard must, in all cases, be had to the resulting signs.

The form of reduction is as follows:

CORNERS.	ORDINATES.	ABSCISSAS.	DIFFERENCE BETWEEN ALTERNATE ABSCISSAS.	DOUBLE AREAS.
L	10.20	2.00	3.10	31.6200
M	1.80	5.50	− 10.00	− 18.0000
N	4.00	12.00	− 9.50	− 38.0000
O	9.40	15.00	3.40	31.9600
P	14.00	8.60	13.00	182.0000
				245.5800
				− 56.
				2)189.5800
				10)94.79 sq. chs.
				9.479 acres.

2. Given the abscissas of the several corners of a field, L, M, N, O, P, Q, R, respectively:

0, 6.50, 14.60, 22.80, 20.00, 16.70, 9.90;

and the corresponding ordinates:

13.20, 3.72, 4.40, 3.90, 17.24, 16.90, and 17.30,

all in chains; to determine the area and make a plot.

3. Given the abscissas of the several corners of a field, A, B, C, D, E, F, G, H, respectively:

100, 300, 360, 290, 400, 250, 120, 0;

and the corresponding ordinates:

0, 0, 160, 300, 380, 520, 520, and 330,

all in feet; to determine the area, and make a plot.

4. Verify Example 3 by a method independent of that given on the preceding page.

5. **Required the area and plot from the following field notes:**

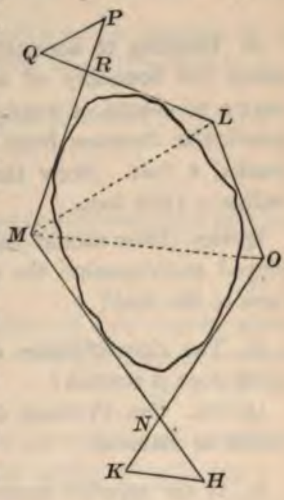
Sides		*A*	
		23.50	
		F	V
		F	
		21.90	
		E	V
		E	
		20.50	
		D	V
Offsets to river bank		*D*	
	0	17.40	
	1.00	15.50	
	1.60	13.00	
	1.75	8.50	
	1.00	6.00	
	1.20	3.50	
	0	*C*	V
		C	On river bank.
	0	18.00	
	2.00	13.50	
	3.50	10.00	
	2.50	5.50	
	0	*B*	V
		B	
		18.50	
		A	

	E		Proof line
	41.10		
Diag. *DF*	25.80		
Diag. *CF*	17.65		
	B		
	F		Diagonals
	30.10		
	D		
	C		
	28.90		
V	*F*		
	F		
	26.75		
	B		

Other examples containing offsets are given in Chapter II.

77. *To find the area of a tract of land when it is impossible to measure the diagonals or perpendiculars, as in the case of a lake or swamp.*

Measure MN and ON, and continue the measurements past their intersection at N, making NH some fractional part of MN, and NK the same part of ON.* Now because of the similarity of the triangles MNO and HKN, MO may be found by measuring a tie-line HK, and dividing it by the fraction used. Similarly, LM may be found. Then OL being measured, the area of the polygon $MNOLR$ can be computed. In case of a pond or lake, if offsets be taken from the sides of the polygon to the edge of the water, and the sum of the areas thus found included between the sides and the lake be deducted from the area of the polygon, the area of the body of water will be shown.



MISCELLANEOUS EXAMPLES.

1. One side of an equilateral triangle measures 18.24 chains. Required the area.

2. The perpendicular of an equilateral triangular piece of ground measures 160 feet. What is the area?

Ans. 14780.16 square feet. What part of an acre?

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* Great care should be exercised in the measurements, since the error is magnified in the computed lines. If the lines are so taken that KH is one-fourth of MO, an error of one link in measuring KH will make a difference of four links in MO.

For methods of performing such work more accurately, see Compass and Transit Surveying, Chapter II. Section IX.

3. It is known that the base of an isosceles triangle is $\frac{6}{5}$ the length of one of its equal sides. The perpendicular measures 80 feet. The sides and area are required.

Ans. Each side, 100 feet; base, 120 feet.
Area, 4800 square feet.

4. Desiring to ascertain the radius of a railroad curve (it being the boundary of a field), a surveyor measured from centre to centre of tracks, a chord of 200 feet; also the perpendicular distance from the centre of chord to the middle of tracks, 4 feet. Show that these measurements indicate the radius = 1252 feet.

QUERY. How should the data obtained in Example 4 be employed to determine the area, assuming that the curve is concave to the field?

5. The circumference of a circle is 100 rods. How many acres does it contain? *Ans.* 4.974.

QUERY. Can Problem 5 be solved without first finding the radius or diameter?

6. If the number expressing the area of an equilateral triangle in square feet is the same as that showing the length of one of its sides in lineal inches, what is its area?

Ans. 332.55.

7. The chord of a circle measures 60 feet, and the height of arc, or versed sine, 10 feet. Find in the same circle the versed sine of a chord of 90 feet. *Ans.* 28.2 feet.

8. The lengths of two chords lying on the same side of the diameter of a circle are 96 and 60, and their distance apart 26. Required the area between them.

SUGGESTION. Let x = perpendicular distance from centre of short chord to the nearest point of circumference, and y = perpendicular distance from centre of long chord to the farthest point of circumference; that is, measured in the opposite direction from the first.

Then
$$x(y+26) = 900.$$
$$y(x+26) = 2304.$$

Whence the diameter is readily determined and thence the area required.

9. Show that the area of the circumscribed hexagon is to the area of the circumscribed equilateral triangle as 2 is to 3.

10. Show that the area of a regular inscribed polygon of n sides $= \frac{n}{2} r^2 \sin \frac{360°}{n}$.

11. Show that the area of a regular circumscribed polygon of n sides $= nr^2 \tan \frac{180°}{n}$.

12. The distance between the centres of two circles, whose diameters are each 50, is equal to 30. What is the area common to the two circles? *Ans.* 559.15.

13. Three equal circles being tangent to each other externally enclose 40 rods. What is the radius of each circle? *Ans.* 15.75 rods.

EXERCISES.

1. Survey a polygon, measure all the sides and necessary diagonals, run test-lines, record the notes, make a plot, and compute the area.

2. Take the boundaries as found above, and complete the survey by measuring one diagonal and perpendicular offsets to the corners. Make record, plot, and computation.

3. Measure a field partly bounded by a creek or lake, rendering it necessary to take offsets thereto. Record the notes, plot, and calculate area.

4. Survey a pond or small lake by tie-lines and offsets.

CHAPTER II.

COMPASS AND TRANSIT SURVEYING.

SECTION I.

DEFINITIONS AND DESCRIPTION OF INSTRUMENTS.

78. The Axis of the earth is the imaginary line about which it rotates.

The Poles are the points where the axis pierces the earth: one the north pole, the other the south pole.

79. A Meridian Plane is a plane embracing the earth's axis.

80. A Meridian Line, or true meridian, is the intersection of a meridian plane with the surface of the earth.

In plane surveying the meridians passing through the extremities of lines surveyed are considered parallel.

81. The Magnetic Needle is a thin bar of strongly magnetized steel, balanced on a pivot, so that it may turn freely, and always come to rest in the direction of the magnetic meridian.

82. The Magnetic Meridian is indicated by the direction of a bar magnet, when horizontal, freely suspended and at rest. It does not in general coincide with the geographic meridian. The angle included between them is called the declination of the needle, or variation of the compass,* and the *change* in this angle is termed the *variation* of the declination.

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* See Chapter III., on Declination of the Needle.

83. The Azimuth of a Line is the angle which the vertical plane containing it makes with the plane of the meridian.

84. The Bearing of a Line, called also the course, is the angle which it forms with the direction of the magnetic needle.

85. The Meridian Distance of a Point is its perpendicular distance from an assumed meridian.

86. The Meridian Distance of a Line is the meridian distance of the middle point of that line.

87. A Horizontal Angle is an angle included between two lines in a horizontal plane.

A Vertical Angle is an angle included between two lines in a vertical plane.

88. An Angle of Elevation is a vertical angle, one side of which is horizontal, and the other inclined upward from the angular point.

89. An Angle of Depression is a vertical angle, one side of which is horizontal, and the other inclined downward from the angular point.

In Compass and Transit Surveying, in addition to the measurement of lines, angles are observed; hence, besides the instruments previously described, we present the following:

The Surveyor's Compass.

90. The Surveyor's* Compass consists essentially of a brass plate carrying a horizontal graduated circle, in the centre of which is suspended, so as to turn freely, a magnetic needle; and at the extremities of the plate are attached vertically two flattened pieces of brass, called sights, having fine slits and

* The Solar Compass is described in Chapter VI.

circular openings in them, by which the instrument is directed upon any object or station.

In addition to the essentials named, this instrument usually has two small spirit levels set on the plate at right angles to each other, a vernier scale for setting off the declination of the needle, a tangent scale for reading vertical angles, and a brass head for mounting the instrument upon a tripod or a single staff called Jacob's Staff.

91. The graduated circle is divided into half-degrees, and is figured from 0 to 90 on each side of the centre line of zeros.

The magnetic needle is from 4 to 6 inches long in the different sizes of compasses, having set in its centre a piece of hardened steel highly polished, which, resting upon the hardened point of the centre-pin, allows the needle to turn freely, horizontally, and to take its direction in the magnetic meridian.

92. The needle is lifted from its support by a concealed spring actuated by a screw. The test of the delicacy of a magnetic needle is the number of vibrations which it will make in a certain arc before coming to rest.

When the compass is not in use, the needle should be screwed up against the glass, and the instrument set so that the north end of the needle points towards the north.

To Adjust the Compass.

93. The Levels. First bring the bubbles into the centre, by the pressure of the hand on different parts of the plate, and then turn the compass half-way around; should the bubbles run to the end of the tubes, it would indicate that those ends were the highest: lower them by tightening the screws immediately under, and loosening those under the lowest ends until, by estimation, the error is half removed; level the plate again, and repeat the first operation until the bubbles will remain in the centre during an entire revolution of the compass.

SURVEYOR'S COMPASS.

94. The Sights may next be tested by observing through the slits a fine hair or thread, made exactly vertical by a plumb. Should the hair appear on one side of the slit, the sight must be adjusted by filing off its under surface on that side which seems the highest.

95. The Needle is adjusted in the following manner: Having the eye nearly in the same plane with the graduated rim of the compass-circle, with a small splinter of wood or a slender iron wire bring one end of the needle in line with any prominent division of the circle, as the zero or ninety-degree mark, and notice if the other end corresponds with the degree on the opposite side: if it does, the needle is said to "cut" opposite degrees; if not, bend the centre-pin by applying a small brass wrench, about one-eighth of an inch below the point of the pin, until the ends of the needle are brought into line with the opposite degrees.

Then, holding the needle in the same position, turn the compass half-way around, and note whether it now cuts opposite degrees; if not, correct half the error by bending the needle, and the remainder by bending the centre-pin.

The operation should be repeated until perfect reversion is secured in the first position.

This being obtained, it may be tried on another quarter of the circle; if any error is there manifested, the correction must be made in the centre-pin only, the needle being already straightened by the previous operation.

96. Electricity. A little caution is necessary in handling the compass, that the glass covering be not excited by the friction of cloth, silk, or the hand, so as to attract the needle to its under surface.

When, however, the glass becomes electric, the fluid may be removed by breathing upon it, or touching different parts of its surface with the moistened finger.

97. The Needle is remagnetized as follows:

The operator, being provided with an ordinary permanent magnet, and holding it before him, should pass with a gentle pressure each end of the needle from centre to extremity over the magnetic pole, describing before each pass a circle of about six inches radius, to which the surface of the pole is tangent, drawing the needle towards him, and taking care that the north and the south ends are applied to the *opposite* poles of the magnet.

Should the needle be returned in a path near the magnetic pole, the current induced by the contact of the needle and magnet, in the pass just described, would be reversed, and thus the magnetic virtue almost entirely neutralized at each operation.

When the needle has been passed about twenty-five times in succession, in the manner just described, it may be considered as fully charged.

A fine brass wire is wound in two or three coils on the south end of the needle, and may be moved back or forth in order to counterpoise the varying weight of the north end.

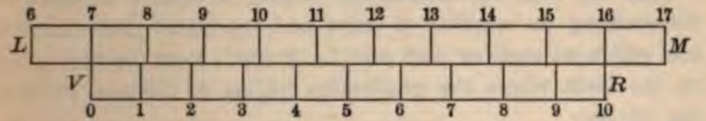
98. The Centre-Pin. This should occasionally be examined, and if much dulled, taken out with a brass wrench or with a pair of pliers, and sharpened on a hard oil-stone — the operator placing it in the end of a small stem of wood or a pin-vise, and delicately twirling it with the fingers as he moves it back and forth at an angle of about 30 degrees to the surface of the stone.

When the point is thus made so fine and sharp as to be invisible to the eye, it should be smoothed by rubbing it on the surface of a soft and clean piece of leather.

99. Weight. The average weights of the different sizes of compasses, including the brass head of the jacob-staff, beginning with the smallest, are respectively $5\frac{1}{2}$, $7\frac{1}{2}$, and $9\frac{1}{2}$ pounds.

The Vernier.

100. A Vernier is an auxiliary scale for measuring smaller divisions than those into which a graduated scale or limb is divided.* The smallest reading of the vernier, or *least count*, is the difference in length between one division on the graduated scale or limb, and one on the vernier. If the divisions on the vernier are smaller than those on the limb, the vernier is *direct;* if the reverse, *retrograde.*



Let LM represent any scale divided into tenths, and we wish to measure or read to tenths of these divisions, *i.e.* to $\frac{1}{100}$. Using a direct vernier, we should have 10 spaces on it equal to 9 on the scale, and each one of them equal to $\frac{9}{10}$ of $\frac{1}{10}$, or $\frac{9}{100}$, of the scale graduation; giving a *least count* of $\frac{10}{100}-\frac{9}{100}=\frac{1}{100}$, as desired. To read to twentieths of the divisions on the scale, we should have 20 divisions on the vernier corresponding to 19 on the scale, or each space on the vernier equal to $\frac{19}{20}\cdot\frac{1}{10}=\frac{19}{200}$, and giving a *least count* of $\frac{20}{200}-\frac{19}{200}=\frac{1}{200}$.

In general, if $s=$ the smallest division of the scale or limb,
$v=$ the smallest division of the vernier,
$n=$ number of divisions on the vernier,

we shall have least count $=s-v=\frac{s}{n}$.

Or, the *least count* of a vernier is equal to the smallest division of the scale or limb divided by the number of divisions on the vernier.†

If $s=\frac{1}{2}$ degree, and $n=30$, as ordinarily found on transit

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* It derives its name from Peter Vernier, 1631.

† It is evidently immaterial whether LM be straight or curved.

plates, the least count will be $\frac{1}{2} \div 30 = \frac{1}{60}$ of a degree = one minute.

If $s = \frac{1}{3}$ degree, and $n = 40$, oftentimes found on vertical arcs to solar attachments, the smallest reading $= \frac{1}{3} \div 40 = \frac{1}{120}$ of a degree $= \frac{1}{2}$ minute.

To space a vernier for a given least count, say 10″, on a limb graduated to 10′, we must have $n = \frac{s}{s-v} = \frac{10}{\frac{1}{6}} = 60$ spaces, covering 59 spaces on the limb.

101. To read an Instrument having a vernier consists in determining the number of units and fractional parts thereof, into which its scale or limb may be divided, from the zero point on the limb, where the graduation begins, to the zero point of the vernier.

It is accomplished as follows: Take the reading of the scale, as shown by the last graduation preceding the zero of the vernier; then find a line on the vernier which coincides with a line on the scale. The number of this line, as indicated by the graduation on the vernier, shows how many units of the *least count* are to be added to the first reading.

EXERCISES.

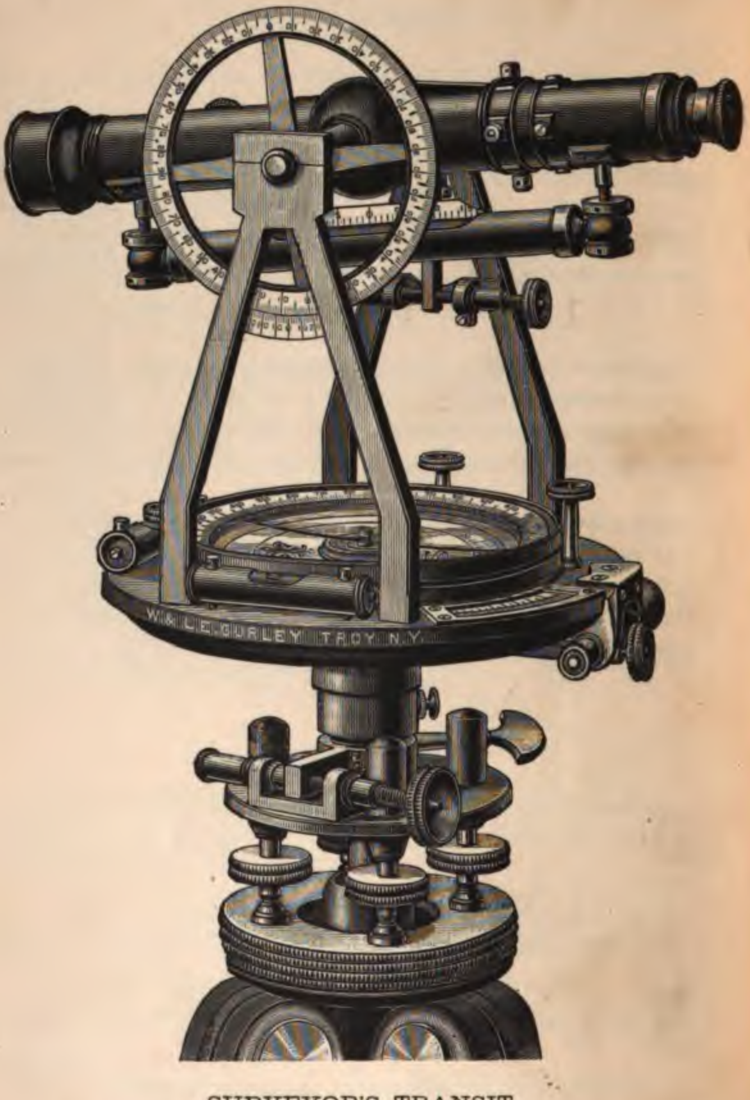
1. A levelling-rod is graduated into feet, tenths, and hundredths. It is required to space a direct vernier so that the rod may be read to thousandths of a foot.

2. An arc is graduated into quarter-degrees, and a vernier of 30 parts covers 29 parts of the arcs; find the least count.

3. A scale is divided into inches and tenths of an inch; plan a direct vernier by means of which the scale may be read to $\frac{1}{150}$ of an inch.

Plan a retrograde vernier to accomplish the same object.

4. Design a vernier which when applied to a limb graduated into 20′ will give a least count of 20″.

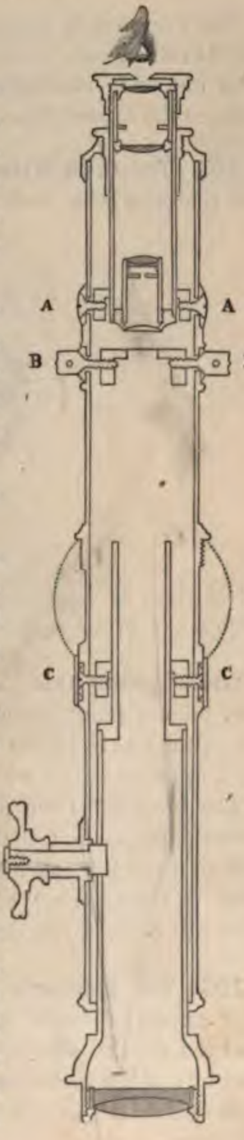


SURVEYOR'S TRANSIT.

NOTE. The principal part of the description of the Compass and Transit, and the plates for the engraving of these instruments, were kindly furnished by Messrs. W. & L. E. Gurley, Troy, N.Y.

SURVEYOR'S TRANSIT.

102. The essential parts of the Transit, as shown in the cut, are the *telescope* with its axis and two supports, the *circular plates* with their attachments, the *sockets* upon which the plates revolve, the *levelling-head*, and the *tripod* on which the whole instrument stands.



The *telescope* is from 10 to 11 inches long, firmly secured to an axis having its bearings nicely fitted in the standards, and thus enabling the telescope to be moved in either direction, or turned completely around if desired.

The different parts of the telescope are shown in the marginal figure.

The object-glass is composed of two lenses, so as to show objects without color or distortion, is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring *CC*, suspended within the tube by four screws, only two of which are shown in the cut.

The object-glass is carried out or in by a pinion working in a rack attached to the slide, and thus adjusted to objects either near or remote as desired.

The eye-piece is made up of four plano-convex lenses, which, beginning at the eye-end, are called respectively the eye, the field, the amplifying, and the object-lenses, the whole forming a compound microscope having its focus in the plane of the cross-wire ring *BB*.

The eye-piece is brought to its proper focus usually by twisting its milled end, the spiral movement within carrying the eye-tube out or in as desired; sometimes a pinion, like that which focuses the object-glass, is employed for the same purpose.

103. The Cross-Wires are two fibres of spider-web or very fine platinum wire, cemented into the cuts on the surface of a

metal ring, at right angles to each other, so as to divide the open space in the centre into quadrants.

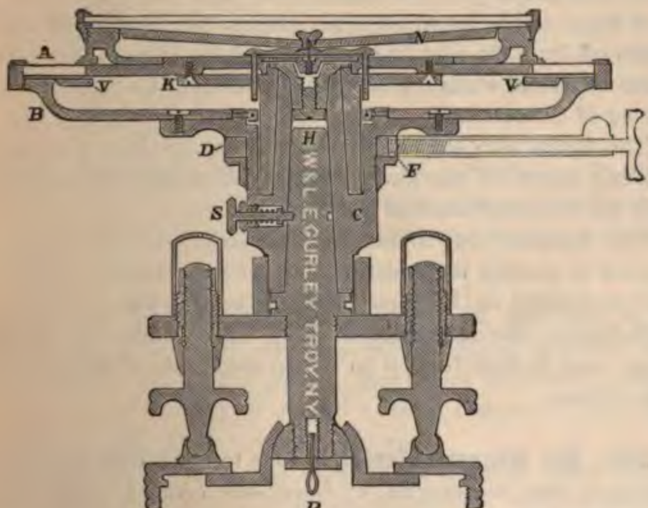
104. Optical Axis. The intersection of the wires forms a very minute point, which, when they are adjusted, determines the optical axis of the telescope, and enables the surveyor to fix it upon an object with the greatest precision.

The imaginary line passing through the optical axis of the telescope is termed the "line of collimation," and the operation of bringing the intersection of the wires into the optical axis, is called the "adjustment of the line of collimation." This will be hereafter described.

105. The Standards of the Transit are firmly attached by their expanded bases to the upper plate, one of them having near the top, as shown in the cut, a little movable box, actuated by a screw underneath, by which the telescope axis is made truly horizontal, as will be hereafter described.

The sectional view here given shows the interior construction of the sockets of the transit, the manner in which it is detached from the spindle, and the means by which it can be taken apart if desired.

In the figure, the limb *BB* is attached to the main socket *C*, which is itself carefully fitted to the conical spindle *H*, and held in place by the spring catch *S*.



The upper plate, *AA*, carrying the compass-circle, standards, etc., is fastened to the flanges of the socket *K*, which is fitted to the upper conical surface of the main socket *C*; the weight of all the parts being supported on the small bearings of the end of the socket, as shown, so as to turn with the least possible friction.

A small conical centre, in which from below is inserted a strong screw, is brought down firmly upon the upper end of the main socket *C*, and thus holds the two plates of the instrument securely together, while at the same time allowing them to move freely around each other in use.

A small disc above the conical centre contains the steel centre-pin upon which rests the needle, as shown; the disc is fastened to the upper plate by two small screws, as represented.

The main socket with all its parts is of the best bell-metal and is most carefully and thoroughly made, the long bearing of the sockets insuring their firm and easy movement, while at the same time they are entirely out of the reach of dust, or other source of wear.

When desired, the whole upper part of the instrument can be taken off from the spindle by pulling out the head of the spring catch at S, and when replaced will be secured by the self-acting spring of the catch.

The figure also shows the covers of the levelling-screws, the shifting centre of the lower levelling-plate, and the screw and loop for the attachment of the plummet.

The compass-box, containing the needle, etc., is covered by a glass to exclude the moisture and air; the circle is silvered, and is divided on its upper surface or rim into degrees and half-degrees, the degree marks being also cut down on its inner edge, and figured from 0 to 90 on each side of the centre or line of zero.

106. The Magnetic Needle is four to five inches long in the different sizes of transits, its brass cap having inserted in it a little socket or centre of hardened steel, perfectly polished, and this resting upon the hardened and polished point of the centre-pin, allows the needle to play freely in a horizontal direction, and thus take its direction in the magnetic meridian. The needle has its north end designated by a scallop or other mark, and on its south end a small coil of fine brass wire, easily moved, so as to bring both ends of the needle to the same level. The needle is lifted from the pin by a concealed spring underneath the upper plate, actuated by a screw shown above, thus raising the button so as to check the vibrations of the needle, or bring it up against the glass when not in use, to avoid the unnecessary wear of the pivot.

107. The Clamp and Tangent Movement, shown in the engraving, page 64, attached to the plates, serves to fasten the two plates together, so that by the tangent screw they can be slowly moved around each other in either direction, or loosened at will and moved by the hand, thus enabling one to direct the telescope rapidly and accurately to the point of sight.

The Two Levels are shown placed at right angles to each other so as to level the plate in all directions, and adjusted by turning the capstan-head screws at their ends, by a small steel adjusting-pin. The glass vials used in the levels are ground on their upper interior surface, so as to make the bubble move evenly and with great sensitiveness.

108. The Lower Plate, or **Limb BB,** is divided on its upper surface — usually into degrees and half-degrees — and generally figured in two rows; viz., from 0 to 360, and from 0 to 90 each way.

109. The Verniers are double, having on each side of the zero mark thirty equal divisions corresponding precisely with twenty-nine half-degrees of the limb; they thus read to single minutes, and the number passed over is counted in the same direction in which the vernier is moved.

The use of two opposite verniers in this and other instruments gives the means of "cross-questioning" the graduations, the perfection with which they are centred, and the dependence which can be placed upon the accuracy of the angles indicated.

Reflectors of silver or celluloid, as in the mountain transit, are often used to throw more light upon the divisions, and more rarely shades of ground glass are employed to give a clear but more subdued light.

110. The Graduations are made commonly on the brass surface of the limb, afterwards filled with black wax, and then finished and silvered. Many instruments, however, have a solid silver plate put over the brass, and the graduations made on the silver itself.

The last is more costly, but insures a finer graduation, with less liability to tarnish or change color.

111. The Sockets of the transit are compound; the interior spindle attached to the vernier plate, turning in the exterior socket C when an angle is taken on the limb; but when the plates are clamped together, the exterior socket itself, and with it the whole instrument, revolves in the socket of the levelling-head.

The sockets are made with the greatest care, ,the surfaces being truly concentric with each other, and the bell-metal or composition of which they are composed, of different degrees of hardness, so as to cause them to move upon each other easily and with the least possible wear.

The levelling-head also consists of two plates connected together by a socket, having at its end a hemispherical nut, fitting into a corresponding cavity in the lower plate.

The plates are inclined to each other or made parallel at will by four levelling-screws, of which only two are shown in the section.

The screws are of bronze or hard composition metal and fitted to long nuts of brass, screwed into the upper parallel plate; and, as will be noticed, have threads only on the upper ends, the lower part of their stems turning closely in the lower unthreaded part of the nuts.

By this arrangement dust is excluded from the lower end of the screws, while the brass cover above equally protects the other end.

The screws rest in little cups or sockets, which are secured to their ends and in which they turn without marring the surface of the lower plate, the cups also permitting the screws to be shifted from side to side, or turned around in either direction on the lower plate.

The clamp and tangent movement of the levelling-head serves to turn the whole instrument upon its sockets, so as to fix the telescope with precision upon any given point, and when un-

clamped allowing it to be directed approximately by hand. The tangent screws, as will be seen, press on opposite sides of the clamp-piece, and thus insure a very fine and solid movement of the instrument.

112. The Lower Levelling-Plate is made in two pieces — the upper one, which is screwed fast to the top of the tripod, having a large opening in its centre, in which the smaller lower one is shifted from side to side, or turned completely around.

By this simple arrangement, termed a *shifting centre*, the instrument is easily moved over the upper plate, and the plummet which hangs from the centre P, set precisely over a point, without moving the tripod.

113. The Levelling-Head of the engineer's transit is attached to the sockets by a screw and washer below; it can be removed for cleaning, oiling, etc., but should be in place when the instrument is in use, or packed for transportation.

114. The Tripod has three mahogany legs, the upper ends of which are pressed firmly on each side of a strong tenon on the solid bronze head by a bolt and nut on opposite sides of the leg; the nut can also be screwed up at will by a wrench furnished for the purpose, and thus kept firm.

The lower end of the leg has a brass shoe with iron point, securely fastened and riveted to the wood.

115. To Adjust the Transit. Every instrument should leave the hands of the maker in complete adjustment; but all are so liable to derangement by accident or careless use, that we deem it necessary to describe particularly those which are most likely to need attention.

The principal adjustments of the transit are:

1. *The Levels.*
2. *The Line of Collimation.*
3. *The Standards.*

116. To Adjust the Levels. ... strument upon its tripod as nearly level as may be, ... ving unclamped the plates, bring the two levels above a... on a line with the two pairs of levelling-screws; then, with the thumb and first finger of each hand clasp the heads of two, opposite; and, turning both thumbs in or out, as may be needed, bring the bubble of the level directly over the screws, exactly to the centre of the opening. Without moving the instrument, proceed in the same manner to bring the other bubble to its centre; after doing this, the level first corrected may be thrown a little out; bring it in again; and when both are in place, turn the instrument half-way around: if the bubbles both come to the centre, they would need no correction, but if not, with the adjusting-pin turn the small screws at the end of the levels until the bubbles are moved over half the error; then bring the bubbles again into the centre by the levelling-screws, and repeat the operation until the bubbles will remain in the centre during a complete revolution of the instrument, and the adjustment will be complete.

117. To Adjust the Line of Collimation. To make this adjustment, — which is, in other words, to bring the intersection of the wires into the optical axis of the telescope, so that the instrument, when placed in the middle of a straight line, will, by the revolution of the telescope, cut its extremities, — proceed as follows:

Set the instrument firmly on the ground and level it carefully; and then, having brought the wires into the focus of the eye-piece, adjust the object-glass on some well-defined point, as the edge of a chimney or other object, at a distance of from 200 to 500 feet; determine if the vertical wire is plumb, by clamping the instrument firmly and applying the wire to the vertical edge of a building, or observing if it will move parallel to a point taken a little to one side: should any deviation be manifested, loosen the cross-wire screws, and by the pressure of the hand on the head outside the tube, move the ring around until the error is corrected.

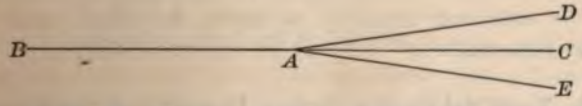
The wires being thus made respectively horizontal and vertical, fix their point of intersection on the object selected; clamp the instrument to the spindle, and having revolved the telescope, find or place some good object in the opposite direction, and at about the same distance from the instrument as the first object assumed.

Great care should always be taken in turning the telescope, that the position of the instrument upon the spindle is not in the slightest degree disturbed.

Now, having found or placed an object which the vertical wire bisects, unclamp the instrument, turn it half-way around, and direct the telescope to the first object selected; having bisected this with the wires, again clamp the instrument, revolve the telescope, and note if the vertical wire bisects the second object observed.

Should this happen, it will indicate that the wires are in adjustment, and the points bisected are with that of the centre of the instrument, in the same straight line.

If not, however, the space which separates the wires from the second point observed, will be double the deviation of that point from a true straight line, which may be conceived as drawn through the first point and the centre of the instrument, since the error is the result of two observations, made with the wires when they are out of the optical axis of the telescope.



For, as in the diagram, let A represent the centre of the instrument, and BC the imaginary straight line, upon the extremities of which the line of collimation is to be adjusted.

B represents the object first selected, and D the point which the wires bisected, when the telescope was made to revolve.

When the instrument is turned half around, and the telescope again directed to B, and once more revolved, the wires will

bisect an object E, situated as far to one side of the true line as the point D is on the other side.

The space DE, is therefore the sum of two deviations of the wires from a true straight line, and the error is made very apparent.

In order to correct it, use the two capstan-head screws on the sides of the telescope, these being the ones which affect the position of the vertical wire.

Remember that the eye-piece inverts the position of the wires, and therefore, that in loosening one of the screws and tightening the other on the opposite side, the operator must proceed as if to increase the error observed. Having in this manner moved back the vertical wire until, by estimation, one-quarter of the space DE has been passed over, return the instrument to the point B, revolve the telescope, and if the correction has been carefully made, the wires will now bisect a point C, situated midway between D and E, and in the prolongation of the imaginary line, passing through the point B and the centre of the instrument.

To ascertain if such is the case, turn the instrument half around, fix the telescope upon B, clamp to the spindle, and again revolve the telescope towards C. If the wires again bisect it, it will prove that they are in adjustment, and that the points B, A, C, all lie in the same straight line.

Should the vertical wire strike to one side of C, the error must be corrected precisely as above described, until it is entirely removed.

118. To Adjust the Standards. In order that the wires may trace a vertical line as the telescope is moved up or down, it is necessary that both the standards of the telescope should be of precisely the same height.

To ascertain this and make the correction if needed, proceed as follows:

Having the line of collimation previously adjusted, set up the instrument in a position where points of observation, such as

the point and base of a lofty spire, can be selected, giving a long range in a vertical direction.

Level the instrument, fix the wires on the top of the object, and clamp to the spindle; then bring the telescope down, until the wires bisect some good point, either found or marked at the base; turn the instrument half around, fix the wires on the lower point, clamp to the spindle, and raise the telescope to the highest object.

If the wires bisect it, the vertical adjustment is effected; if they are thrown to either side, this would prove that the standard opposite that side was the highest, the apparent error being double that actually due to this cause.

To correct it, one of the bearings of the axis is made movable, so that by turning a screw underneath this sliding piece, as well as the screws which hold on the cap of the standard, the adjustment is made with the utmost precision.

Other Adjustments of the Transit.

Besides the three adjustments already described — which are all that the surveyor will ordinarily have to make — there are those of the needle and the object-glass slide which may sometimes be required.

The first is given with the description of the compass; the last will now be described.

119. To Adjust the Object-Slide. Having set up and levelled the instrument, the line of collimation being also adjusted for objects from 300 to 500 feet distant, clamp the plates securely, and fix the vertical cross-wire upon an object as distant as may be distinctly seen; then, without disturbing the instrument, throw out the object-glass, so as to bring the vertical wire upon an object as near as the range of the telescope will allow. Having this clearly in mind, unclamp the limb, turn the instrument half-way around, reverse the eye-end of the telescope, clamp the limb, and with the tangent-screw bring the vertical

wire again upon the near object; then draw in the object-glass slide until the distant object first sighted upon is brought into distinct vision. If the vertical wire strikes the same line as at first, the slide is correct for both near and remote objects; and, being itself straight, for all distances.

But if there be an error, proceed as follows: first, with the thumb and forefinger twist off the thin brass tube that covers the screws *CC* shown in the sectional view of the telescope, p. 65. Next, with the screw-driver, turn the two screws *CC* on the opposite *sides* of the telescope, loosening one and tightening the other, so as apparently to increase the error, making, by estimation, one-half the correction required.

Then go over the usual adjustment of the line of collimation, and having it completed, repeat the operation above described; first sighting upon the distant object, then finding a near one in line, and then reversing, making correction, etc., until the adjustment is complete.

120. To Use the Transit. The instrument should be set up firmly, the tripod legs being pressed into the ground, so as to bring the plates as nearly level as convenient; the plates should then be carefully levelled and properly clamped, the zeros of the verniers and limb brought into line by the upper tangent-screw, and the telescope directed to the object by the tangent-screws of levelling-head.

The angles taken are then read off upon the limb, without subtracting from those given by the verniers, in any other position.

Before an observation is made with the telescope, the eyepiece should be moved in or out, until the wires appear distinct to the eye of the operator; the object-glass is then adjusted by turning the pinion-head until the object is seen clear and well-defined, and the wires appear as if fastened to its surface.

The intersection of the wires, being the means by which the optical axis of the telescope is defined, should be brought precisely upon the centre of the object to which the instrument is directed.

The needle is used, as in the compass, to give the bearing of lines, and as a rough check upon the angles obtained by the verniers and limb; but its employment is only subsidiary to the general purposes of the transit.

121. Attachments of Transits. The engraving of the Surveyor's Transit represents the attachments often applied to the Engineer's Transit, viz.: vertical circle, level on telescope, and clamp and tangent to telescope axis. They are of use where approximate levelling and vertical angles are to be taken in connection with the ordinary use of the transit, and with their adjustments, etc., will now be described.

122. The Vertical Circle firmly secured to the axis of the telescope is $4\frac{1}{2}$ inches in diameter, plated with silver, divided to half-degrees, and with its vernier enables the surveyor to obtain vertical angles to single minutes.

123. The Level on Telescope consists of a brass tube about $6\frac{1}{2}$ inches long, each end of which is held between two capstan-nuts connected with a screw or stem attached to the under side of the telescope tube.

124. The Clamp and Tangent consists of an arm at one end encircling the telescope axis, and at the other connected with the tangent-screw; the clamp is fastened at will to the axis by a clamp-screw, inserted at one side of the ring, and then by turning the tangent-screw the telescope is raised or lowered as desired.

125. To Adjust the Vertical Circle. Having the instrument firmly set up and carefully leveled, bring into line the zeros of the circle and vernier, and with the telescope find or place some well-defined point or line, from 200 to 300 feet distant, which is cut by the horizontal wire.

Turn the instrument half-way around, revolve the telescope, and fixing the wire upon the same point as before, note if the zeros are again in line.

If not, loosen the capstan-head screws, which fasten the vernier, and move the zero of the vernier over half the error;* bring the zeros again into coincidence, and proceed precisely as at first, until the error is entirely corrected, when the adjustment will be complete.

This method is not applicable when only an *arc* of a circle is attached. The adjustment may then be made as follows: Observe successively from each of the two points to the other, and as before use half the error in adjusting the vernier. Verify by repetition.

A slight error may be most readily removed by putting the zeros in line and then moving the wire itself over half the interval.

126. The Level is Adjusted by bringing the bubble carefully into the centre by the nuts at each end; and when there is a vertical circle on the instrument, this should be done when the zeros of circle and vernier are in line and in adjustment; when there is no vertical circle, proceed as follows:

127. To Adjust the Level on Telescope. Choose a piece of ground nearly level, and having set the instrument firmly, level the plates carefully, and bring the bubble of the telescope into the centre with the tangent-screw. Measure in any direction from the instrument, from 100 to 300 feet, and drive a stake, and on the stake set a staff, and note the height cut by the horizontal wire; then take the same distance from the instrument in an opposite direction, and drive another stake.

On that stake set the staff, and note the height cut by the wire when the telescope is turned in that direction.

The difference of the two observations is evidently the difference of level of the two stakes.

Set the instrument over the lowest stake, or that upon which

* Called *Index Error*. It may be rectified as here shown, or each observation corrected by this amount.

the greatest height was indicated, and bring the levels on the plates and telescope into adjustment as at first.

Then, with the staff, measure the perpendicular distance from the top of the stake to the centre of one of the horizontal cross-wire screw-heads; from that distance subtract the difference of level between the two stakes and mark the point on the staff thus found; place the staff on the other stake, and with the tangent-screw bring the horizontal wire to the mark just found, and the line will be level.

The telescope now being level, bring the bubble of the level into the centre, by turning the little nuts at the end of the tube, and noting again if the wires cut the point on the staff; screw up the nuts firmly and the adjustment will be completed.

128. To Take Apart the Surveyor's Transit. When it is necessary to separate the plates of the transit, proceed as follows:

(1) Remove the clamp-screw and take off the head of the pinion, both on the north end and outside the compass circle.

(2) Unscrew the bezel ring containing the glass cover of the compass, remove the needle and button beneath it, and take out the two small screws so as to remove the disc.

(3) Take the instrument from its spindle, and with a large screw-driver take out the screw from the underside of the conical centre (see figure, p. 67).

(4) Drive out the centre from below by a round piece of wood, holding the instrument vertical so that the centre will not bruise the circle.

(5) Set the instrument again upon its spindle, take out the clamp-screw to the tangent movement of the limb, and the work is complete. To put the transit together again, proceed exactly the reverse of the operation thus described.

129. The Solar Attachment is essentially the solar apparatus of Burt placed upon the cross-bar of the ordinary transit, the polar axis only being directed above instead of below, as in the solar compass.

A little circular disc of an inch and a half diameter, and having a short, round pivot projecting above its upper surface, is first securely screwed to the telescope axis.

Upon this pivot rests the enlarged base of the polar axis, which is also firmly connected with the disc by four capstan-head screws passing from the under side of the disc into the base already named.

These screws serve to adjust the polar axis, as will be explained hereafter.

130. The Hour Circle surrounding the base of the polar axis is easily movable about it, and can be fastened at any point desired by two flat-head screws above. It is divided to five minutes of time; is figured from I. to XII., and is read by a small index fixed to the declination circle, and moving with it.

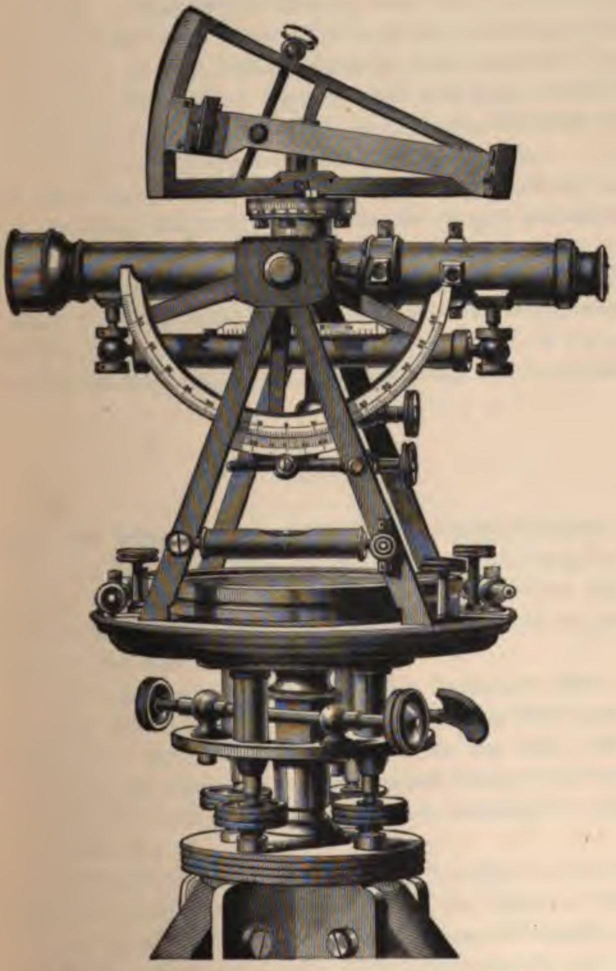
A hollow cone, or socket, fitting closely to the polar axis, and made to move snugly upon it, or clamped at any point desired by a milled-head screw on top, furnishes by its two expanded arms below a firm support for the declination arc, which is securely fastened to it by two large screws, as shown.

131. The Declination Arc is of about 5 inches radius, is divided to quarter degrees, and reads by its vernier to single minutes of arc, the divisions of both vernier and limb being in the same plane.

The declination arm has the usual lenses and silver plates on the two opposite blocks, made precisely like those of the ordinary solar compass, but its vernier is outside the block, and more easily read.

The declination arm has also a clamp and tangent movement, as shown in the cut. The arc of the declination limb is turned on its axis, and one of the other solar lens used, as the sun is north or south of the equator; the cut shows its position when it is north.

The Latitude is set off by means of a large vertical limb having a radius of 2½ inches; the arc is divided to twenty minutes,



TRANSIT WITH SOLAR ATTACHMENT.

is figured from the centre, each way, up to 80°, and is read by its vernier to single minutes.

It has also a clamp-screw inserted near its centre, by which it can be set fast to the telescope axis in any desired position.

The vernier of the vertical limb is made movable by the tangent-screw attached, so that its zero and that of the limb are readily made to coincide when, in adjusting the limb to the level of the telescope, the arc is clamped to the axis.

The usual tangent movement to the telescope axis serves, of course, to bring the vertical limb to the proper elevation, as hereafter described.

A level on the under side of the telescope, with ground vial and scale, is indispensable in the use of the solar attachment.

The divided arcs, verniers, and hour circle, are all on silver plate, and are thus easily read and preserved from tarnishing.

The Adjustments.

132. The Solar Lenses and Lines are adjusted precisely like those of the ordinary solar, the declination arm being first detached by removing the clamp and tangent screws, and the conical centre with its two small screws, by which the arm is attached to the arc.

The adjuster, which is a short bar furnished with every instrument, is then substituted for the declination arm, the conical centre screwed into its place at one end, and the clamp-screw into the other, being inserted through the hole left by the removal of the tangent-screw, thus securing the adjuster firmly to the arc.

The arm is then turned to the sun, as described in the article on the Solar Compass, and reversed by the opposite faces of the blocks upon the adjuster, until the image will remain in the centre of the equatorial lines. This adjustment is very rarely needed, as the lenses are cemented in their cells, and the plates securely fastened.

133. The Vernier of the Declination Arc is adjusted by setting the vernier at zero, and then raising or lowering the telescope by the tangent-screw, until the sun's image appears exactly between the equatorial lines.

Having the telescope axis clamped firmly, carefully revolve the arm until the image appears on the other plate.

If precisely between the lines, the adjustment is complete; if not, move the declination arm by its tangent-screw, until the image will come precisely between the lines on the two opposite plates; clamp the arm and remove the index error by loosening two flat-head screws on the back, which fasten the movable arc to the declination limb; place the zero of the limb and vernier in exact coincidence and the adjustment is finished.

134. To Adjust the Polar Axis. First level the instrument carefully by the long level of the telescope, using in the operation the tangent movement of the telescope axis in connection with the levelling screws of the parallel plates, until the bubble will remain in the centre during a complete revolution of the instrument upon its axis.

Place the equatorial sights on the top of the blocks as closely as is practicable with the distinct view of a distant object; and having previously set the declination arm at zero, sight through the interval between the equatorial sights and the blocks at some definite point or object, the declination arm being placed over either pair of the capstan-head screws on the under side of the disc.

Keeping the declination arm upon the object with one hand, with the other turn the instrument half around on its axis, and sight upon the same object as before. If the sight strikes either above or below, move the two capstan-head screws immediately under the arm, loosening one and tightening the other as may be needed, until half the error is removed.

Sight again and repeat the operation, if needed, until the sight will strike the same object in both positions of the instrument, when the adjustment of the axis in one direction will be complete.

Now turn the instrument at right angles, keeping the sight still upon the same object as before; if it strikes the same point when sighted through, the axis will be truly vertical in the second position of the instrument.

If not, bring the sight upon the same point by the other pair of capstan-head screws now under the declination arc, reverse as before, and continue the operation until the same object will keep in the sight in all positions, when the polar axis will be made precisely at right angles to the level and to the line of collimation of the transit.

It should here be noted that as this is by far the most delicate and important adjustment of the solar attachment, it should be made with the greatest care, the bubble kept perfectly in the centre and frequently inspected in the course of the operation.

135. To Adjust the Hour Arc. Whenever the instrument is set in the meridian, as will be hereafter described, the index of the hour arc should read apparent time.

If not, loosen the two flat-head screws on the top of the hour circle, and with the hand turn the circle around until it does, fasten the screws again, and the adjustment will be complete.

To obtain mean time, of course the correction of the equation for the given day, as given in the Nautical Almanac, must always be applied.

136. To Find the Latitude. First level the instrument very carefully, using, as before, the level of the telescope until the bubble will remain in the centre during a complete revolution of the instrument, the tangent movement of the telescope being used in connection with the levelling screws of the parallel plates, and the axis of the telescope firmly clamped.

Next clamp the vertical arc so that its zero and that of its vernier coincide as near as may be, and then bring them into exact line by the tangent-screw of the vernier.

Then, having the declination of the sun for 12 o'clock of the given day as affected by the meridional refraction carefully set

off upon the declination arc, note also the equation of time and fifteen or twenty minutes before noon, the telescope being directed to the north, and the object-end lowered until, by moving the instrument upon its spindle and the declination arc from side to side, the sun's image is brought nearly into position between the equatorial lines. Now bring the declination arc directly in line with the telescope, clamp the axis firmly, and with the tangent-screw bring the image precisely between the lines and keep it there with the tangent-screw, raising it as long as it runs below the lower equatorial line, or, in other words, as long as the sun continues to rise in the heavens.

When the sun reaches the meridian the image will remain stationary for an instant, and then begin to rise on the plate.

The moment the image ceases to run below is of course apparent noon, when the index of the hour arc should indicate XII, and the latitude be determined by the reading of the vertical arc.

It must be remembered, however, that the angle through which the polar axis has moved in the operation just described is measured from the zenith instead of the horizon, as in the ordinary solar, so that the angle read on the vertical limb is the complement of the latitude.

The latitude itself is readily found by subtracting this angle from 90°; thus at Troy, the reading of the limb being found as above directed to be 47° 16′, the latitude will be

$$90° - 47° 16' = 42° 44'.$$

It will be noticed that with this apparatus the latitude of any place can be most easily ascertained without any index error, as in the usual solar compass.

137. To Use the Solar Attachment. From the foregoing description it will be readily understood that good results cannot be obtained from the solar attachment unless the transit is of good construction,—furnished with the appliances of a level on telescope, clamp and tangent movement to axis, and vertical

arc with adjustable vernier, and the sockets or centres in such condition that the level of the telescope will remain in the centre when the instrument is revolved upon either socket.

138. To Run Lines with the Solar Attachment. Having set off the complement of the latitude of the place on the vertical arc, and the declination for the given day and hour as in the solar, the instrument being also carefully levelled by the telescope bubble, set the horizontal limb at zero, and clamp the plates together, loosen the lower clamp so that the transit moves easily upon its lower socket, set the instrument approximately north and south, the object-end of the telescope pointing to the north, turn the proper solar lens to the sun, and, with one hand on the plates and the other on the revolving arm, move them from side to side, until the sun's image is brought between the equatorial lines on the silver plate.

The lower clamp of the instrument should now be fastened, and any further lateral movement be made by the tangent-screw of the tripod. The necessary allowance being made for refraction, the telescope will be in the true meridian, and being unclamped, may be used like the sights of the ordinary solar compass, but with far greater accuracy and satisfaction in establishing meridian lines. Of course when the upper or vernier plate is unclamped from the limb, any angle read by the verniers is an angle from the meridian, and thus parallels of latitude or any other angles from the true meridian may be established as with the solar compass.

The bearing of the needle, when the telescope is on the meridian, will also give the variation of the needle at the point of observation.

The declination of the needle being set off, and the needle kept then at zero, or "with the sun," lines may be run by the needle alone when the sun is obscured.

Though when not inconsistent with the remarks following the table on page 95, the sun should be observed for direction at every station.

The Saegmuller Attachment.

139. As seen in the engraving on the opposite page, it consists essentially of a small telescope and level, the telescope being mounted in standards, in which it can be elevated or depressed. The standards revolve around an axis, called the polar axis, which is fastened to the telescope axis of the transit instrument. The telescope, called the "Solar Telescope," can thus be moved in altitude and azimuth. Two pointers, attached to the solar telescope to approximately set the instrument, are so adjusted that when the shadow of the one is thrown upon the other the sun will appear in the field of view.

140. Adjustments. When the apparatus is attached to the transit, which instrument must be in good adjustment, its polar axis *should be at right angles both to the horizontal axis of the main telescope and to the line of collimation.*

Test. Level the transit, and bring the bubble of each telescope to the centre of its run. Revolve the solar telescope about its polar axis, and if its bubble remains central, this adjustment is complete. If not, correct half the movement by the adjusting screws at the base of the polar axis, and the other by revolving the solar telescope on its horizontal axis.

141. Second. *The line of collimation of the solar telescope and the axis of its attached level must be parallel.*

Test. Bring the telescopes into the same vertical plane, and the large bubble to the middle of its run. Direct then the transit telescope to a mark at a convenient distance away, say 100 feet; point also the "solar" to a mark above this equal to the distance between their axes. If now the bubble of the solar telescope is not in the middle of the tube, make it so by the adjusting screws, and the instrument will be in adjustment.

When the combined instrument is in proper adjustment the bubbles of the telescopes and plates will be in the middle of their tubes, and the lines of collimation parallel.

TRANSIT WITH SOLAR ATTACHMENT,

AS MADE BY FAUTH & Co., WASHINGTON, D.C.

All the adjustments, including those of the transit, should be *frequently* examined, and kept as nearly perfect as possible.

142. The advantages of solar attachments over the ordinary solar compass consist principally in the telescopic sight, and the use of a vertical limb to set off declination and co-latitude.

LATITUDE.

By the Sun. — With Saegmuller's Attachment.

143. Level the transit carefully, point the telescope south, and elevate or depress the object-end, according as the declination of the sun is south or north, an amount equal to the declination.* Bring the solar telescope into the vertical plane of the main telescope, level it carefully, and clamp it. With the solar telescope observe the sun a few minutes before his culmination, bring the horizontal middle wire tangent to the upper limb by moving the transit telescope in altitude and azimuth, and keep it so by the slow-motion screws until the sun ceases to rise. Then take the reading of the vertical arc, correct for index error, if any, for refraction due to altitude,† as per table below; diminish the result by the sun's semi-diameter, and subtract the result from 90° for the latitude.

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* For declination, consult a nautical almanac.

† Corrected for index error, the arc reading would be the sum of the co-latitude and refraction. The refraction being due to the meridian altitude of the sun, which altitude in the United States is equal to the algebraic sum of the declination and co-latitude.

TABLE OF MEAN REFRACTIONS OF CELESTIAL OBJECTS FOR TEMPERATURE 50°, AND BAROMETER 29.6 INCHES.

ALTITUDE.	REFRACTION.	ALTITUDE.	REFRACTION.
10°	5′ 15″	20°	2′ 35″
11	4 47	25	2 02
12	4 23	30	1 38
13	4 03	35	1 21
14	3 45	40	1 08
15	3 30	45	0 57
16	3 17	50	0 48
17	3 04	60	0 33
18	2 54	70	0 21
19	2 44	80	0 10

By interpolation, the refraction due to any altitude within the limits of the table may be found.

LATITUDE BY CIRCUMPOLAR STAR.

144. The arc measuring the angle of elevation of the pole at any station indicates the latitude of that station. If, then, the place of the pole were indicated by a heavenly body, its altitude measured and corrected for refraction would give at once the latitude.

There being no such body, a circumpolar star may be used. Take its altitude at either culmination, subtract refraction due to altitude, and the remainder, increased or diminished by the polar distance according as the lower or upper culmination was observed, will give the latitude.

Better, when practicable, to observe both culminations, correct for refraction, and take the arithmetical mean of the result. Still greater accuracy would be obtained by taking the mean of observations at upper and lower transit of several circumpolar stars.

If A and A' respectively denote the angles measuring, from the north, the altitudes of a circumpolar star at its upper and lower culminations, and r and r' the corresponding refractions, then,

$$\text{latitude} = \tfrac{1}{2}[A + A' - (r + r')].$$

To Find the Meridian and Declination of the Needle, using the Attachment.*

145. *First*. Take the declination of the sun as given in the Nautical Almanac for the given day, and correct it for refraction and hourly change. Incline the *transit telescope* until this amount is indicated by its vertical arc. If the declination of the sun is north, depress the object-end; if south, elevate it. Without disturbing the position of the transit telescope, bring the solar telescope into the same vertical plane, and make it horizontal by means of its level. The two telescopes will then form an angle which equals the amount of the declination, and the inclination of the solar telescope to its polar axis will be equal to the polar distance of the sun.

Second. Without disturbing the *relative* positions of the two telescopes, incline them and set the vernier to the co-latitude of the place.

By moving the transit and the solar attachment around their respective *vertical* axes, the image of the sun will be brought into the field of the solar telescope, and after accurately bisecting it the *transit telescope must be in the meridian, and the compass-needle indicates its deviation at that place.*

The vertical axis of the solar attachment will then point to the pole, the apparatus being in fact a small equatorial. Revolve the main telescope on its horizontal axis, and set a mark at a convenient distance,— 1000 feet if practicable.

Make a *reverse* observation as follows: Turn the transit 180° in azimuth, and set off the declination, elevating or depressing now the *eye*-end, according as the declination is south or north; bring the object-end of the solar telescope to point in the direction of the eye-end of that of the main instrument, and level it. Set the vertical arc to the co-latitude of the place, and complete the observation as before. Reverse the large telescope on its

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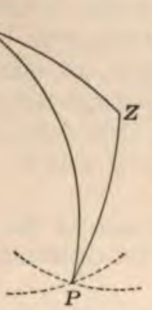
* For other methods, see Chapter III., p. 218, and Chapter VI., Solar Compass.

horizontal axis, and see if it points to the mark set by the *direct* observation; if it do not, take the mean of the two pointings for the meridian.

If greater accuracy is required, make other observations at different hours of the day, under different conditions of the atmosphere, and compare results with those given in Chapters III. and VI.



146. Time and azimuth are calculated from an observed altitude of the sun by solving the spherical triangle formed by the sun, the pole, and the zenith of the place. The three sides, SP, PZ, ZS, complements respectively of the declination, latitude, and altitude are given, and we hence deduce SPZ, the hour angle, from apparent noon, and PZS the azimuth of the sun.*

The "Solar Attachment" solves the same spherical triangle by construction, for the second process brings the vertical axis of the solar telescope to the required distance ZP from the zenith, while the first brings it to the required distance SP from the sun.

If the two telescopes, both being in position — one in the meridian, and the other pointing to the sun — are now turned on their *horizontal* axes, the vertical remaining undisturbed, until each is level, the angle between their directions — found by sighting on a distant object — is SPZ, the time from apparent noon.

This gives an easy observation for correction of time-piece.

147. An error either in the declination or latitude will cause an error in the azimuth.

These errors in azimuth corresponding to one-minute error in declination or latitude, for various hours and half-hours of the

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* A Table of Equation of Time is given at the end of this book which will be useful in solving analytically the spherical triangle PZS for time.

day, and for different latitudes, have been computed and tabulated.*

The following table exhibits these errors in latitude 40°.

For latitude 50° the errors are one-fifth greater, and for latitude 30° the errors are about one-ninth less than those given.†

By interpolation, those corresponding to other latitudes and fractional parts of the hour may be obtained.

TABLE OF ERRORS IN AZIMUTH FOR ONE MINUTE ERROR IN LATITUDE OR DECLINATION ON THE PARALLEL OF 40°.

Hours	11.30 A.M. 12.30 P.M.	11 A.M. 1 P.M.	10 A.M. 2 P.M.	9 A.M. 3 P.M.	8 A.M. 4 P.M.	7 A.M. 5 P.M.	6 A.M. 6 P.M.
For one min. error in dec.	10.00′	5.05′	2.61′	1.85′	1.51′	1.35′	1.30′
For one min. error in lat.	9.92′	4.87′	2.26′	1.30′	0.75′	0.35′	0.00′

The table indicates the best time to observe the sun for meridian, or to determine the true bearing of a line, to be soon after sunrise or just before sunset.

However, on account of refraction at these times being great and very uncertain, it is best in general not to make the observation when the sun is nearer the horizon than about 15 degrees. Moreover, the solar apparatus should not be relied on for very accurate work between 10 A.M. and 2 P.M.

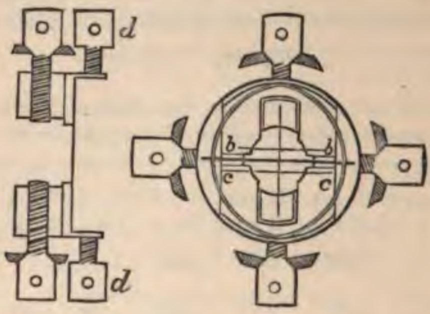
An error in latitude does not cause an error in azimuth when the sun is in the pole of the meridian.

148. The Stadia, or **Micrometer**, is a compound cross-wire ring or diaphragm, shown below, having three horizontal wires, of which the middle one is cemented to the ring as usual, while the others, *bb* and *cc*, are fastened to small slides, held apart by

* By Professor Johnson, C.E., Washington University, Mo.; and by R. T. Stewart, C.E., Instructor in Mathematics and Engineering, Western University of Pennsylvania.

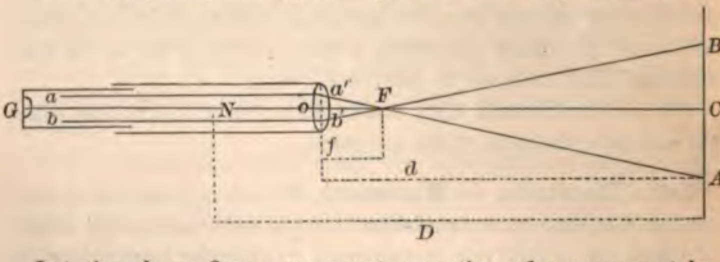
† More accurately, 11⅑ per cent.

a slender brass spring hoop, and actuated by independent screws *dd*, by which the distance between the two movable wires can be adjusted to include a given space; as, 1 foot on a rod 100 feet distant. These wires will in the same manner include 2 feet on



a rod 200 feet distant, or half a foot at a distance of 50 feet, and so on in the same proportion; thus furnishing a means of measuring distances — especially over broken ground — much more easily, and even more accurately, than with a tape or chain.

149. Its principles may be explained more fully as follows:



Let the above figure represent a section of a common telescope with but two lenses, between which the diaphragm with the stadia wires is placed, and assume that

f = the focal distance of the object-glass;
p = the distance of the stadia wires a and b from each other;
d = the horizontal distance of the object-glass to the stadia;

a = stadia reading (BA);

D = horizontal distance from middle of instrument to stadia.

The telescope is levelled and sighted to a levelling or stadia rod, which is held vertically, hence at right angles with the line of sight. According to a principle of optics, rays parallel to the axis of the lens meet, after being refracted, in the focus of the lens. Suppose the two stadia wires are the sources of those rays, we have, from the similarity of the two triangles $a'b'F$ and FAB, the proportion

$$d - f : a = f : p.$$

The quotient $f:p$ is, or at least can be made, constant, and may be designated by k; hence we may write

$$d - f = FC = ka.$$

To get the distance from the centre N of the instrument there must be added to FC the value

$$c = OF + ON.$$

ON is mostly equal to half the focal length of the object-glass; hence,

$$c = 1.5f.$$

Therefore the formula for the distance of the stadia from the centre of instrument, when that stadia is at right angles to the level line of sight, is

$$D = ka + c. \tag{1}$$

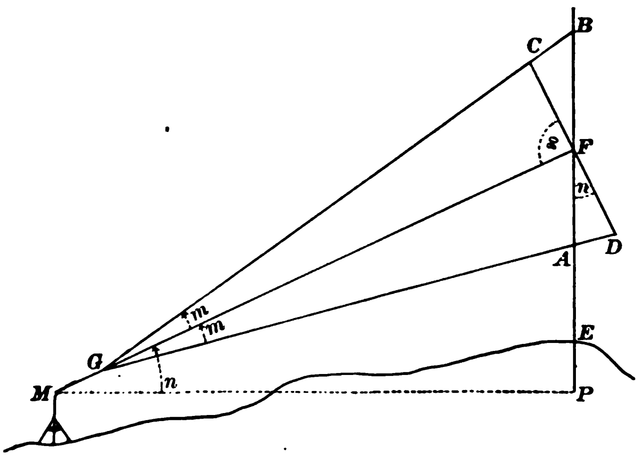
150. When the line of sight is not level, it is impracticable, especially in long distances, to hold the rod in a vertical plane, and at the same time perpendicular to the line of sight; hence it is customary to hold the rod vertical, as in the preceding case, and obtain the true distance by applying a correction depending upon the angle of inclination of the sight.

This correction is deduced as follows:

Let $AGB = 2m$;

n = the angle of inclination;

$MF = c + GF = c + k \times CD = D'$;
CD must be expressed by AB;
$MP =$ the horizontal distance $= D' \cos n = D$;
$AB = a$.



Now the angle

$$BAG = 90 + (n - m);$$
$$\therefore\ ABG = 90 - (n + m).$$

Hence, $$\frac{AF}{GF} = \frac{\sin m}{\sin[90 + (n - m)]};$$

or, $$AF = \frac{GF \sin m}{\cos(n - m)},$$

and $$\frac{BF}{GF} = \frac{\sin m}{\sin[90 - (n + m)]};$$

or, $$BF = \frac{GF \sin m}{\cos(n + m)}.$$

$$\therefore\ AF + BF = GF \sin m\left[\frac{1}{\cos(n - m)} + \frac{1}{\cos(n + m)}\right].$$

But $AF + BF = a$,

and $$GF = \frac{CD}{2 \tan m} = \frac{CD \cos m}{2 \sin m}.$$

Substituting this value of GF in the equation above, we obtain

$$a = \frac{CD \cos m \sin m \cos(n+m) + \cos(n-m)}{2 \sin m \cos(n+m) \cos(n-m)};$$

or, $$CD = a\,\frac{\cos^2 n \cos^2 m - \sin^2 n \sin^2 m}{\cos n \cos^2 m},$$

and $$D' = c + ka\,\frac{\cos^2 n \cos^2 m - \sin^2 n \sin^2 m}{\cos n \cos^2 m}.$$

Whence,

$$D = c \cos n + ka \cos^2 n - ka \sin^2 n \tan^2 m.$$

The third member of this equation may be safely neglected, as it is very small, even for long distances and large angles of elevation (for 1500′, $n = 45°$ and $k = 100$, it is but 0.07′); therefore the final formula for distances, with a stadia kept vertical, and with wires equidistant from the centre wire, is the following:

$$D = c \cos n + ak \cos^2 n. \qquad (2)$$

The value of $c \cos n$ is usually neglected, as it amounts to but 1 or 1.5 feet; it is exact enough to add always 1.25′ to the distance as derived from the formula

$$D = ak \cos^2 n. \qquad (2a)^*$$

151. The focal length f of the object-glass may be found by focussing the instrument upon some distant object, say a heavenly body, and measuring then the distance between the plane of the cross-wires and that of the objective. ON, being equal to the distance between the objective and the intersection of a plumb-line with the horizontal axis of the telescope, may be obtained by direct measurement.

The distance p, between the stadia wires, may be determined as follows:

Set up the instrument on level ground, or nearly so, and measure forward from the plumb-line a distance equal to c, and

* The above explanation of the stadia is substantially that given by Mr. G. J. Specht, published by Van Nostrand, 1884, though corrected and simplified.

mark the point; measure onward from the mark any convenient distance d, 400 or 500 feet, as a base. The telescope being level, observe carefully the space a intercepted by the stadia wires on a levelling-rod held vertically at the farther extremity of the base.

Then from the proportion $d-f:a=f:p$ the required distance p may be obtained.

EXAMPLES.

1. Given $f=8$ inches, base $=500$ feet, and $a=5.25$ feet. Find $p=.084$ inches.

2. At what fractional part of the focal length must the stadia wires be separated so that one foot on the rod will correspond to 100 feet base? State also the distance between the wires in terms of the focal length, when one foot on rod corresponds to 66 feet base.

3. Measure with a stadia one or more sides of a field, also the distance across a valley, or from one ridge to another, and compare the results with chain measurement between the same points.

4. Measure with the stadia up or down a hillside, and chain between the same points. Compare results.

GRADIENTER.

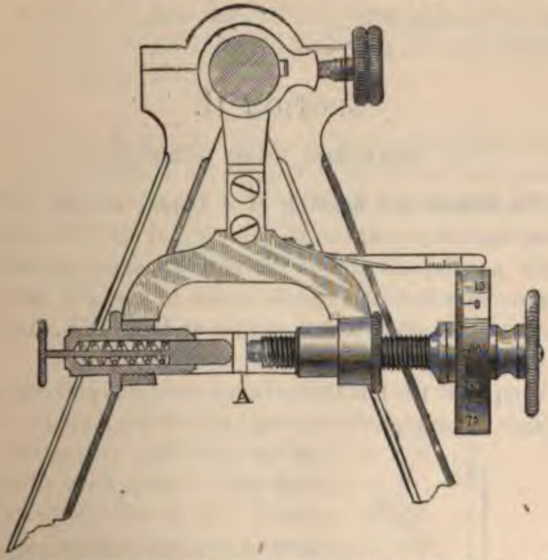
152. This attachment, as shown on next page, is often used with transits for fixing grades, determining distances, etc.

It consists mainly of a screw attached to the semicircular expanded arm of the ordinary clamp of the telescope axis; the screw is accurately cut to a given number of threads, and passing through a nut in one side of the arm, presses against a little stud A fixed to the inside surface of the right-hand standard.

In the other side of the semicircular arm is inserted a hollow cylinder containing a pin actuated by a strong spiral spring, the end of the pin pressing against the side of the stud opposite that in contact with the screw.

Near the other end of the screw, and turning with it, is a wheel, or micrometer, the rim of which is plated with silver, and divided into one hundred equal parts.

A small silver scale, attached to the arm and just above the micrometer wheel, is divided into spaces, each of which is just equal to one revolution of the screw; so that by comparing the edge of the wheel with the divisions of the scale, the number of complete revolutions of the screw can be easily counted.



It will be seen that when the clamp is made fast to the axis by the clamp-screw, and the gradienter-screw turned, it will move the telescope vertically, precisely like the tangent-screw ordinarily used.

And as the value of a thread is such that a complete revolution of the screw will move the horizontal cross-wire of the telescope over a space of one foot on a rod at a distance of one hundred feet, it is clear that when the screw is turned through fifty spaces on the graduated head, the wire will pass over fifty one-hundredths, or one-half a foot on the rod, and so on in the same proportion.

In this way the gradienter can be used in the measurement of distances, precisely like the stadia just described.

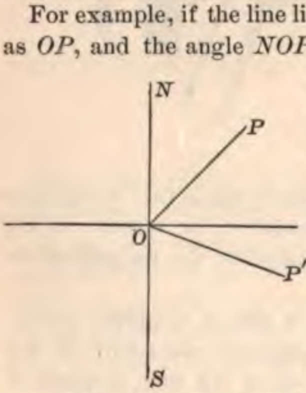
Grades can also be established, with great facility, as follows: First, level the instrument; bring the telescope level to its centre by the clamp and gradienter screw; move the graduated head until its zero is brought to the edge of the scale; and then turn off as many spaces on the head as there are hundredths of feet to the hundred in the grade to be established.

SECTION II.

A. BEARINGS WITH COMPASS.

153. To Obtain the Bearing of a Line. At one end of the line, or at any other point in it, set up and level the compass, loosen the needle, and direct the sights toward the other end. The degree on which the needle comes to rest will indicate the angle between the magnetic meridian and the direction of the line, or the bearing.

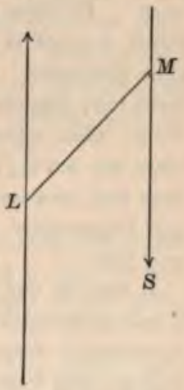
For example, if the line lies between the north and east points, as OP, and the angle NOP being, say 42 degrees, the bearing of the line OP is written, N. 42° E., and read, "north forty-two degrees east." If, as OP', it lies between south and east, and the angle SOP' is, say 74 degrees, it is written, S. 74° E., and read, "south seventy-four degrees east"; in like manner for lines in other quadrants.



It will be observed that the bearing of a line does not exceed 90°. A line which might be read "N. 90° W." or "S. 90° W." is recorded as *west*. The bearing can be read most accurately by placing the eye over one end of the needle and taking the reading from the other end.

Since the graduations are usually made to half-degrees, the bearing can be taken quite accurately to quarter-degrees, and by practice, even closer, without the use of the vernier. In fact, the principal use of the vernier on a compass is to facilitate the running of lines from old deeds, where, when the declination is ascertained, it is turned off on the vernier, and the surveyor may use then the bearings as given in the deed by which he is surveying the tract, without making a calculation for the bearing of each line. The vernier cannot be relied on to read bearings to minutes, on account of the difficulty of accurately manipulating it.

154. Reverse Bearings. Since in plane surveying the meridians passing through the extremities of a line are considered parallel, the direct and reverse bearings should indicate the same angle. That is to say, a line, as LM, the bearing of which, taken at L, called also *fore-sight*, is N. 40° E., when taken at M, *back-sight*, should be S. 40° W.; the degrees being the same, the letters indicating the opposite cardinal points.



When surveying a tract of land with the compass, the instrument should be set up at every corner, and the bearing and reverse bearing of every line taken, as a check on the observer's reading and the working of the needle, since a disagreement in the angle thus measured would be evidence sufficient to warrant a review of the work.

155. Local Attraction. If the readings of the needle of the fore-sight and back-sight have been correctly made, and there is found a disagreement, local attraction exists. It is usually caused by the presence of ferruginous matter. It may exist at both stations or at only one of them.

Assuming that the direct and reverse bearings of the preceding line agree, then the difference in the reading at the two ends

of the line, when the attraction exists, will show the local variation at the last station, and this correction must be applied to the reading of the needle for the bearing of the next line. If, however, the needle will not reverse on the first line of a survey, then it will be necessary to *set up* at some other point of the tract; or, if this is impracticable, select one or more stations near the suspected points, and by taking the bearings of these from the stations, and also the reverse bearings, the intensity and position of the attraction may be determined.

156. Proof Bearings and Tests of Accuracy. In any important compass survey it is well to check the work by sighting to distant prominent objects, such as buildings, trees, etc., and noting the readings. Since *two* bearings are required to locate each object, — and until it is located it cannot serve as a check, — it will be necessary to take at least *three* bearings to each. If, then, when plotting, the three lines intersect in a point, a proof is given of the correctness of the measurements thus connected. The lengths and bearings of diagonals of the tract may likewise be taken as checks on the accuracy of the work; also, when in plotting, if the last bearing and distance *close* the survey, it is considered a proof of the work. The best test, however, of the accuracy of the survey is by Latitudes and Departures, which is explained in Section VI. Articles 207 and 208.

It may be well to caution the student against the fallacy of a test sometimes given, — that if the sum of the interior angles, determined from the bearings, equals twice as many right angles, less four, as the figure has sides, it proves the work. This "test," while it furnishes proof for a transit survey in which the interior *angles* have been measured, will *not* show that the *bearings* of a tract have been correctly taken. The student will readily perceive the truth of this statement if he makes or imagines a plot of a field with a certain side the meridian, then conceives the whole plot turned around so that another side comes to the meridian, it will be evident that

though the bearings are changed, the sum of the interior angles is unaffected. The *so-called* test would prove the work in either case.

157. Suggestions. Test frequently to see that the instrument is in proper adjustment. Keep the same end ahead. Read from the same end of needle. Sight as low on the flagstaff as possible. Make the line of sight as nearly horizontal as practicable. When reading near the cardinal points, be careful that the bearing is not read in the wrong quadrant, also that the common error of reading 56° for 44° is not committed. See that the instrument is set precisely over the station from which the measurements are to be made; that the flagstaff is exactly on the proper point, and that it is held plumb. Level the instrument carefully; especially see that it is level across the line of sight. Take the bearing and measure the distance on the *true* line when practicable; when not, because of a high fence, bushes, etc., set off the least perpendicular distance therefrom at both ends which will afford a clear view, and take the bearing and distance of the extremities of these perpendiculars.

EXERCISES.

1. With a surveyor's compass, by a constant and direct bearing only, run a line, say 40 chains in length, over hilly ground, and part of it, if possible, through brush; then return, using the reverse bearing only.

2. With the same instrument run another line equally difficult, using both direct and reverse bearings forward and back.

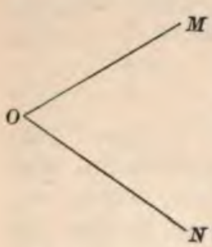
3. Make a survey of a lot one side of which is near to a railroad track. If local attraction is found, determine its intensity.

4. Determine the magnetic bearing of each part of a broken line of several turns along a railroad track, or where local attraction is known to exist.

B. ANGLES WITH TRANSIT.

158. With the Transit the survey of a line or the measurement of an angle can be made with greater accuracy than with the compass, since the reading of the plates to minutes supplants the reading of the needle to quarter or half-quarter degrees, and the pointing power of the transit greatly exceeds that of the compass.

159. To measure a horizontal angle, as *MON*. Set up the instrument precisely at *O*; level it and direct the intersection of the wires to either point, say *N*. Clamp the instrument firmly to the spindle, note the reading of the vernier, then loosen the vernier plate and bring the telescope quite near the other line so that its extremity *M* is in the field of view. Clamp the plate, and with its tangent or slow-motion screw bring the line of collimation precisely on *M*. Again take the reading.



The difference of the two readings will be the angle required. It is more convenient to make the first sight, *ON*, with the zero of the limb and plate coincident, since then the reading of the plates after observing *M* gives at once the angle. If at each observation but one vernier is read, it is best to read every time from the same one; it is better at each observation, though, to read both verniers and take the mean of these, thereby eliminating eccentricity. If, however, great accuracy is required, the measurement of the angles should be taken more than once, by the method of repetition or by series.

160. By Repetition. Make an observation upon any point, and read both verniers; clamp the lower plate to the spindle, direct the telescope to another point, and, as a check, again read the verniers.

Now, keeping the index at the last reading, turn both plates

back, and observe again on the first point; clamp, as before, the lower plate, and turn the upper one so as to sight on the second point. It is perceived that by this operation the angle has been measured twice, but on different parts of the limb. An angle may obviously be repeated any number of times: the mean of the several readings gives more nearly than a single measurement the true angle. The reading at each observation serves as a check on the work. An angle may be repeated by simply noting the reading at the first and last observation, taking their difference, and dividing by the number of repetitions. It must be noted, however, how often, if at all, the 360° point is passed. Now, if the telescope is plunged, the plates turned 180° in azimuth, and repetitions of the angle again be made, beginning at the second point, the mean of the two sets of readings will give still more nearly the true angle, since the errors of adjustment and twist of station are thus lessened and those of observation reduced.

161. By Series. Observe as before upon any point, and read the verniers, clamp the lower plate, turn the vernier plate until the telescope may be fixed upon another point, and again read; thus continue to make observations upon each point desired in their order, sweeping round the horizon, and make the last observation upon the first point. The last reading should be the same as the first. Plunge the telescope, move the plates in azimuth, and observe on the points again, proceeding in the contrary direction. Several series of observations may thus be made, as in the method by repetition. The magnitude of each angle is obtained from the mean of its reading.

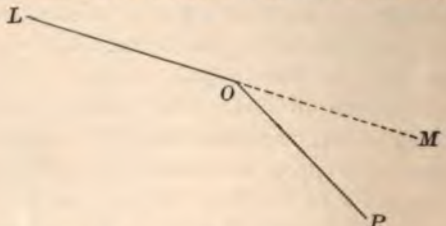
Remark. Care should be exercised to have the instrument properly centred, that is, set precisely over the centre of the station, especially if the object sighted is near the observer. The error arising from an eccentric setting is inversely as the distance of the object sighted; an eccentric setting of one inch producing an error of nearly three (3′) minutes of arc in sight-

ing 100 feet, while the error arising from a sight of 900 feet is less than one-third ($\frac{1}{3}'$) of a minute.

Read both verniers to eliminate eccentricity. See that the reading is not made from the wrong end of the vernier, and that a half-degree is not omitted, calling the reading, say, 36° 15′, instead of 36° 45′. If great accuracy is required when running a straight or broken line, lessen errors of adjustment by reversing the instrument in altitude and azimuth, making two sets of observations at each station, and take the mean of their readings. See Article 157.

If it is desired to locate the lines surveyed with reference to the meridian, the bearing of one of them should be taken by the needle of the instrument; the bearings of the others may be deduced therefrom. See Article 167.

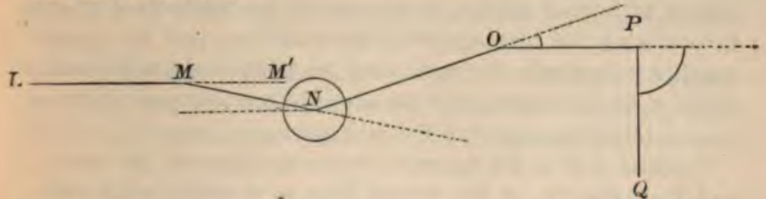
162. Angle of Deflection. The amount of divergence which a line makes with the preceding is called the deflection, and the angle which measures it is termed the deflection angle.



In the figure *POM* is the deflection angle: it is evidently the supplement of *LOP*. To measure it, set the transit at *O*, sight to *L*, clamp the limb to the spindle and the plates together, then plunge the telescope: it will point to *M*. Take the reading, unclamp the vernier-plate and move it until the wires intersect *P*. The difference between the reading now and the first reading is the deflection angle. If, when making the first observation, the vernier was at zero, the reading, after sighting *P*, would indicate at once the angle.

163. Traversing, or surveying by the back angle, is a method

of surveying by which the direction of each line of a survey is compared with the first as a meridian or reference line. It is effected as follows:



Let it be required to traverse the broken line $LMNOPQ$. Set up the instrument at M, clamp the vernier at zero, for convenience, and, with the lower motion, sight L, clamp below, transit the telescope, loosen above and observe N: the reading will show the angle $M'MN$ which the line MN forms with LM. Clamp the plates, move to N, plunge the telescope, and, with the lower motion, sight M, the index remaining as at M; then clamp below, loosen above, transit the telescope, and direct it to O: the index will show the angle which the line NO makes with LM. And so continue until the end of the line.

To guard against mistakes in reading, and to avoid recording whether the deflection is right or left, it is well to assume all angles measured in the same direction. In the figure the readings are all to the right, or clockwise, as indicated by the circular arcs, and the record is as follows:

Stations.	Azimuths with LM.	Bearings with LM.	Magnetic Bearings assuming Bearing of LM N. 50° E.
L	0°	North.	N. 50° E.
M	18°	N. 18° E.	N. 68° E.
N	340°	N. 20° W.	N. 30° E.
O	360° or 0°	North.	N. 50° E.
P	90°	East.	S. 40° E.

From the nature of the operation it may be perceived that, algebraically, the azimuth of any line is equal to its deflection

plus the azimuth of the preceding line. This method is particularly adapted to surveying roads, streets, water courses, etc., and even in farm surveying it possesses an advantage over the survey by interior angles, on account of the readiness it affords in obtaining the bearings from the azimuths, and the greater rapidity with which the work may be plotted, since the angle which each line makes with the assumed meridian, or reference line, is taken at once from the field notes.

Suppose LM in the figure to be the meridian of the survey, and the azimuths of the several lines as recorded in the table. Now, assuming the direction of LM to be north, it is evident that MN will be in the northeast quadrant 18° from the north point, or N. 18° E; NO will be 20° to the west of north, or N. 20° W.; OP, making no angle with the meridian, will have a bearing north, and PQ east.

So that, in general,

When the azimuth is less than 90°, it equals the bearing, and the line is in the northeast quadrant.

When the azimuth is between 90° and 180°, the bearing is southeast, and is the supplement of the azimuth.

When the azimuth is between 180° and 270°, the bearing is southwesterly, and may be found by subtracting 180° from the azimuth.

When the azimuth is between 270° and 360°, the bearing is northwesterly, and is the difference between 360° and the azimuth.

When the azimuth is 90°, the bearing is due east.

When the azimuth is 180°, the bearing is due south.

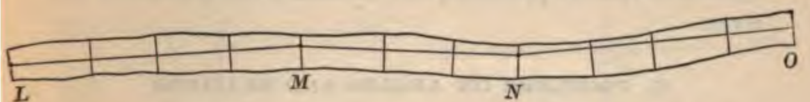
When the azimuth is 270°, the bearing is due west.

When the azimuth is 360°, the bearing is due north.

If it is required to find the magnetic or true bearing of any or all the lines, take the magnetic or true bearing of the meridian of the survey and apply it, by addition or subtraction, according as the bearing of the assumed meridian, or standard line, is northeast or southwest. In the example given, suppose the bearing of the assumed meridian LM to be N. 50° E.: then the bearing

of the second line MN will be recorded 18° to the east of the reference line, or N. 68° E.; the line NO, having a deflection of 20° to the left of the reference line will be recorded N. 30° E.; and OP, N. 50° E. Thus the fourth column is added to the table.

164. To Traverse a Road, as $LMNO$. Proceed as indicated in the last article, and in addition measure the lines LM, MN, NO, and perpendicular offsets thereto, at proper distances.



If the road deviates much from a straight line, it will be necessary, in order to obtain more correctly the area, to take two offsets at M, one perpendicular to LM, the other to MN; and also two at N, one perpendicular to MN, and the other perpendicular to NO.*

Likewise to Survey a Small Stream. Traverse and measure the distances between assumed stations, as L, M, N, O, P, so chosen as to make no more of them than is consistent with few and short offsets to the various bends of the stream. If the

stream is small, not exceeding 10 feet in width, or even wider if shallow, and it is desired to survey it between X and Y, a good plan is to run a straight line between these points and measure offsets therefrom to the stream; or, if such a line will make the offsets rather long, run RQ, and measure offsets from it to X and Y and intermediate points. If, however, the stream is wide

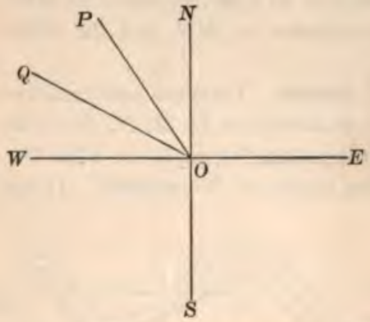
* Article 234.

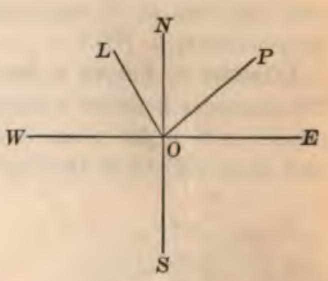
and the crossing difficult, it will probably be better to use more stations, as shown in the figure. If a compass is used, the bearings may be taken instead of the angles.

If a river of considerable width is to be surveyed, it will be necessary, in addition to the measurement of broken lines on each side from which offsets are taken, to make a series of angular measurements connecting the lines on one side with those on the other, and thence by trigonometrical calculations determine their relative positions, and ultimately the surface of the river.

C. PROBLEMS ON ANGLES AND BEARINGS.

165. Angles between Lines. To determine the angle between two lines, meeting at a point, given by their bearings.





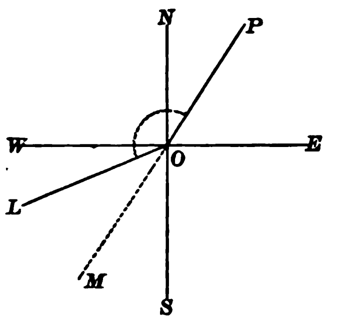
1. If the lines run between the same cardinal points, that is, in the same quadrant, take the difference of their bearings.

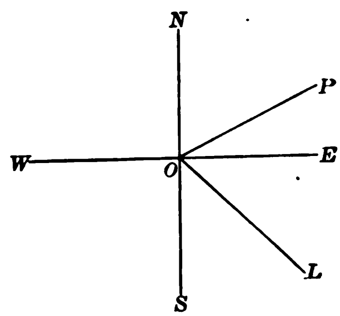
Suppose the bearing of OP is N. 32° W. and that of OQ N. 60° W.; the angle between them is obviously $NOQ - NOP$; or, $60° - 32° = 28°$.

2. When the lines run in different quadrants and both above or both below the horizontal or E. and W. line, take the sum of their bearings. If OP bears N. 60° E. and OL N. 20° W., the angle $POL = PON + NOL = 60° + 20° = 80°$.

3. If the lines run in diagonally opposite quadrants, subtract the difference of the bearings from 180°. Assuming the bearing of OP N. 28° E. and of OL S. 58° W., the angle

$$POL = 180° - LOM = 180° - (58° - 28°) = 150°.$$





4. When the lines are in different quadrants, and both to the right or both to the left of the vertical or N. and S. line, subtract the sum of the bearings from 180°. If OP bears N. 65° E. and OL S. 42° E., the angle

$$POL = 180° - (NOP + SOL) = 180° - (65° + 42°) = 73°.$$

ADDITIONAL EXAMPLES.

1. A line OP bears N. 40° W. and OL N. 40° E.; required the angle POL.

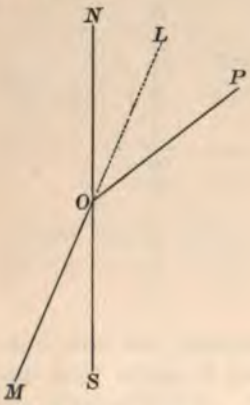
2. Find the angle POL, when OP bears S. 50° E. and OL N. 89° E.

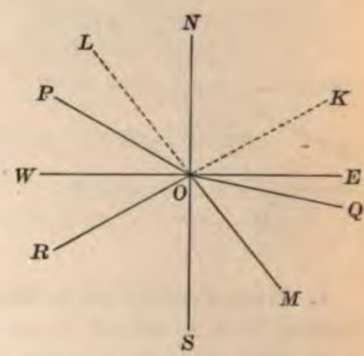
3. Required the angle at O, when OP bears N. 80° W. and OL S. 79° E.

4. What is the angle O, if OP runs S. 89¾° W. and OL N. 89½° E.?

5. A line OP runs S. 70° W. and OL S. 45° W. Find the angle O.

166. There may be given the bearing of a line, as MO, and the deflection angle LOP, to the right or left of the direction of MO, to find the bearing of OP; or, the bearings of MO and OP may be given to determine the magnitude of the deflection angle LOP.



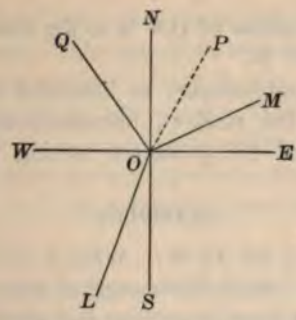


a. Given the bearing of a line and the deflection of the next, to find its bearing.

Suppose MO bears N. 32° W., and the deflection of $OP = 20°$ to the left; the bearing of OP is evidently 20° farther towards the west than MO or its prolongation OL. It is therefore N. 52° W. Again, assuming RO bears N. 60° E. and the deflection of OQ 40° to the right, it is evident that OQ is in the southeast quadrant, 10° from the east point; or, its bearing is S. 80° E.

b. When the bearings of the lines are given, to determine the deflection.

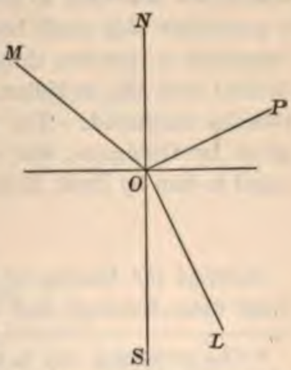
Suppose LO (p. 115) bears N. 20° E. and OM N. 70° E.; the deflection of OM from LO, or its prolongation OP, is evidently $70° - 20° = 50°$ to the right. Again, the bearing of LO remaining the same, and that of OQ N. 30° W., then it is readily seen that the deflection angle is $20° + 30° = 50°$ to the left.



General rules might be given for the cases under the above heads, corresponding to those in the preceding article, but they are deemed unnecessary, as a little reflection will enable the student to determine the required bearing, or angle, in any given case.

167. *Given the angle between two lines, and the bearing of one line, to find the bearing of the other.*

The solution of this problem is ordinarily required in transit surveying, for, when surveying with that instrument, it is common to take the bearing of only one line, and deduce the courses of the others from that bearing and the measured angles. Suppose LO bears N. 24° W. and the angle $LOP = 82°$, to find the bearing of OP. It is evident that the bearing of OP or the angle NOP, which gives the degrees in the bearing,



$$= 180° - (SOL + LOP)$$
$$= 180° - (24° + 82°) = 74°.$$

Hence the bearing of OP is N. 74° E.

Assume the angle $POM = 100°$, and the bearing of OP as found above; then, since there are 100° − 74°, or 26°, more in the angle than lies between OP and the

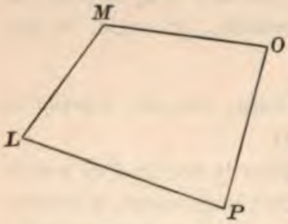
north point, the position of OM is to the west of north 26°, or its bearing is N. 26° W.

Some simple combinations, as indicated in the illustrations given, will enable the student, unencumbered with *rules*, to readily solve any of the problems coming under this head.

EXAMPLES.

1. A line bears S. 89° 15′ W. What is the bearing of a line perpendicular to it? Also, the bearing of a line making an angle of 135° with it? Is there more than one answer to the last?

2. If OP bears S. 36° W., and the angle $OPL = 68°$, what is the bearing of PL? *Ans.* N. 32° W.



SUGGESTION. Pass a meridian through the angle, and consider the given bearing reversed.

3. The angles L, M, O, P, of the trapezium are respectively 62°, 130°, 80°, and 88°, and the bearing of LM N. 70° E.; find the other bearings.*

168. To Change the Bearings of the Sides of a Survey. It is sometimes desirable to change the bearings of a survey so that a particular side shall become a meridian. The whole plat is conceived to revolve through an angle sufficient to make the desired side the meridian; the relative position of the sides remains unaltered. The following rule is substantially that given by Gummere, who states that the method was communicated to him by Prof. Robert Patterson, late of Philadelphia.

RULE.

Subtract the bearing of the side that is to be made a meridian from those bearings that are between the same points that it is,

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* The calculation may be tested, after having deduced the bearings of all the sides, by taking the last bearing found, as PL, applying the angle L, and observing if it gives the proper bearing of LM.

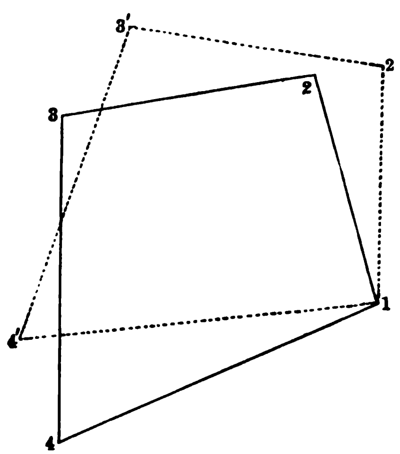
and also from those that are between points directly opposite to them. If it is greater than any of those bearings, take the difference, and change west to east or east to west.

Add the bearing of the side which is to be made a meridian to those bearings which are neither between the same points that it is nor between the points directly opposite to them. If either of the sums exceed 90°, *take the supplement, and change south to north or north to south.*

The accompanying diagram of full and dotted lines exhibits the positions of the sides of the following described farm, before and after turning through 16¼° to the right:

(1) N. 16¼° W., 24.63 chains; (3) S. ¼° W., 34.28 chains;
(2) S. 79° W., 27.00 chains; (4) N. 65° E., 37.20 chains,

to the place of beginning. The bearings are changed so as to make the first side a meridian.



EXAMPLES.

1. Given the bearings of a tract of land:

(1) S. 10° E.; (2) S. 30° W.; (3) N. 60° W.;
(4) N. 20° W.; (5) N. 80° E.,

to the place of beginning. Required the changed bearings that the fourth side may be a meridian.

(1)	S. 10° E.	(4)	North.
	20		
Changed bearing,	S. 10° W.		
(2)	S. 30° W.	(5)	N. 80° E.
	20		20
Changed bearing,	S. 50° W.		100
(3)	N. 60° W.		180
	20	Changed bearing,	S. 80° E.
Changed bearing,	N. 40° W.		

The student who avails himself of the hints and methods referring to the manipulation of angles and bearings as given in the preceding articles, will have no difficulty in determining the changed bearings direct from the data, without the use of *rules*. Thus in the example above it will be observed that each line is turned through 20° to the right; that is, the fourth course is made due north. The next side to it going round to the right, N. 80° E., will be turned the same number of degrees (20), which places it 10° from the east point in the southeast quarter, or its bearing is S. 80° E.; the first side turning through the same angle (20°) will be thrown 10° west of the south point, or S. 10° W.; the second course will be 20° farther to the southwest, or S. 50° W.; and the third course turned toward the north point 20° will be N. 40° W.

2. Find the bearings of all the sides of the following described tract of land when the second side is made a meridian:

(1) N. $68\frac{1}{2}$° E., 8.42 chains; (3) S. $78\frac{3}{4}$° W., 4.90 chains;
(2) N. 27° W., 10.25 chains; (4) S. $\frac{1}{2}$° E., 4.40 chains;
(5) S. 12° E., 7.04 chains,

to the place of beginning.

3. Given the bearings of a tract of land as follows:

(1) S. $39\frac{1}{2}°$ W.; (3) N. 15° W.; (5) N. 2° E.;
(2) East; (4) N. $79\frac{1}{4}°$ E.; (6) S. $73\frac{3}{4}°$ W.,

to find the bearings of all the sides when the first becomes a meridian.

4. Given the bearings of a tract of land as follows:

(1) S. 79° W.; (3) N. $89\frac{1}{2}°$ E.; (5) S. $80\frac{3}{4}$ E.;
(2) S. $\frac{1}{4}°$ W.; (4) N. $1\frac{3}{4}°$ E.; (6) S. $58\frac{1}{2}°$ E.;
(7) N. 39° E.; (8) N. $16\frac{1}{4}°$ W.,

to find the bearings when the eighth side becomes a meridian.

EXERCISES.

1. With a transit, using back and fore sights, run a tangent forward and back over hilly and brush land requiring six or eight settings of the instrument. The last two points set forward will give the direction back. Note the distance, if any, between the corresponding positions occupied by the instrument.

2. Traverse, or survey by the back angle, a broken line of six stations, using the first line as the meridian, or reference line, of the survey. Record the notes, indicating the azimuthal angles and bearings.

3. Measure the three angles of a triangular piece of land, the corners being visible from each other; see how much, if any, their sum differs from two right angles.

4. Traverse a pentagonal field, the index at the beginning being set at zero, and see if, when finally sighting on the station first occupied, the reading is zero.

SECTION III.

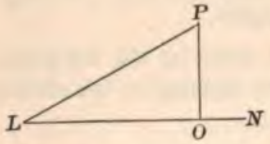
Obstacles.

A. PROBLEMS ON PERPENDICULARS AND PARALLELS.

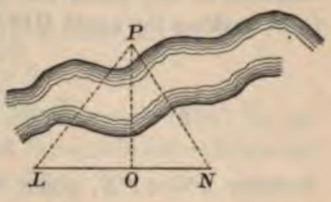
169. The Obstacles which occur in field work are more easily and expeditiously overcome with the compass, or transit, and chain, than with the chain alone. Methods for the latter were given and illustrated in Chapter I. Section II., Chain Surveying.

To erect a perpendicular to a line at any given point. Set up the instrument over the point; if a compass is used, take the bearing of the line, and then move the instrument in azimuth until a bearing differing 90° from the first is observed. The line of sights will then indicate the direction of the required perpendicular. If a transit is employed, centre on the point, sight to a point in the line, clamp to spindle, and turn the vernier plate 90° either way; then the line of collimation will show the direction of the perpendicular sought. Of course by the methods explained above, a line can be run with either instrument from any given point and making *any* given angle thereat with a line.

170. *To let fall a perpendicular from a given point to a line.* Let P be the point, and LN the line. If the compass is used, take the bearing of LN, remove the instrument to P, and with a bearing differing 90° from the first, run PO for the required perpendicular. With a transit centre on L, measure the angle OLP, remove to P, and make the angle LPO equal to the complement of L; the line of sight of the instrument will then be in the direction of the required perpendicular.



171. *To let fall a perpendicular to a line from an inaccessible point.* Measure the distance between any two points, as L and N, in the line; also the angles PLN and LNP. Then in the triangle PLN we have given the side LN and the angles to find PL or PN. Computing PL, the distance



$$LO = PL \cos PLO.$$

Or we may deduce an expression for LO in terms of the measured line and the observed angles, thus:

$$LO = PO \cot PLO.$$

$$NO = PO \cot PNO.$$

Hence $\quad LO : NO = \cot PLO : \cot PNO,$

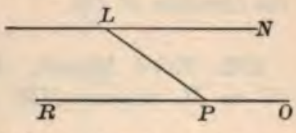
and $\quad LO : LO + NO = \cot PLO : \cot PLO + \cot PNO;$

but $\quad LO + NO = LN,$

therefore $\quad LO = \dfrac{LN \cot PLO}{\cot PLO + \cot PNO}.$

QUERY. Could a line be run not perpendicular as above through an inaccessible point, making *any* angle with the given line?

172. *To run a line through a given point parallel to a given line.* With the compass obtain the bearing of the line, and then from the given point run a line with the same bearing. With the transit, LN being the line and P the point, centre on L, measure the angle NLP, remove to P, and make the angle LPR equal to NLP; the line of collimation will then be in the required parallel.



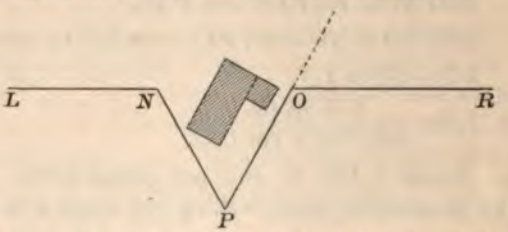
B. PROBLEMS ON ALIGNMENT.

173. *To prolong a line, as LN, beyond a tree, a building, or any obstacle.*

First Method. By Deflection Angles. Set up the instrument at any point of the line, as N, and deflect, sufficient to pass the obstacle, to any point P. Measure NP, remove to P, deflect to O, making the angle QPO double the angle at N.

Measure $PO = PN$, place the instrument at O, observe P, plunge the telescope and deflect to R, so that $SOR = \frac{1}{2} OPQ$; the telescope will then be in the prolongation of LN.

174. *Second Method. By Equilateral Triangle.* Deflect 60° from the direction of the line at N; measure to P a distance

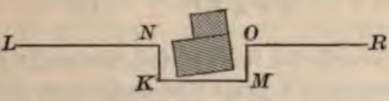


sufficient that PO, making an angle of 60° with PN, will clear the obstacle. Measure $PO = PN$, and turn the telescope in the direction of OR, the prolongation of LN, by deflecting 60° from the direction of PO.

175. *Third Method. By Isosceles Triangle.* Deflect at N 45° to M, measure NM, make NMO a right angle, and MO

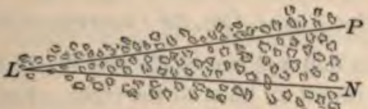
$= MN$; at O turn into OR by deflecting from the direction of OM 45°.

176. *Fourth Method. By Perpendiculars.* Erect a perpendicular NK of sufficient length that a line passing through



K parallel to LN will clear the obstacle; run KM; lay off $MO = NK$, and a right angle turned from MO will indicate the direction of LN, or its prolongation OR.

177. Random Line. When brush, wood, or any obstruction prevents N being seen from L, run a line LP as nearly as may



be judged in the direction of LN: when opposite N, as at P, measure the shortest distance from P to N, call it d; then the angle PLN in degrees $= \dfrac{57.3 \times d}{LP}$.

Setting up again at L, and applying the correction thus found in a proper manner to the angle or bearing before used, the line LN may be traced.

Demonstration. When the distance PN does not exceed 5 per cent of the length of PL, PN and PL may be regarded as radii of a circle, and PN coincident with the arc which subtends the angle PLN; then

$$2\pi LP : 360 = PN : PLN,$$

or
$$PLN = \frac{360 \times PN}{2\pi \times LP} = \frac{57.3 \times PN}{LP}.$$

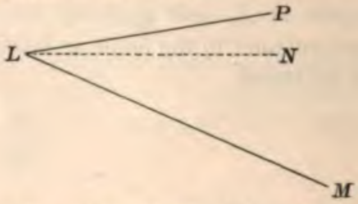
When PN exceeds the limit stated, the angle PLN should be found by measuring PN perpendicularly from PL, and dividing this by the length LP for the tangent of the angle PLN.

EXAMPLES.

1. A random line was run N. 41° 15′ E. 18.34 chains, when the nearest distance to the desired corner, which was to the left, was found to be 16 links. Required the correction and the bearing of the true line. *Ans.* Cor. 30′; bearing of line, N. 40° 45′ E.

2. A random line was run S. 89° 45′ W. 24.80 chains, when the corner was found 22 links to the right. Find the correction and the bearing of the line.

3. The length of a random line is 16.64 chains, and a perpendicular from its extremity to the desired point equals 96 links. What correction is needed?



4. A random line LP, 25.12 chains long, run by transit, makes an angle of 27° with LM, and the point P is 18 links to the left of N; LN being the true line. Determine the proper angle to turn off at L with which to trace LN.

C. PROBLEMS ON MEASUREMENT.

178. *a.* **When the Ends of the Line are Accessible and Visible from Each Other.**

The methods indicated in Problems on Alignment will be found useful in many instances for the determination of the lengths of lines, the direct measurements of which are impracticable. Thus, in the figure in Article 176, the distance NO will be found by measuring KM.

In figure accompanying Article 174 the measurement of either NP or PO will give the side NO.

Otherwise (Article 175). Measure NM, and multiply it by $\sqrt{2}$, or extract the square root of twice the square of NM for the required length NO.

By random line, as in Article 177, when the shortest distance PN is taken, the length of the true line will equal the measured or random line.

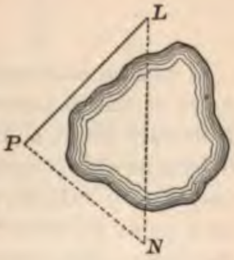
If the perpendicular from P is used, then the length of the true line will equal the square root of the sum of the squares of LP and PN; that is,

$$LN = \sqrt{\overline{LP}^2 + \overline{PN}^2}.$$

To ascertain the horizontal measurement of a hillside, take the angle of its slope, measure up or down it (preferably down), and the product of this distance and the cosine of the angle will be the horizontal distance required.*

179. *By Triangulation.* Measure LP and the angles L and P; the sine proportion may then be employed to determine

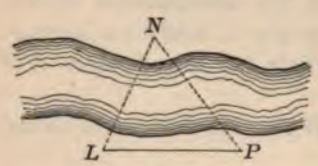
$$LN = \frac{LP \times \sin P}{\sin (L + P)}.$$



180. *Otherwise.* Measure LP, PN, and the angle P. Then having two sides and the included angle of the triangle, the third side LN may be computed.

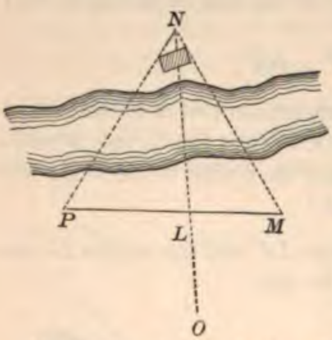
* When measuring the angle of elevation, the surveyor should sight to a point on the rod a distance above ground equal to the height of the line of collimation of his instrument.

181. *b*. **When One End of the Line is Inaccessible.** Let N represent the inaccessible but visible end of the line LN, the length of which is desired. Measure LP of such length, if possible, that none of the angles will be less than 30°; the nearer LNP is equilateral, the better. Observe the angles L and P. Then, by the sine proportion,



$$LN = \frac{LP \times \sin P}{\sin(L + P)}.$$

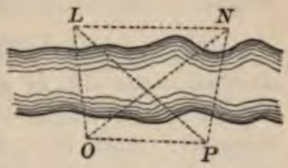
182. When the Points are not Visible from Each Other. In the figure let N represent the invisible point in the line LN, the length of which is required. Measure a line in any convenient direction through L, as MP, noting the distances ML and LP, of such a length that the point N may be seen from each extremity. Observe the angles P and M. In the triangle PMN, find, by the sine proportion, the length of PN. Then in PNL are known two sides and the included angle, with which may be found LN.



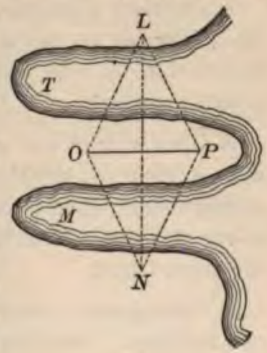
It will be observed that the problem requires the measurement of the distance between *two points*, L and N, invisible from each other, and direction unknown. If it were simply to determine the distance from L to an *invisible point* in the *prolongation of* OL, we should measure perpendicularly from OL to a point P, from which the point N could be seen, observe the angle LPN; then $LN = PL \times \tan LPN$.

Query. What would be the best method of solving the problem under the last supposition, if it were impracticable to measure a perpendicular from OL?

183. *c.* **When the Ends of the Line are Inaccessible.** Let it be required to determine the length of the inaccessible line LN. Measure OP, and observe the angles LON, NOP, OPL, and LPN; then in the triangle LOP compute LO, and in NOP, ON. There will then be given two sides and the included angle of the triangle LON to find LN.



184. The same general method would apply if the base intersected the line the length of which is desired. Suppose it is required to determine the distance between L and N, points on opposite sides of two inlets, M and T. Measure OP and take the angles at the extremities on both sides of the base. There will then be data sufficient to find OL and ON, and finally LN.



QUERY. Would it be practicable in any case to make OP perpendicular to LN? If so, would it be necessary to measure the distance OP and all the angles, as above? Why?

EXAMPLES.

1. To determine the distance between two points L and N, on opposite banks of a stream, I measured a base $LP = 300$ feet, and observed the angles which N made with L and P to be 58° 45′ and 64° 50′, respectively. Required LN.

2. If LP in Example 1 were taken at right angles to LN, the angle P being 40° 30′, what would be the length of LN?

3. To ascertain the distance LN between two inaccessible points invisible from each other, I measured a line MP through

L, from the extremities of which N could be seen. $ML = 160$ feet; $LP = 200$ feet; angle at $M = 65° 30'$; angle $P = 69° 15'$. What is the length of LN?

4. To determine the distance between two points L and N, situated on the side of a river opposite to where I was, a base line OP 400 feet long was measured, and the following angles observed: $LON = 68° 30'$; $NOP = 32° 45'$; $NPL = 50° 30'$; $LPO = 40° 15'$. Required LN.

EXERCISES.

1. Prolong a line beyond a house, tree, or other obstruction, using any one of the methods herein given. Return, pass the obstruction by some other method. See how near the starting-point is reached.

2. Run a trial line of considerable length through a wood, with a view of sighting a stake previously set. Make the proper measurements and calculation to correct the angle and re-run the line. Note the distance, if any, from the stake after the second trial.

3. Triangulate across a creek or small lake. Use at least two methods. See how near the results agree.

4. By triangulation determine the distance between two points without going near them. Verify the result by subsequent measurement.

5. Measure the distance between two points in a given line, invisible and assumed inaccessible from each other. Compare the results of two methods. Verify subsequently by direct measurement.

6. Run a trial line between two points which are invisible from each other, on account of an intervening ridge. Correct the angle and re-run the line. If the proper point is not reached, should the angle be again corrected?

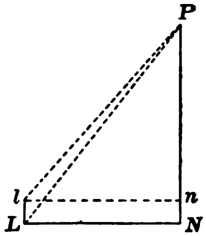
SECTION IV.

Heights and Distances.

A. ACCESSIBLE HEIGHTS.

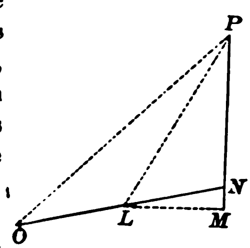
185. *Let it be required to determine the height P above a horizontal plane LN.* Measure the distance LN and the angle of elevation L. Then

$$PN = LN \tan L.$$



If the ground is level, or nearly so, the telescope cannot be placed at L, in the horizontal plane with N, but at some point l, and the angle Pln is measured instead of PLN. In such a case Nn must be added to the calculated height.

186. *Let it be required to find the height of an object standing on an inclined plane ON.* Measure the distances NL and LO, and the angles NLP and NOP. In the triangle OLP, by the sine proportion, find PL. Then in the triangle PLN, having two sides and the included angle, PN may be determined.



187. *Otherwise.* Measure NL, and at L the angles of elevation of N and P. Then the projection of LN on the horizontal plane

$$= LM = LN \cos NLM,$$

and $\quad MN = LN \sin NLM;$

$$PM = LM \tan PLM;$$

whence $\quad PN = PM - NM$; or, expressed in a single equation,

$$PN = LN \times \cos NLM \times \tan PLM - LN \times \sin NLM.$$

EXAMPLES.

1. At 120 feet distance from the centre of the foot of a liberty pole, the angle of elevation of its top was 38° 40′. Required its height.

2. The distance LN (see Article 185) measures 90 feet, the angle of elevation l is 42° 30′, the telescope being 4.8′ above the horizontal plane LN. Determine height of the point P.

3. To determine the height of an object on an inclined plane, two stations, L and O (marginal figure, Article 186), were selected, one 50 feet and the other 110 feet, measured on the slope from N. The angle $NLP = 40° 15′$, and $NOP = 22° 30′$. Required the height.

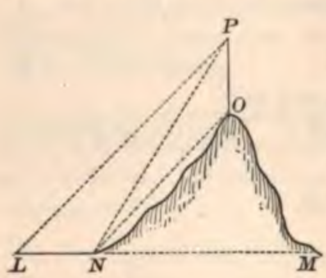
QUERIES. Practically, is it necessary to know the height of instrument * in such cases?

If there was a change of slope at L, would any other measurement be necessary to calculate the required height?

4. Suppose NL (figure, Article 186) measures 60 feet, and the angles of elevation at L, of N and P, are respectively 12° 30′ and 59° 20′. Determine the height of P above N.

B. INACCESSIBLE HEIGHTS.

188. *To determine the height of an object situated on an inaccessible hill.*



Measure in the same vertical plane with P a horizontal line LN, and observe at N the angles of elevation of the points O and P, and at L the angle of elevation of P. In the triangle LNP, by the sine proportion, calculate PN.

By the same method, find NO from the triangle PON. Then

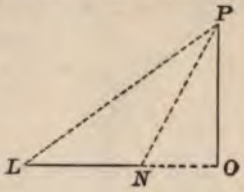
* Height of instrument is the height of the line of sight above the the ground, or any other assumed horizontal plane.

$$PO = PN \sin PNM - NO \sin ONM.$$

The student may show, after finding PN and NO as above, a different method of finding PO than that indicated.

Ex. At a certain station the angle of elevation of the base of a tower on a hill-top was 38° 40′, and that of the top 50° 15′; 190 feet more remote, the angle to the top was 36° 20′. The stations being in the same horizontal plane, required the height of tower and of the hill.

189. *Let PO be an object whose height is required.* Measure in the same vertical plane with P a horizontal base line LN, and observe the angles of elevation PLN and PNO. Then, by the sine proportion, find PN, and



$$PO = PN \sin PNO.$$

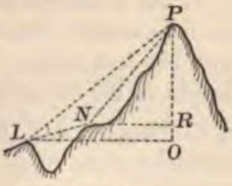
190. *Otherwise.*

$$PO \cot L = LO,$$
$$PO \cot N = NO,$$
$$LO - NO = LN;$$
$$\text{or, } PO(\cot L - \cot N) = LN.$$
$$\therefore PO = \frac{LN}{\cot L - \cot N}.$$

Ex. If $LN = 120$ feet, and the angles at L and N respectively 27° 50′ and 45° 19′,

$$PO = \frac{120}{\cot 27° \, 50' - \cot 45° \, 19'} = 136.6 \text{ feet.} \quad \textit{Ans.}$$

191. If it is impracticable to locate the base line in a horizontal plane, measure from L in the direction of P any line LN, and at L take the angles of elevation of N and P. Observe also the angle at N. By the sine proportion obtain LP. Then

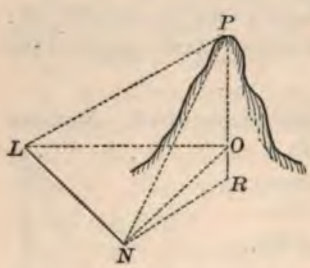


$$PO = LP \sin PLO,$$

and $$PR = PO - RO = LP \sin PLO - LN \sin NLO.$$

QUERY. May the observed angle at N be either LNP or PNR?

192. *Otherwise.* L and N being in different planes, measure the horizontal distance between them. Observe the angle of elevation PLO and the horizontal angles OLN and ONL. By the sine proportion find LO. Then



$$PO = LO \tan PLO,$$

or, expressed in a single equation,

$$PO = \frac{LN \sin LNO \tan PLO}{\sin NOL},$$

which equals the height of P above the horizontal plane through L.

If it is required to find the height of P above the horizontal plane through N, proceed as follows: Assuming N to be below* L, observe at N the angle of elevation of P; then find the horizontal distance between N and O by the sine proportion, using the triangle NLO; thus, $\sin O : \sin L = LN :$ fourth term. This fourth term will not be NO, since the measurement of the distance and angles employed in the computation is referred to a horizontal plane, and hence the fourth term will express the *horizontal* distance between N and O, which equals NR, R being a point in the prolongation of the vertical PO. Whence,

$$PR = NR \tan PNR.$$

EXAMPLES.

1. At a certain station the angle of elevation of the top of an inaccessible object situated on a horizontal plane was 60° 50′, and 120 feet farther away the angle was 29° 10′. Required the height of the object and its distance from the first station.

2. Suppose LN (figure, Article 191) is 140 feet, the angles of elevation at L, of N and P, are respectively 9° 25′ and 30° 16′,

* It may obviously be above or below L; the same reasoning will hold.

and the angle $PNR = 42°$. Find the height of P above O and R.

3. In figure, Article 192, suppose

$LN = 1000$ feet; angle $PLO = 26° 18'$;
angle $OLN = 36° 20'$; angle $PNR = 55° 10'$.
angle $ONL = 95° 40'$;

Find PO and RO.

193. *To determine the perpendicular distance from a given horizontal plane of an inaccessible object situated below it.*

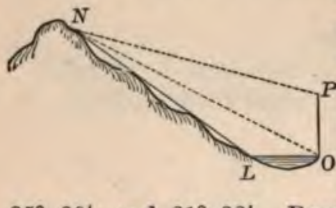
Let P be the point whose perpendicular distance from a horizontal plane through L is required. Select two points L and N visible from each other, and from which P can be seen. Measure the horizontal distance between them; observe also the horizontal angles PLN and PNL, and the angle of depression of the point P, at L. By the sine proportion calculate the *horizontal* distance from L to P; this multiplied by the tangent of the angle of depression observed at L will give the perpendicular distance required.

If L and N are not in the same horizontal plane, observe at N the angle of depression of P, and calculate as above the perpendicular distance between the point and the horizontal plane through N. The difference of these perpendicular distances will also give the difference in height of L and N. A check on the work may be had by determining from more direct methods already given the difference in elevation of L and N.

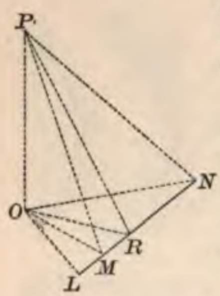
EXAMPLES.

1. At L and N (last figure) the horizontal angles measure respectively $67° 40'$ and $43° 10'$; and sighting P, the angles of depression taken in the same order are $32° 18'$ and $21° 42'$. The distance between the stations being 1200 feet; required the difference in height of P, L, and N.

2. To find the height of an object, PO, standing on the edge of a lake and inaccessible to L, a station on the opposite rocky shore, a distance of 500 feet was measured from L up the slope to N, where the angles of depression of L, O, and P were observed respectively, 39° 40′, 25° 20′, and 21° 32′. Required the height of PO.



194. *To determine the height of an object, and its distance from three observing-stations situated in a straight line and in the horizontal plane through the foot of the object.*



Let PO represent the required height; L, R, and N the stations; the angles of elevation of P taken at each and in the order named α, β, and θ. The distance $LR = a$, $RN = b$, and the unknown height $= x$. It is evident that the triangles POL, POR, and PON are right-angled at O, and therefore

$$OL = x \times \cot \alpha.$$
$$OR = x \times \cot \beta.$$
$$ON = x \times \cot \theta.$$

Again, drawing OM perpendicular to LN, we shall have from the acute-angled triangle LOR,

$$\overline{OL}^2 = \overline{OR}^2 + \overline{RL}^2 - 2RL \times RM,$$

and from the obtuse-angled triangle NOR,

$$\overline{ON}^2 = \overline{OR}^2 + \overline{RN}^2 + 2RN \times RM;$$

or, substituting the proper values for the lines represented, we shall have

$$x^2 \cot^2 \alpha = x^2 \cot^2 \beta + a^2 - 2a\, MR,$$
$$x^2 \cot^2 \theta = x^2 \cot^2 \beta + b^2 + 2b\, MR.$$

Eliminating MR by multiplying the first by b, the second by a, adding and factoring, we obtain

$$x^2(b\cot^2\alpha + a\cot^2\theta)$$
$$= x^2\cot^2\beta(a+b) + ab(a+b).$$

Whence $$x = \sqrt{\frac{ab(a+b)}{b\cot^2\alpha + a\cot^2\theta - \cot^2\beta(a+b)}}$$

If the stations are equidistant, the formula reduces to

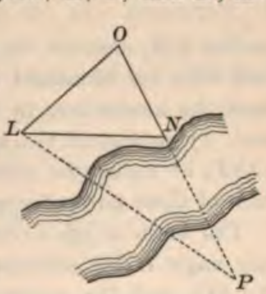
$$x = \sqrt{\frac{2a^2}{\cot^2\alpha + \cot^2\theta - 2\cot^2\beta}}.$$

Or, $$x = \frac{a}{\sqrt{\frac{\cot^2\alpha}{2} + \frac{\cot^2\theta}{2} - \cot^2\beta}}.$$

Having obtained the height of P above the plane, the horizontal distance from the object to either station may be determined by multiplying this height by the cotangent of the angle of elevation at the station. The oblique distance from either station to P is given by the product of PO and the cosecant of the angle of elevation at the station.

INACCESSIBLE DISTANCES.

195. *The distance apart of three objects*, L, O, and N, inaccessible from P are known, viz.: $LO = 2000$ feet, $ON = 1800$ feet, and $LN = 2400$ feet. At P, situated in the prolongation of ON, the observed angle $= 21°\ 48'$; how far is it from station P to each object?



First calculate angle O; then in the triangle POL there will be known all the angles and one side, whence the required distances may be readily found.

Usually the station P cannot be chosen so as to fall in ON or OL produced; then the measurement of *two* angles will generally be sufficient, with the known distances to locate the

point of observation. For example, suppose the distances and angles are as follows:

$$NO = l = 3000 \text{ feet};$$

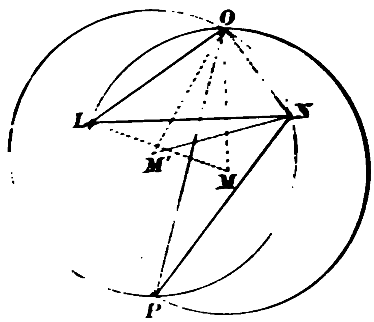
$$OL = n = 3900 \text{ feet};$$

$$LN = o = 4500 \text{ feet};$$

$$\text{angle } NPO = \alpha = 25° 40';$$

$$\text{angle } LPO = \beta = 29° 30'.$$

By construction, the point P may be found as follows: Subtract from 180° 2 LPO, and from LO lay off at L and O the angles LOM and OLM, each equal to half the remainder. From the point M thus determined as a centre, and with a

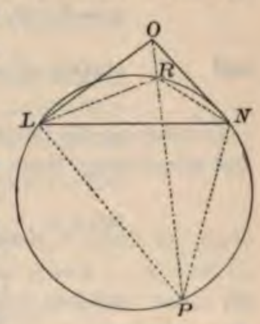


radius LM, describe the circumference OLP. The angle LPO will then be contained in the segment LPO, and the point P must be somewhere in the circumference OLP. In like manner, by means of the angle OPN, find another circumference ONP, in which the point P must be situated. The intersection of these circumferences indicates its position.

The angle at the circumference being half that at the centre, the angle LMO, subtended by the same chord as LPO, will be 2 LPO, and the angles OLM and LOM being equal and together the supplement of LMO, each angle will

$$= \frac{180° - 2\,LPO}{2} = 90° - LPO.$$

Otherwise. Construct an angle NLR equal to OPN; also LNR equal to OPL, and describe a circumference through the points L, R, and N. The point P must lie in the circumference, and also in the line drawn from O through R. Their point of intersection therefore will indicate its position.



The student may give the reason.

196. *By Calculation.* Pass a circle through the points L, N, P, and join L and N with R, thus forming a triangle in which the angles RLN and RNL are equal, respectively, to the observed angles RPN and RPL, and these, with the known side LN, furnish data sufficient to compute the sides LR and RN. Next calculate the angle ONL, whence, by subtraction, the angle ONR is found. Now, in the triangle NOR there are given two sides and the included angle to find NOR and ORN, or its supplement PRN, and by means of the sine proportion and the triangles PON and POL the distances PN, PO, and PL may be obtained.

Otherwise. After finding the angle O, obtain an expression for either OLP or ONP, and then, by the sine proportion, the required distances.

Denote the angle OLP by ϕ, ONP by ψ, and the other parts as before; then

$$\sin\beta : \sin\phi = n : OP, \text{ or } OP = \frac{n \sin\phi}{\sin\beta};$$

$$\sin\alpha : \sin\psi = l : OP, \text{ or } OP = \frac{l \sin\psi}{\sin\alpha}.$$

Whence $$\frac{n \sin\phi}{\sin\beta} = \frac{l \sin\psi}{\sin\alpha};$$

and $$\sin\phi = \frac{l \sin\psi \sin\beta}{n \sin\alpha}.$$

Again, $\phi = 360 - \alpha - \beta - O - \psi$;

or, putting $360 - \alpha - \beta - O = \theta$,

$$\phi = \theta - \psi, \text{ in which } \theta \text{ is known;}$$

and
$$\sin(\theta - \psi) = \frac{l \sin\psi \sin\beta}{n \sin\alpha}.$$

Developing the left-hand member, dividing through by $\cos\psi$, and simplifying, there results

$$\tan\psi = \frac{n \sin\alpha \cos\theta \, *}{l \sin\beta + n \sin\alpha \cos\theta};$$

or,
$$\cot\psi = \frac{l \sin\beta}{n \sin\alpha \sin\theta} + \cot\theta.$$

There are therefore but three steps in the solution:

1. Calculate the angle O, and thence obtain θ.
2. Find $\tan\psi$, or $\cot\psi$.
3. By sine proportion, calculate PN, PO, and PL.

In the example given, since the sides are in the proportion $5:6:8$, the angle O may be readily found from the well-known formula for the cosine of an angle,

$$\cos O = \frac{25 + 36 - 64}{60} = -.05 = 92° \ 52',$$

and
$$\theta = 213° \ 38';$$

whence
$$\psi = 109° \ 53',$$
$$\phi = 103° \ 45'.$$

$\sin\ 23°\ 40'$	Ar. co. $= 0.396406$
$: \sin 109°\ 53'$	$= 9.973307$
$:: 3000$	$= 3.477121$
$: PO = 7028$	$= 3.846834$

* Regard must be given to the signs of the trigonometrical functions.

$\sin 23° 40'$	Ar. co. $= 0.396406$
$: \sin 46° 27'$	$= 9.860202$
$:: 3000$	$= 3.477121$
$: PN = 5417$	$= 3.733729$

$\sin 29° 50'$	Ar. co. $= 0.303225$
$: \sin 46° 25'$	$= 9.859962$
$:: 3600$	$= 3.556303$
$: PL = 5242$	$= 3.719490$

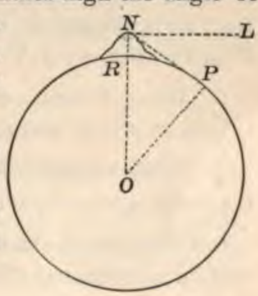
If the supplement of the observed angles at P equals the angle at O, the circle will pass through the three points L, N, and O, and P may be *anywhere* on the circumference, and hence its distance is indeterminate by the first method given above; and, substituting in the formula the proper values to find $\cot \psi$ by the second method, the numerator of the fraction will become infinite, as also the $\cot \theta$; hence, such an observation will fail in both cases to locate the point P.

EXAMPLE.

Suppose $LN = 960$ rods, NO 576 rods, LO 640 rods, the angle $LPO = 19°$, and $NPO = 25°$. Find the distances PO, PN, and PL.

Ans. $PL = 758$ rods; $PO = 1310$ rods; $PN = 1350$ rods.

197. From the top of a mountain m miles high the angle of depression of a line tangent to the earth's surface is α degrees; it is required thence to find an expression for the radius of the earth, assuming it to be a sphere.



Let O represent the centre of the earth; N the mountain top; P the point of tangency; OP and OR radii of the earth; RN the height of mountain and prolongation of OR.

Draw NL perpendicular to ON, and denote the radius of the earth by r; then, since NL and NP are respectively perpendicular to NO and OP, the angle $NOP =$ the angle of depression $LNP = a$.

Hence $$(r + m) \cos a = r.$$

$$\therefore r = \frac{m \cos a}{1 - \cos a}. \quad Ans.$$

MISCELLANEOUS PROBLEMS.

1. Determine the height of a hill, knowing that the angle of elevation of its top from a certain station $= 50°$, and at a station 800 feet more remote the angle of elevation $= 36° 20'$.

2. The angle of depression, taken from a balloon to a station whose horizontal distance is known $= 18° 40'$. Find the height of the balloon.

3. Two war vessels, desiring to ascertain their distances from a fort, remove from each other 2000 feet, and measure the angle between each other and the fort; the angles being $79° 40'$ and $82° 20'$, what were their distances?

4. Two observers on the same horizontal plane, 1500 feet apart, and in a vertical plane with a balloon, observe its angles of elevation to be $62° 40'$ and $71° 10'$. Required the height of the balloon.

5. The passage between two objects L and N being obstructed by a swamp, the lines $LP = 420$ feet, and $PN = 540$ feet, were measured, and the angle LPN observed $= 86° 42'$. Find the distance LN.

6. What distance can a person whose eye is $5\frac{1}{2}$ feet above the ocean see its surface? Assume radius $= 3960$ miles.

7. If the sun subtend an angle of $32° 2'$, and his distance from the earth is 93,000,000 miles, what is his diameter?

8. What is the altitude of the sun when the shadow of a staff cast on a horizontal plane is to the height of the staff as 7 to 5?

9. If the horizontal parallax * of the moon be $56' \, 50''$ and the diameter of the earth 7920 miles, what is the distance of the moon from the earth?

10. If the moon subtend an angle of $31' \, 14''$, when its distance is 240,000 miles, what is its diameter?

11. When the meridian altitude of the sun is $50°$, the shadow cast by the peak of a mountain reaches a certain point on a horizontal plain; but when his mid-day altitude is $60°$, the shadow strikes a point 2000 feet nearer the base of the mountain. Determine the height of the mountain above the plain.

Queries. If on the same day two observations were made on the sun for altitude, one or both when he was not on the meridian, and the length of the shadow measured as in Ex. 11, would sufficient data be thus obtained to determine the height of the mountain?

Would it be possible with data obtained, as in the first query, to ascertain the height of the mountain if the sun was vertical over the mountain at noon?

12. If the height of a mountain is m miles and its top is visible d miles, find an expression for the diameter of the earth, assuming it to be a sphere.

13. The angle of depression taken on the top of Peak of Teneriffe, which is two and a half miles high, to the farthest visible point was $2° \, 2'$. It is required to determine the observed distance and the diameter of the earth, assuming it to be a sphere. Dist., 140,876 miles; Diam., 7936 miles. *Ans.*

EXERCISES.

1. Measure the height of a flagstaff or church spire above the street.

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* The angle at the moon, or other heavenly body, subtended by the semi-diameter of the earth.

2. Measure the height of a monument, tower, or some other prominent building upon a hill, without obtaining the distance to the foot of the object. Also, if practicable, measure the distance to the foot of the object and the proper angles. Compute and compare results with each other, and with the actual height, if it can be ascertained.

SECTION V.

RECORDING THE FIELD NOTES.

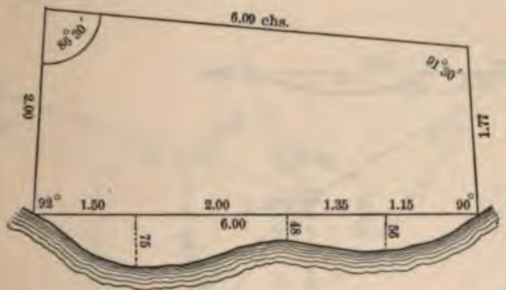
198. The Field Notes may be recorded in various ways, depending upon the instrument used, and the extent and intricacy of the survey.

First Method. If the compass is employed, the bearings simply to be taken, distances measured, and the tract bounded by straight lines (no offsets), the simplest, most compact, and also most convenient form for the subsequent calculation of the area is to write the stations, bearings, and distances in three columns, thus:

STATIONS.	BEARINGS.	DISTANCES.	REMARKS.
1	S. 21° 53′ E.	13.11	To a maple.
2	N. 48° 12′ E.	13.70	" birch.
3	N. 43° 40′ W.	4.73	" stake and stones.
4	N. 45° 08′ W.	4.75	" white oak.
5	S. 51½° W.	2.53	" sandstone.
6	S. 72½° W.	6.53	" red oak, beginning.

199. *Second Method.* If the tract is not large, and there are offsets in addition to the bearings and distances, or if simply the angles and distances are measured, a very good method, especially for a beginner, is to make a rough plat of the survey,

and indicate in their corresponding places on the sketch the bearings, or angles, and the lengths of the lines and offsets, as shown below:



The above is a sketch of a small field, showing offsets to stream, etc. The following are hasty surveys of boundaries, etc., of land for proposed park in City of Wilmington, Del., July, August, and September, 1885:

Instruments: Transit. Chesterman's 100-foot steel tape.

Work:

Lines run with transit, and carefully measured with steel tape from station to station.

Angles between these lines taken, always from left to right.

Magnetic bearings of lines taken.

Stations numbered or lettered in regular order.

Offsets (sometimes angles and distances) taken to locate houses, corners of fences, etc., offsets made at right angles with lines joining stations.

Notes:

Taken free-hand in small note-books (size $5\frac{3}{4}'' \times 3\frac{1}{2}''$).

Sketches made to suit the page and to make the matter clear for plotting.

The usual checks used on field and office work.

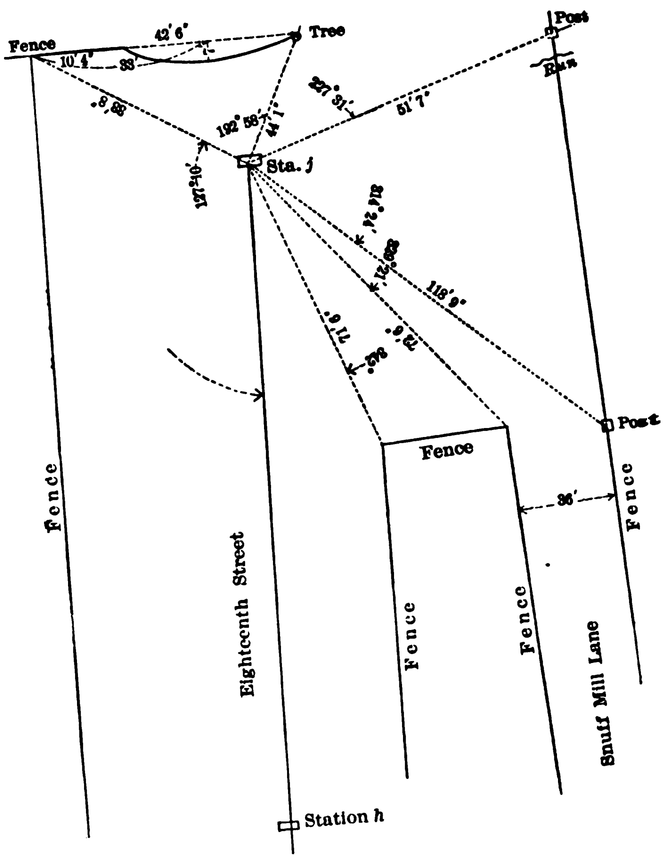
Explanation of Sketches:

No. 1. Single page of note-book. Location of fences on boundary of land proposed for park.

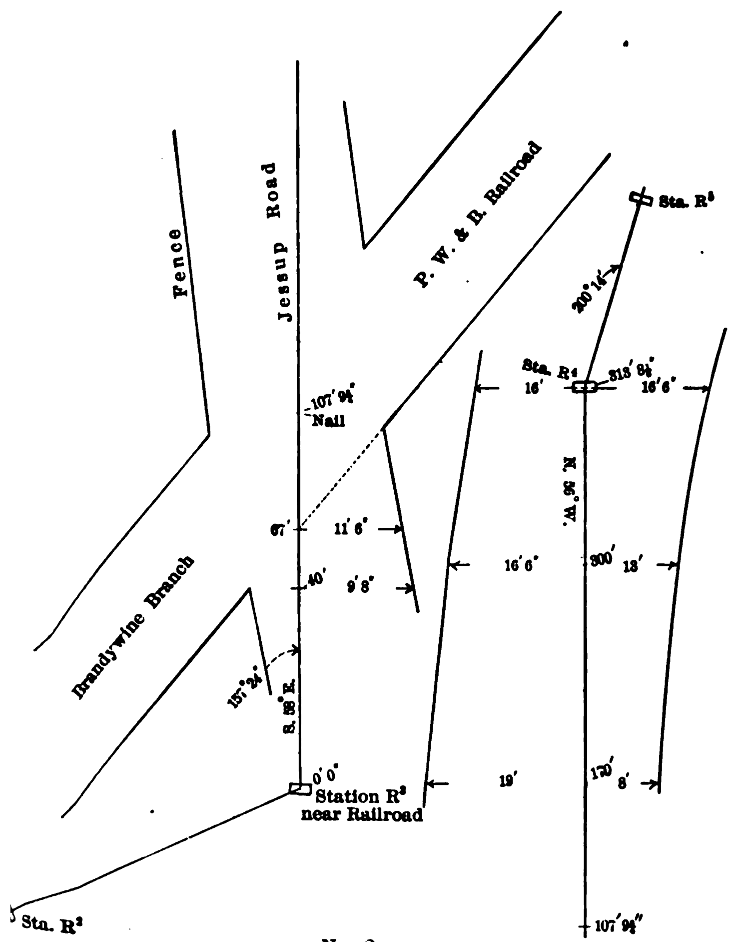
No. 2. Two opposite pages of note-book. Location of road through land proposed for park, showing railroad crossing.

No. 3. Two opposite pages of note-book. Location of run between two adjoining owners of land proposed for park.

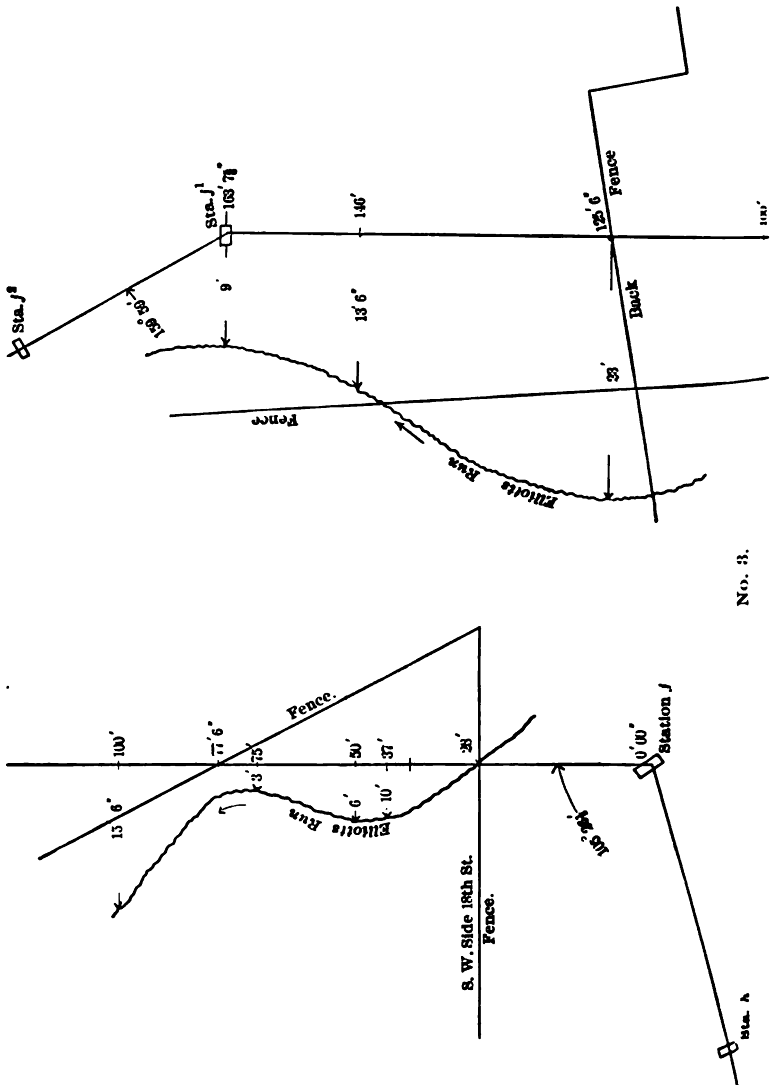
No. 4. Two opposite pages of note-book. Location of houses, etc., in land proposed for park.


Fence
42' 6"
Tree
Post
10' 4"
33'
Run
8' 88
227° 31'
51' 7"
192° 58'
44' 1"
Sta. j
127° 10'
118' 9"
Post
Fence
36'
Fence
Fence
Eighteenth Street
Fence
Fence
Snuff Mill Lane
Station h

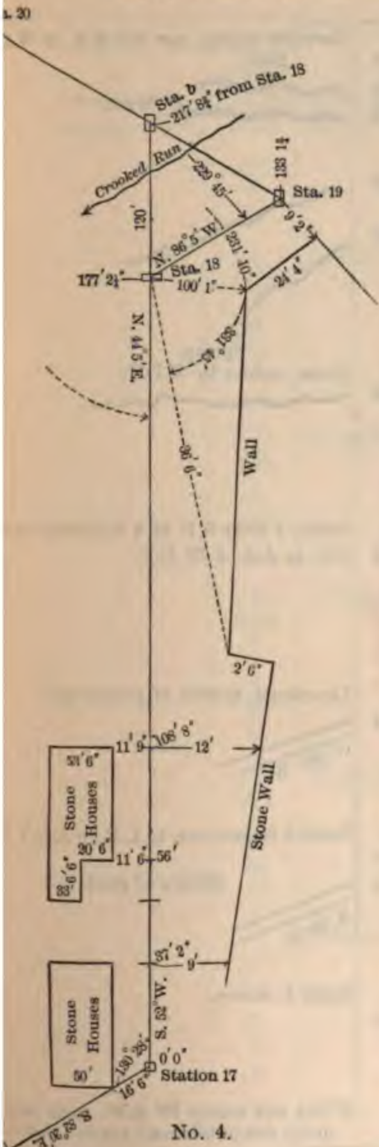
No. 1.


Fence
Jessup Road
P. W. & B. Railroad
Brandywine Branch
Sta. R⁵
Sta. R⁴
Station R³
near Railroad
Sta. R²
Nail
N. 36° W.
S. 33° E.

No. 2.


Sta. j²
Sta. j¹
Fence
Back
Fence
Elliotts Run
Fence.
Elliotts Run
S. W. Side 18th St.
Fence.
Station j
Sta. h

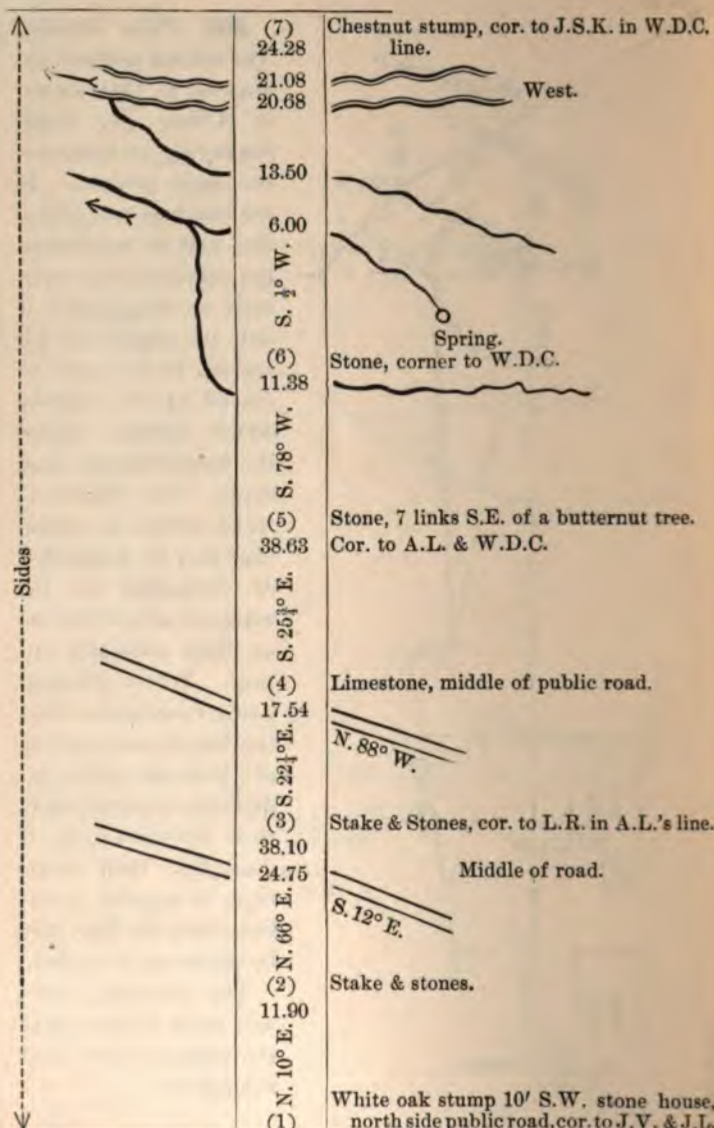
No. 3.



No. 4.

200. *Third Method.* The column method, analogous to that shown in Article 38, Chain Surveying, is, however, the most general. If the bearings are taken, they may be inserted in the column either vertically or diagonally; if only the angles are observed, they should be placed at the stations which indicate where the measurements were made. The objects to which offsets are measured may be designated or delineated on the marginal side of the line as they naturally appear. Where streams, roads, fences, etc., cross the line, representations of them are made, indicating approximately their direction; or, if desirable, their bearings, or angular deviations from the line, may be taken and recorded.

The following notes will more fully explain the method under consideration:

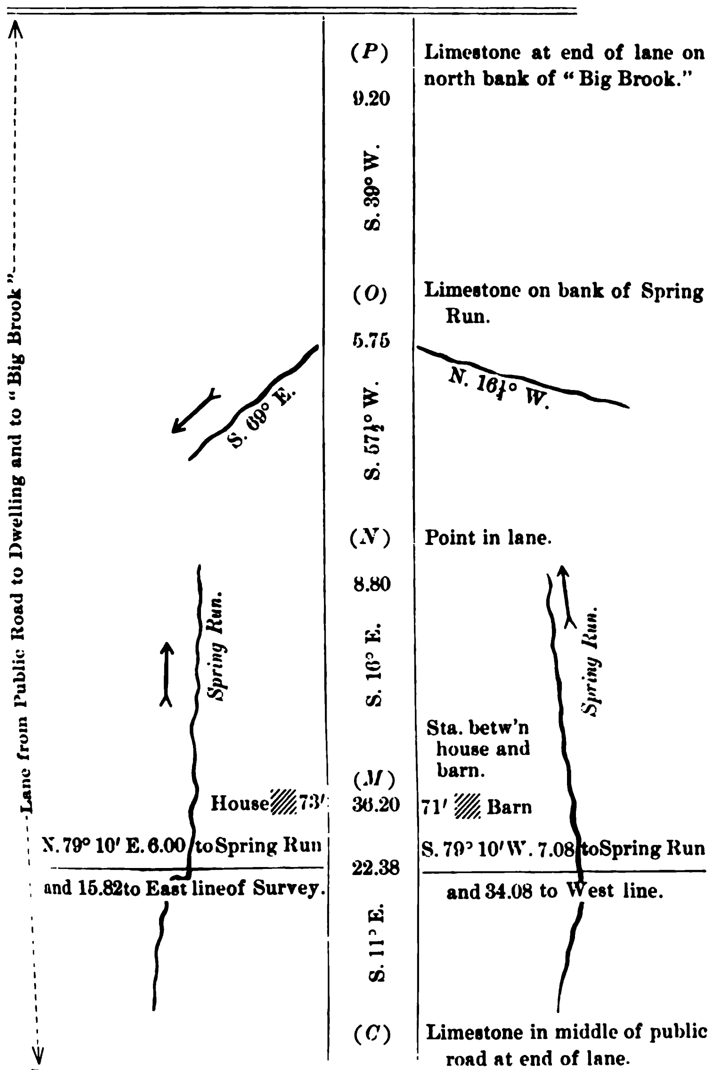

Sides
(7) 24.28
Chestnut stump, cor. to J.S.K. in W.D.C. line.
21.08
20.68
West.
13.50
6.00
S. ½° W.
Spring.
(6) 11.38
Stone, corner to W.D.C.
S. 78° W.
(5) 38.63
Stone, 7 links S.E. of a butternut tree. Cor. to A.L. & W.D.C.
S. 25¼° E.
(4) 17.54
Limestone, middle of public road.
N. 88° W.
S. 22¼° E.
(3) 38.10
Stake & Stones, cor. to L.R. in A.L.'s line.
24.75
Middle of road.
S. 12° E.
N. 60° E.
(2) 11.90
Stake & stones.
N. 10° E.
(1)
White oak stump 10′ S.W. stone house, north side public road, cor. to J.V. & J.L.

(1)	
53.39	
14.52	
14.12	East.
N. ¼° E.	
(12)	**Limestone, cor. to W.V. in C.C.'s line.**
10.47	
S. 89¼° W.	
(11)	**Limestone.**
8.00	
N. 1¼° E.	
(10)	**Limestone, cor. to J.S.K. in C.C.'s line.**
30.60	
S. 89¾° W.	
(9)	**Limestone.**
8.31	
S. ¼° W.	
(8)	**Limestone.**
4.45	
N. 88¼° W.	
(7)	

Sides

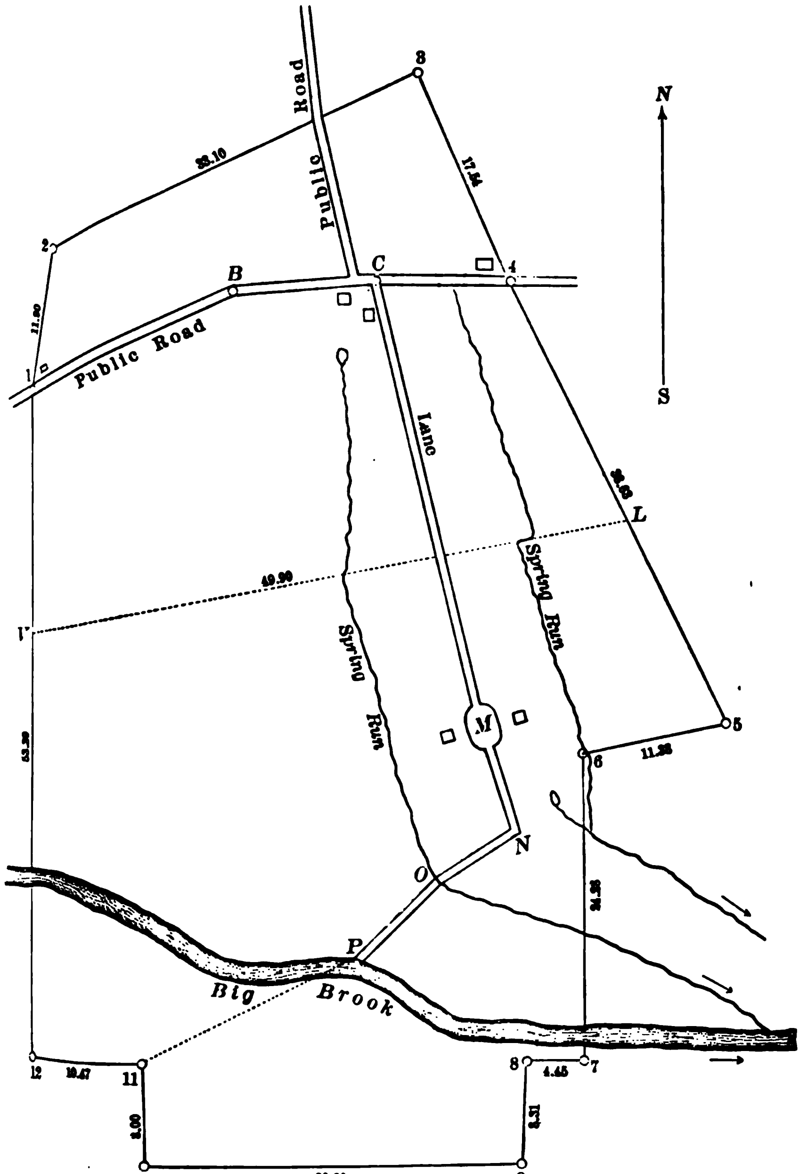
	(4)	Limestone. Sta. (4) in foregoing description.
	10.49	
Stone house 100′ from line.	9.00	
	S. 88° E.	
	(*C*)	Limestone. Lane leading to dwelling, S. 11° E.
N. 12° W.	10.41	
Road	9.00	Barn.
Public Road	6.40	House 60′ from line.
	N. 85¾° E.	
	(*B*)	
	20.38	
	N. 65¼° E.	
Stone house near corner.	(1)	White oak stump. Sta. (1) in foregoing description.

The bearing and distance of proof-line from *P* to Station (11) = S. 62½° W. 19.10.



The notes show that the sides of the tract were first surveyed; which, with their bearings and distances, include also the location and general direction of road-crossings, streams, etc., a description of the corners, and the names of owners of property adjoining the survey. Next to traversing the bounding lines, the survey of the public road, crossing the farm from east to west, was made. This road enters the tract at station (1); at 6.40 chains from (B) it passes a house which is 60 feet to the right; at 9.00 chains a road to the left, the bearing of which is given; at 10.41 chains is a corner at end of lane leading to dwelling; near the east end of road a stone house is located, at 100 feet north of the line; and at 10.49, station (4) of sides survey is reached, at which point the road leaves the farm. The survey of the lane to the dwelling, and thence to the creek, is next recorded. Here are noted the intersection of a line S. 79° 10′ W., and the distances on this, east and west, to spring runs, as well as the distances to the east and west sides of the tract;* the dwelling and barn are located, and the limestone on the north bank of Big Brook reached. A line was run from this last point to station (11), which, in connection with the survey of the lanes, the public road, and the cross-line from L to V, gave proof of the accuracy of the work.

* This line was made a boundary in the subsequent division of the land.


Public Road
Public Road
N
S
B
C
L
M
N
O
P
Lane
Spring Run
Spring Run
Big Brook
38.10
17.54
11.90
49.90
11.38
30.80
4.45
10.47
1
2
3
4
5
6
7
8
9
10
11
12

SECTION VI.

LATITUDES AND DEPARTURES.

201. The Difference of Latitude of the two ends of a line is the perpendicular distance between the parallels of latitude which pass through them, and is reckoned north or south, according as the bearing is northerly or southerly.

The Difference of Longitude of the two ends of a line is the perpendicular distance between the meridians which pass through them, and is reckoned east or west, according as the bearing is easterly or westerly.

The difference of latitude of a line is often called briefly the *latitude*, or northing or southing; and the difference of departure, its *departure*, or easting or westing.

It will be perceived from the definitions just given that, when a line bears either due north or south, the distance equals the latitude, and the departure is nothing; but if the bearing is east or west, the distance and departure are equal, and the latitude is zero. Furthermore, it will be seen that in all other cases except those just cited, the latitude, departure, and distance form the three sides of a right triangle: the distance being the hypotenuse, and the latitude and departure the sides about the right angle.

Let LP represent a line given by its bearing and distance; it is required to determine its latitude and departure.

Let OL and PM represent parallels of latitude, and LM and OP meridians. The lengths of $LM = OP$ and $LO = MP$ are required.

The problem stated simply is: Given in a right triangle LMP the hypotenuse LP (distance), the angle L (bearing), to find the side LM (latitude), and MP (departure).

From Trigonometry, $LM = LP \cos L$,

$$MP = LP \sin L.$$

So it is seen that the latitude of a line is obtained by taking the product of the distance and the *cosine* of the bearing, and the departure is equal to the product of the distance and *sine* of the bearing.

202. The case just treated is the principal one which the surveyor will use, since it is necessary — as will subsequently be seen — in computing areas, to determine the latitudes and departures; and by these formulas he will generally obtain them, having taken in the field the bearings, or angles, and distances.

Other cases, however, will occur in practice referring to the triangle LMP, and for convenience they are here subjoined.

Designating the length of the line, or distance, by s, the bearing by b, the latitude and departure respectively by l and d, then we may write the following formulas:

Case.	Given.	Required.	Formulas.	
1	b, s.	l, d.	$l = s \cos b$,	$d = s \sin b$.
2	b, l.	s, d.	$s = \dfrac{l}{\cos b} = l \sec b$,	$d = l \tan b$.
3	b, d.	s, l.	$s = \dfrac{d}{\sin b}$,	$l = \dfrac{d}{\tan b} = d \cot b$.
4	s, l.	b, d.	$\cos b = \dfrac{l}{s}$,	$d = \sqrt{s^2 - l^2}$.
5	s, d.	b, l.	$\sin b = \dfrac{d}{s}$,	$l = \sqrt{s^2 - d^2}$.
6	l, d.	b, s.	$\tan b = \dfrac{d}{l}$,	$s = \sqrt{l^2 + d^2}$.

EXAMPLES.

1. Given the bearing and distance of a line, N. 23° 54′ W. 18.25 chains; required its latitude and departure.

2. Given the bearing of a line N. 87° 40′ E., and the departure 2640 feet; find its distance and latitude.

3. Given the length of a line 24.60 chains, and the departure 17.40; find its bearing and latitude.

4. Given the latitude 23.76 chains south, and the departure 0.94 chains west; required the bearing and distance.

5. Given the distance 1886 feet, and the latitude 943; determine its bearing and departure.

6. It is required to find the distance and departure of a line, given the bearing S. 30′ W., and latitude 10.80 chains.

203. The Traverse Table. By the use of Formula 1, last article, latitudes and departures have been calculated for every quarter-degree of the quadrant, corresponding to distances from 1 to 10, and even from 1 to 100; these results tabulated constitute the *traverse table.* Such a table was considered quite indispensable when the compass was the principal surveying instrument, but since the more accurate transit has to a great extent superceded the compass, and surveyors are now reading to *minutes* instead of *quarter-degrees*, the common traverse table reading only to quarter-degrees is of little practical value.

When, therefore, the bearings are read to minutes, the latitudes and departures are generally best obtained from a table of natural sines and cosines.*

However, for the benefit of those engaged in compass surveying, and for those who, though reading to minutes, prefer to obtain by interpolation the latitudes and departures from the traverse table, one is given near the end of this volume.

* A traverse table in which the calculations are made to every *minute* of bearing for distances from 1 to 10 and extending to five decimal places, would answer the purpose admirably. Such a table is in existence, but it is not common. The common tables of natural sines and cosines are simply tables of latitudes and departures corresponding to a unit's distance. With a distance 2, the latitude and departure are twice those in the table; when the distance is 3, three times; when n, n times.

Explanation of the Traverse Table. The number of degrees in the bearing if it does not exceed 45 is found in the left-hand column of the page, and the latitudes and departures, as indicated at the top, may be taken under the proper distance; if the number of degrees is greater than 45, it is found in the right-hand column of the page, and the columns of latitudes and departures are indicated at the bottom. For example:

1. Let it be required to find the latitude and departure corresponding to a bearing N. 34° 30′ E. and distance 5 chains.

We find in the table, opposite 34° 30′ and under "distance 5," in the column headed "Lat.," 4.121, and in the column headed "Dep.," 2.832. Hence the latitude and departure are respectively 4.12 N. and 2.83 E.

2. Required the latitude and departure of a line bearing N. 72¼° W. 9 chains.

Looking in the column at the right of the page for 72° 15′, and under "distance 9," we find, reading at bottom,

in the Lat. column, 2.744;
in the Dep. column, 8.572.

Hence the latitude is 2.74 chains N., and the departure 8.57 chains W.

204. The table may be used to find the latitude and departure for any distance however great. If, in first example above, we suppose the bearing to remain the same, but the distance to be 50 chains; then, since for the same bearing the latitudes and departures vary directly as the distances, the latitude, or departure, for 50 chains is 10 times that for 5; and, as multiplying by 10 is in effect removing the decimal point one place to the right, we may take directly from the table opposite 5 the latitude and departure of 50, or 41.21 N. and 28.32 E.

If the distance is not a multiple of 10, but made up of units and tens, we may take out of the table the latitude and departure for the *units*, and for the *tens* as indicated above. The sum of these will evidently be the latitude and departure required.

3. Let it be required to find the latitude and departure of a line S. 40° E. 34 chains.

Looking in the table opposite 40° and under "distance 3," take out at once, by conceiving the decimal point removed one place to the right.

	For 30 chains,	Lat. 22.98	Dep. 19.28
Then	" 4 "	" 3.06	" 2.57
	34 chains,	Lat. 26.04 S.	Dep. 21.85 E.

By an extension of the above principle, the table may be used to obtain the latitude and departure when the distance is composed of chains and links.

4. Given the bearing of a line S. 28° 45′ W. 26.58 chains, to find its latitude and departure.

For 20 chains,	Lat. = 17.53	Dep. = 9.62
6 "	" = 5.26	" = 2.89
.5 "	" = .44	" = .24
.08 "	" = .07	" = .04
26.58 chains,	Lat. = 23.30 S.	Dep. = 12.79 W.

5. Find by the traverse table the latitude and departure of a line bearing N. 41° 45′ E. 17.29 chains.

6. Given the bearing of a line S. ½° W., distance 23.48 chains, to find its latitude and departure.

7. What are the latitude and departure of a line bearing S. 85° 30′ E. 135.42 chains?

8. If the bearing and distance are N. 89¾° W. 20.09 chains, what are the latitude and departure?

205. By means of interpolation the traverse table may be used to find the latitude and departure when the bearing is given to minutes. Thus, the bearing being N. 34° 20′ E. any given distance, take out the latitude and departure corre-

sponding to 34° 15′ and the given distance, and *add* * to that *departure* $\frac{5}{15}$, or $\frac{1}{3}$, of the difference between it and that corresponding to 34° 30′ and the given distance, for the departure required. Likewise obtain $\frac{5}{15}$ of the difference between the *latitudes* corresponding to 34° 15′ and 34° 30′ and the distance, and *subtract* * from the latitude first found for the latitude required.

For a bearing 34° 23′, the fractional part to be taken of the difference between 34° 15′ and 34° 30′ would be $\frac{8}{15}$; the numerator being the excess in minutes above the quarter, and the denominator 15.

206. In the absence of a traverse table calculated to minutes, the table of natural sines and cosines, as before stated, is the best to use when the bearings are given to minutes.

It is shown in Article 201 that the cosine of the bearing multiplied by the distance gives the latitude, and the product of the distance and sine of bearing gives the departure.

EXAMPLES.

1. The bearing and distance of a line are N. 37° 43′ W. 24.29 chains; required its latitude and departure.

Four places of decimals from the table will usually be sufficient.

The cosine of 37° 43′ true to four places = .7911.
The sine of 37° 43′ true to four places = .6118.

$$.7911 \times 24.29 = 19.21 \text{ N. Lat.}$$
$$.6118 \times 24.29 = 14.86 \text{ W. Dep.}$$

The following contracted form of multiplication, using five decimal places, gives practically the same result:

Cosine of bearing = .79105; sine of bearing = .61176.

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* The departure increases with an increase of the bearing; the latitude diminishes.

Distances	20 chains,	Lat. = 15.8210	Dep. = 12.2352
	4 "	" = 3.1642	" = 2.4470
	.2 "	" = .1582	" = .1224
	.09 "	" = .0712	" = .0551
	24.29 chains,	Lat. = 19.21 N.	Dep. = 14.86 W.

2. Find the latitude and departure of a line bearing S. 62° 17′ E. 37.18 chains.

3. Required the latitude and departure of a line N. 88° 57′ W. 28.97 chains.

4. Required the latitude and departure of a line bearing S. ½° E. 2640 feet.

5. Given the bearings and distances of two lines running from the same point *P*, as follows: *PO*, N. 38° 37′ E. 1760 feet, and *PL*, N. 71° 54′ E. 1320 feet; to find by means of latitudes and departures the distance *OL*.

6. Assuming *PO* bears N. 48° 17′ W. 27.42 chains, and *PL* S. 36° 28′ W. 19.24 chains, find, as in the last example, the distance *OL* between the extremities of the lines.

207. Testing a Survey. It is evident that when a surveyor has passed completely round a tract of land and returned to the place of beginning, he has gone in a northerly direction just as far as he has gone in a southerly direction, and as far easterly as westerly. Hence the sum of the north latitudes should equal the sum of the south latitudes, and the sum of the east departures equal the sum of the west departures.*

In practice, this degree of accuracy is seldom attained, for various causes incident to the manipulation of the instruments, their inherent defects, imperfect chaining, etc.

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* If the survey is effected by traversing (Article 163), the reading at the last station should be 360° or 0°. If the interior angles are measured, their sum should equal twice as many right angles less four as the figure has sides. If a small error exists, it must be distributed evenly among the angles, unless on account of the difficulty of observing one or more of the angles, these should have a larger share of the error. See, also, Article 156.

On account of the varying conditions in different surveys, it is impracticable to state precisely how great an error should be allowed without a re-survey of the tract. A rule usually followed by compass surveyors is to allow an error of 1 link for every 5 chains, 1 : 500.

This is perhaps a fair average for ordinary farm surveying. If the ground is exceptionally clear, and quite level, an error of 1 : 1000 is not too great; if, on the other hand, the ground is uneven, rocky, and brushy, 1 : 300, or even 1 : 200, might be allowed. The error resulting from a transit survey of the same ground should be much less. For the average case given above, instead of 1 : 500 it should not be much less than 1 : 1200.

The above rules are cited simply as guides to the young surveyor to aid him in forming a standard for himself, based on his own experience.

208. Correcting Latitudes and Departures, or Balancing the Survey. (1) A survey is *balanced* when the northings equal the southings, and the eastings equal the westings. When these equalities do not exist, the *error* is distributed among the lines, proportioned to their lengths. This operation is called *correcting the latitudes and departures*. It is best illustrated by an example:

Stations.	Bearings.	Dists.	Latitudes.		Departures.		Corrections.		Corrected Latitudes.		Corrected Depart's.	
			N.	*S.*	*E.*	*W.*	*Lat.*	*Dep.*	*N.*	*S.*	*E.*	*W.*
1	S. 20° 53' E.	13.11	. . .	12.25	4.67	. . .	2	1	. . .	12.27	4.68	. . .
2	N. 48° 10' E.	13.62	9.08	. . .	10.15	. . .	2	1	9.06	. . .	10.16	. . .
3	N. 43° 40' W.	4.73	3.42	. . .	. . .	3.26	1	. .	3.41	. . .	. . .	3.26
4	N. 45° 08' W.	4.75	3.35	. . .	. . .	3.36	1	1	3.34	. . .	. . .	3.35
5	S. 51° 30' W.	2.53	. . .	1.57	. . .	1.98	. .	. .	. . .	1.57	. . .	1.98
6	S. 72° 30' W.	6.56	. . .	1.96	. . .	6.26	1	1	. . .	1.97	. . .	6.25
		45.30	15.85	15.78	14.82	14.86	7	4	15.81	15.81	14.84	14.84
			15.78			14.82						
Error in latitude, 7 links.						4 links, Error in departure.						

In the table the latitudes and departures corresponding to the several bearings and distances are obtained by means of a table of sines and cosines, and placed in their proper columns.

The first course being between the south and east, the latitude found is written in the column headed *S.*, the departure in column *E.*, and so on, the letters of the course indicating the columns in which to place the latitudes and departures. The difference of the sums in the latitude columns is then taken, and found to be 7 links: this is the error in latitude.

The error in departure, found in a corresponding manner, is 4 links.

The total distance round the field is shown by the footing of the distance column to be 45.30 chains. The distribution of the error is effected then by the proportions:

For the Latitude.

Sum of the sides	:	length of any side	= error	:	correction for that side.
45.30	:	13.11	= 7	:	2
45.30	:	13.62	= 7	:	2

For the Departure.

45.30	:	13.11	= 4	:	1

It is unnecessary usually to make but one proportion each for the latitude and departure correction, since the error for any other side may be found mentally by comparing its length with that of the side used in the proportion. Whole links only are used. The latitude correction for the second side is a little greater than 2, but it is nearer 2 than 3, and is therefore called 2.

The corrections thus found are written in their proper columns, headed "Correction, Lat. Dep.," opposite the sides to which they refer, and are so applied by addition or subtraction as may be required to reduce the errors to zero. The quantities thus obtained are placed in the columns of corrected latitudes and departures to the right of the corrections.

Since the southings are too small, the correction 2 is added to 12.25, making 12.27, for the first entry in the column of corrected latitudes. The eastings being too small, the correction 1 is added to 4.67, making 4.68, to be written under *E.* in the corrected departures; and so on for the rest.

If the corrections have been properly applied, the northings will equal the southings, and the eastings the westings, and the survey is *balanced.*

In the example just given, the difference of latitude is 7 and the departure 4 links; hence, the length of a line to *close* the survey $= \sqrt{7^2 + 4^2} =$ about 8 links; and as the perimeter of the tract $= 45.30$ chains, the "error of the survey," or "error of closure," $= 1$ link for 5.66 chains, or 1 : 566.

Some surveyors prefer a more compact table than that given above, and instead of a double set of latitudes and departures, use but one, and write in ink of different colors the *corrected* latitudes and departures over the first. Others, again, prefer *two* columns instead of *four* for the latitudes and departures, using the plus (+) sign to indicate north latitudes and east departures, and the minus (−) sign to indicate south latitudes and west departures.

The form given above is, however, preferable to either, since a mistake in the application of the corrections is in that more easily detected, the footings are more expeditiously and accurately obtained, and the subsequent part of the work referring to the area is thereby facilitated.

If a side of the survey passes over very rough ground, or through a dense wood, or for any reason it is rendered more difficult to measure than any of the others, the surveyor should exercise his judgment in deciding how much more of the error than the rule would indicate should be applied to that side.

Regard must also be had to the probability of error in the bearings; hence, when a side of considerable length is aligned through a thicket, or over very uneven ground, and where oftentimes the observations are made to top of rod, if it is found that a slight change in the bearing will diminish materially the error, the change should be made.

The diurnal variation of the needle is not unfrequently a source of error in compass surveys. A range of 10 minutes is quite common, and even 15 minutes is occasionally noted. This error may be avoided by measuring the *angles* of the tract, or testing the compass every two or three hours by setting up and sighting on some line as standard.

Some authors and surveyors affirm that when the bearing of a line is due east or due west, the error in latitude is nothing, and therefore such a line needs no correction. Likewise a due north and south line has no error in departure. The writer does not concur in this view; for the errors in compass work are not confined to the chaining, and in transit surveying there is frequently considerable error in the angles. In the application of the rule these facts are assumed; indeed, as soon as a correction, made in the usual manner, is applied to any side, a change of bearing results, for the corrected latitudes and departures no longer belong to the original bearing, but to some other. Moreover, there is no more reason for supposing a line runs due *north* because it is so read than that a line runs N. $\frac{1}{4}°$ E. or N. $89\frac{3}{4}°$ E. being so read; yet no surveyor would hesitate to apply the rule to either of these, thus assuming that an error in bearing as well as in chaining was committed; and this is the correct assumption on which, without excepting any side, the distribution of the error, except as follows, should be made.

(2) If, however, a survey is made with a transit in good adjustment, the angles, either interior or deflection, being carefully observed, and the test hereinbefore mentioned when applied giving the inference that the angles were accurately measured, and the error of closure therefore due to erroneous chaining, then the correction which should be applied is obtained as follows:

Add up the columns of latitudes, and also those of departures, and say, as the arithmetical sum of all the $\left\{\begin{matrix}\textit{latitudes}\\ \textit{departures}\end{matrix}\right\}$ *is to any particular* $\left\{\begin{matrix}\textit{latitude}\\ \textit{departure}\end{matrix}\right\}$, *so is the error in* $\left\{\begin{matrix}\textit{latitude}\\ \textit{departure}\end{matrix}\right\}$ *to the correction to be applied to that* $\left\{\begin{matrix}\textit{latitude}\\ \textit{departure}\end{matrix}\right\}$.

(3) If greater accuracy is required than can be attained by the preceding methods, each side should be weighted; that is to say, the surveyor determines the relative difficulties in measurement and alignment of the boundaries, considering some one side the standard. Calling the error probably made in the side chosen as standard one (1), another side, which in the judgment of the surveyor was, per unit, twice as difficult to measure, would be multiplied by 2, or, as it is termed, have a weight of 2; another multiplied by 3, or $1\frac{1}{2}$, etc. Then, instead of taking the perimeter for the divisor, as was done in the first case above, the sum of the sides thus multiplied or weighted is used, and the proportion is as follows:

As the sum of the multiplied distances is to any particular multiplied distance, so is the error in $\left\{\begin{array}{l}\textit{latitude}\\ \textit{departure}\end{array}\right\}$ *to the correction to be applied to that* $\left\{\begin{array}{l}\textit{latitude}\\ \textit{departure}\end{array}\right\}$.*

The following illustrates the method of balancing a survey when the sides are weighted:

STATIONS.	BEARINGS.	DISTANCES.	WEIGHTS.	MULTIPLIED DISTANCES.	LATITUDES.		DEPARTURES.		CORRECTIONS.		CORRECTED DISTANCES.		CORRECTED DEPART'S.	
					N.	*S.*	*E.*	*W.*	*Lat.*	*Dep.*	*N.*	*S.*	*E.*	*W.*
1	N. 9° W.	15.50	1	15.50	15.31	. . .	. . .	2.43	1	2	15.32	. . .	. . .	2.41
2	N. 31° E.	25.40	3	76.20	21.77	. . .	13.09	. . .	6	9	21.83	. . .	13.18	. . .
3	S. 71° E.	10.00	3	30.00	. . .	3.17	9.48	. . .	3	4	. . .	3.14	9.52	. . .
4	S. $10\frac{1}{4}$° E.	19.70	2	39.40	. . .	19.37	3.59	. . .	3	5	. . .	19.34	3.64	. . .
5	S. $10\frac{3}{4}$° W.	14.60	$1\frac{1}{2}$	21.90	. . .	14.34	. . .	2.72	2	2	. . .	14.32	. . .	2.70
6	S. 89° W.	21.25	1	21.25	. . .	0.37	. . .	21.25	2	2	. . .	0.35	. . .	21.23
				204.00	37.08	37.25	26.16	26.40			37.15	37.15	26.34	26.34
						37.08		26.16						

Error in latitude, 17 links. 24 links, error in departure.

* Weights could be applied to the correction of the chaining in the second case, by multiplying the latitudes and departures instead of the lengths of the sides.

EXAMPLES.

Correct the latitudes and departures in the following examples by the first method:

1.		2.	
(1) S. $\frac{1}{4}°$ E.	22.45 chains;	(1) South	22.45 chains;
(2) N. $89\frac{3}{4}°$ E.	67.10 "	(2) East	67.10 "
(3) N. $\frac{1}{4}°$ W.	23.85 "	(3) North	23.85 "
(4) S. $89\frac{3}{4}°$ W.	66.30 "	(4) West	66.30 "
(5) S. $21\frac{3}{4}°$ W.	1.30 "	(5) S. 22° W.	1.30 "

EXERCISES.

A few surveys should now be made, and the methods above given employed in balancing.

SECTION VII.

SUPPLYING OMISSIONS.

209. When, for any cause, it is impracticable to obtain the direction or the length, or both, of a side of a tract of land, these may be obtained by calculation. Even the lengths or bearings of *two* sides may in general be supplied.*

The determination, however, of these sides or bearings is based upon the measurements of the other bounding lines and angles; but as these are not usually *precisely correct*, and as there are no means of testing them in their application to the solution of problems under this head, it is earnestly recommended that *all measurements*, if possible, be made.

There are four cases.

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* If the two omitted sides are parallel and equal, their bearings cannot be supplied; or if they are parallel and of equal or unequal lengths, their distances cannot be computed.

Case I.

210. *Given the bearings and distances of all the sides of a tract of land except the bearing and distance of one side, to determine these.*

Find the latitudes and departures of the given sides. The difference of the northings and southings will show the latitude of the line omitted, and the difference of the eastings and westings its departure. Then

$$\text{Length of line} = \sqrt{\text{lat.}^2 + \text{dep.}^2}$$

$$\text{Tan angle of bearing of line} = \frac{\text{dep.}}{\text{lat.}}.$$

The cardinal points between which the line runs are indicated by the deficiency in the latitude and departure columns.

EXAMPLES.

1. Given (1) N. $24\frac{1}{2}°$ E. 23.75 chains;
   (2) S. $81\frac{1}{4}°$ E. 11.70 "
   (3) S. $1°$ E. 12.64 "
   (4) S. $11\frac{1}{2}°$ W. 14.50 "

To find the length and bearing of a line connecting the extremity of the fourth side with the first corner.

Stations.	Bearings.	Dists.	N.	S.	E.	W.
1	N. $24\frac{1}{2}°$ E.	23.75	21.61	. . . .	9.85	. . .
2	S. $81\frac{1}{4}°$ E.	11.70	. . .	1.78	11.56	. . .
3	S. $1°$ E.	12.64	. . .	12.64	.22	. . .
4	S. $11\frac{1}{2}°$ W.	14.50	. . .	14.21	. . .	2.89
			21.61	28.63	21.63	2.89
				21.61	2.89	
				7.02 N.	18.74 W.	

$$\text{Length of line} = \sqrt{(7.02)^2 + (18.74)^2} = 20.01 \text{ chains}.$$

$$\text{Tan bearing} = \frac{18.74}{7.02}.$$

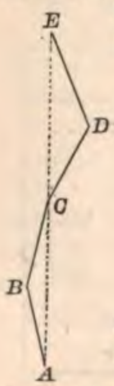
$$\text{Bearing} = \text{N. } 69^\circ\ 28'\ \text{W.}$$

2. Given the bearings and distances of the sides of a tract of land as follows; it is required to find the length and bearing of the fourth side.*

(1) N. $11\frac{3}{4}^\circ$ E. 12.69 chains;
(2) S. $87\frac{3}{4}^\circ$ W. 8.50 "
(3) N. $85\frac{1}{2}^\circ$ W. 11.70 "
(5) S. $82\frac{1}{2}^\circ$ E. 10.53 "

The foregoing case may be employed to overcome an obstacle in a line, as *LN*. Thus, surveying *LOPN*, there will be given all the sides except *LN*, which can be determined as above. If it is desired to straighten an old road, the length and direction of the new road may be computed from the distances and deflections, or bearings of the old.

For example, let *ABCDE* be a crooked road which it is desired to replace by a straight one, *AE*. The bearings and distances being as follows, the length and bearing of *AE* are required.



AB, N.	12.70	chains;
BC, N. 20° E.	13.25	"
CD, N. 35° E.	12.75	"
DE, N. 10° W.	16.90	"

Ans. N. 9° 41′ E. 52.98 chains.

EXAMPLE 2. Given the following as the bearings and distances of a road, it is desired to straighten, to find the length and bearing of the new road.

* In practice, the result should be checked by making a plot of the field.

(1) N. 12° W. 13.10 chains;
(2) N. 8° E. 16.20 "
(3) N. $2\frac{1}{2}$° W. 14.40 "
(4) N. $40\frac{1}{2}$° E. 15.08 "
(5) N. $60\frac{1}{4}$° W. 16.12 "

EXAMPLE 3. In last figure but one, suppose LO bears N. 44° 20′ W., distance 3.95 chains. Deflection at O from OL 30°, and $OP = 6.90$ chains. Deflection at P from OL 100°, and $PN = 5.40$ chains. It is required to find the length and bearing of NL. *Ans.* Bearing south. Length, 12.55 chains.

CASE II.

211. *Given the bearings and distances of all the sides of a tract of land, except the distances of two sides not parallel, to determine these.*

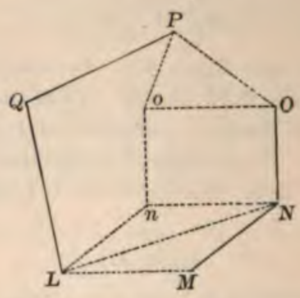
By Article 168, change all the bearings so that one of the sides, whose direction only is known, shall become a meridian. Tabulate the latitudes and departures corresponding to the changed position of the sides. The side made meridian will have no departure, and the difference of the eastings and westings, therefore, will be the departure of the other unknown side. Now with this departure and the changed bearing the distance and difference of latitude of this side may be found, and should be inserted in their proper places in the table. Then the difference between the northings and southings will be the latitude, or length of the side made a meridian.*

212. *Otherwise. If the deficient sides adjoin.*

If a line† be drawn connecting L and N, a figure, $LNOPQ$, will be shown, in which all the sides are given except LN: the bearing and distance of this side may, therefore, be calculated by the preceding case. This line and the two sides, LM

* It is immaterial whether or not the deficient sides adjoin.
† Called a *closing line* since it closes the survey $LQPON$.

and MN, whose bearings only are given, will form a triangle, in which will be known one side and all the angles, whence the unknown distances, LM and MN, may be computed.



213. *If the sides do not adjoin.*

In the figure suppose that the distances LM and PO are wanting. Draw Ln and no parallel and equal respectively to MN and NO. Then by joining oP, a closed figure will be formed, all the bearings and distances of which are known except the bearing and distance of the closing line, Po, and these may be found by Case I. Po thus determined, there will be known in the triangle PoO all the angles and one side, to find PO, and Oo, which is equal to LM.

EXAMPLES.

1. Given the following bearings and distances of the sides of a tract of land, to find the length of the 3d and 6th sides. (See last figure.)

(1)	N. $6\frac{1}{2}°$ W.	9.38 chains;
(2)	N. $65\frac{1}{2}°$ E.	8.25 "
(3)	S. 39° E.	Unknown;
(4)	S. 2° W.	4.45 chains;
(5)	S. 46° W.	5.00 "
(6)	N. 88° W.	Unknown.

STATIONS.	BEARINGS.	DISTS.	N.	S.	E.	W.
1	N. 6½° W.	9.38	9.32	...	...	1.06
2	N. 65½° E.	8.25	3.42	...	7.51	...
3	S. 39° E.	...	...	...	...	...
4	S. 2° W.	4.45	...	4.45	...	0.16
5	S. 46° W.	5.00	...	3.47	...	3.60
			12.74	7.92	7.51	4.82
			7.92		4.82	
			4.82		2.69	

Tan bearing $= \dfrac{2.69}{4.82}$, or Po bears S. 29° 10′ W.

Length of $Po = \sqrt{(4.82)^2 + (2.69)^2} = 5.52$.

Angle P therefore $= 68°\ 10'$.
Angle O " $= 49°$.
Angle o " $= 62°\ 50'$.

sin. 49°	Ar. co.	= 0.122220
: sin. 62° 50′		= 9.949235
:: 5.52		= 0.741939
: PO (3d side) = 6.51		= 0.813394

sin. 49°	Ar. co.	= 0.122220
: sin. 68° 10′		= 9.967674
:: 5.52		= 0.741939
: $Oo = LM$ (6th side) = 6.79		= 0.831833

EXAMPLE 2. Given the following data to supply the omissions.

(1) N. 8½° E. 9.80 chains;
(2) N. 31½° E. Unknown;
(3) S. 70° E. "
(4) S. 5½° W. 17.70 chains;
(5) N. 87° W. 18.75 chains, to the beginning.

EXAMPLE 3. In the last example insert the distances found, and suppose the first and fourth sides are wanting; determine these by either or both methods.

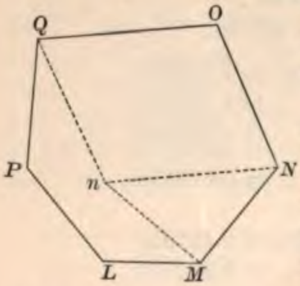
CASE III.

214. *Given the bearings and distances of all the sides of a tract of land, except the bearings of two sides, to determine these.*

Tabulate the latitudes and departures of the sides completely given; obtain the difference of the northings and southings, and of the eastings and westings. These differences will be the latitude and departure of a *closing line*.

The bearing and distance of the closing line may hence be computed; then in the triangle formed by this line and the two sides whose distances are given, determine the angles; and thence, with a proper application of them to the bearing of the closing line, the wanting bearings may be found.

In the figure let $PQONML$ represent a tract of land in which all the bearings and distances are known except the bearings of QO and NM.



Drawing nN parallel and equal to QO, and joining Qn and nM, a closed figure, $PQnMLP$, will be formed, in which the bearing and distance of nM, the closing line, may be calculated by Case I. Then in the triangle MnN, having all the sides, the angles are readily found, and by proper application of these with the bearing of Mn the bearings of NM, and $nN = QO$ may be obtained.

NOTE. — If the sides whose bearings are required adjoin, the reasoning is evident. If they do not adjoin, a transposition of some of the sides may be made, as in the preceding case, without changing the direction or length of any of them, making the unknown sides adjoin, and with the closing line form the triangle referred to in the last paragraph. The rule is, therefore, applicable to either.

EXAMPLES.

1. Given the following data of a survey, to supply the omissions. Referring to the last figure:

the bearing of *PQ*,	N. 3° E.	dist. 4.57	chains.
" " *QO*,		" 6.25	"
" " *ON*,	S. $23\frac{1}{2}$° E.	" 5.50	"
" " *NM*,		" 4.33	"
" " *ML*,	N. 87° W.	" 2.97	"
" " *LP*,	N. 43° W.	" 3.33	"

Stations.	Lines.	Bearings.	Dists.	N.	S.	E.	W.
P	*PQ*	N. 3° E.	4.57	4.56	. . .	0.24	. . .
Q	*QO*	. . . .	6.25	. . .	. . .	. . .	. . .
O	*ON*	S. $23\frac{1}{2}$° E.	5.50	. . .	5.05	2.19	. . .
N	*NM*	. . . .	4.33	. . .	. . .	. . .	. . .
M	*ML*	N. 87° W.	2.97	0.15	. . .	. . .	2.97
L	*LP*	N. 43° W.	3.33	2.43	. . .	. . .	2.28
				7.14	5.05	2.43	5.25
				5.05			2.43
			Deficiency, 2.09 S.			Deficiency, 2.82 E.	

Tan of bearing of $nM = \frac{2.82}{2.09}$, and bearing = S. 53° 28′ E.

$$\text{Dist. } nM = \sqrt{(2.09)^2 + (2.82)^2} = 3.51.$$

To find the angle of nMN:

$$\begin{array}{lrl}
\log 4.33 & \text{Ar. co.} = & 9.363512 \\
\log 3.51 & " \;\; = & 9.454693 \\
\log 7.045 & = & 0.847881 \\
\log\ .795 & = & \underline{1.900367} \\
 & & 2)\underline{19.566453} \\
\log \text{cosine } \frac{1}{2}\, nMN = & & 9.783226 \\
 & \text{and } \frac{1}{2} < = & 52^\circ\ 37' \\
 & & \underline{\quad\ 2} \\
 & > nMN = & 105^\circ\ 14'
\end{array}$$

Now, since Mn bears N. 53° 28′ W., and the angle $nMN = 105° 14'$, the line MN is in the northeast quadrant, and makes an angle with the meridian $= 105° 14' - 53° 28' = 51° 46'$, or its bearing is N. 51° 46′ E.; and hence, reading in the order the measurements were made, the bearing of $NM =$ S. 51° 46′ W.

To find the angle nNM, and thence the bearing of QO:

$$\begin{array}{lll} 6.25 & \text{Ar. co.} & = 9.204120 \\ : 3.51 & & = 0.545307 \\ : : \sin 105° 14' & & = 9.984466 \\ \hline : \sin 32° 48' \ (< nNM) & & = 9.733893 \end{array}$$

$$\begin{array}{ll} \text{Bearing of } NM & = \text{S. } 51° 46' \text{ W.} \\ < nNM \ 32° 48' \text{ on west side, add} & \quad 32° 48' \\ \hline \text{Bearing of } Nn = OQ & = \text{S. } 84° 34' \text{ W.,} \\ \quad \text{or bearing of } QO & = \text{N. } 84° 34' \text{ E.} \end{array}$$

2. Supply the omissions from the following data:

(1)	N. 34° W.	13.00	chains;
(2)	S. 41½° W.	12.90	"
(3)	S. 50° E.	8.20	"
(4)		2.56	"
(5)		6.90	"
(6)	N. 26° E.	9.95	"

Case IV.

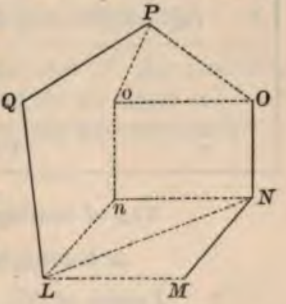
215. *Given the bearings and distances of all the sides of a tract of land except two, one of which has only its bearing given, and the other the distance, to supply the omissions.*

Make a meridian the side whose bearing only is given. Tabulate the latitudes and departures corresponding to the changed position of the survey. The side made meridian will have no departure, and the difference of the eastings and westings, therefore, will be the departure of the side whose bearing is unknown. With the given distance and this departure the

changed bearing and difference of latitude of this side may be found, and should be inserted in their proper places in the table. Then the difference of the northings and southings will be the latitude, or length, of the side made a meridian.

216. *Otherwise. When the deficient sides adjoin.*

Let the bearing of MN and the distance LM be wanting. Calculate by Case I. the direction and length of the closing line LN. A triangle, LMN, may then be formed in which will be given the lengths of LN and MN, and the angle NLM. The distance LM and the angle N may therefore be computed, and the angle N thus found properly applied to the bearing of the closing line will give the bearing of MN.



217. *When the deficient sides do not adjoin.*

Referring to the same figure as before, suppose the bearing of LM and the distance OP wanting. Transpose the sides as there shown, and calculate, as in Case I., the direction and length of the closing line Po. Then, as in the preceding article, there will be given a triangle, OPo, in which are known two sides Po and Oo, and the angle P, whence the bearing of Oo, or LM, and the distance PO, may be determined.

EXAMPLES.

1. Given the following notes, to supply the omissions.

QP.	N. 10°	E.	13.71 chains;	
PO.	S. $88\frac{1}{2}$°	E.	18.75	"
ON.	S. $16\frac{1}{2}$°	E.	16.50	"
NM.	S.	W.	13.00	"
ML.	N. 80°	W.;		
LQ.	N. 36°	W.	10.00 chains.	

STATIONS.	LINES.	BEARINGS.	DISTS.	N.	S.	E.	W.
Q	QP	N. 10° E.	13.71	13.50	...	2.38	...
P	PO	S. 88½° E.	18.75	...	0.49	18.74	...
O	ON	S. 16½° E.	16.50	...	15.82	4.68	...
N	NM	....	13.00	...	...	...	...
M	ML	N. 80° W.	...	...	...	...	...
L	LQ	N. 36° W.	10.00	8.09	...	...	5.88
				21.59	16.31	25.80	5.88
				16.31		5.88	
			Deficiency, 5.28 S.		Def., 19.92 W.		

$$\text{Tan of bearing of closing line} = \frac{19.92}{5.28},$$

or bearing of NL, S 75° 09′ W.

$$\text{Length of } NL = \sqrt{(5.28)^2 + (19.92)^2} = 20.61,$$

and angle $MLN = 24° \ 51'$.

To find angle LMN:

$$\begin{array}{lll} 13.00 \quad (NM) & \text{Ar. co.} & = 8.886057 \\ : \sin. 24° \ 51' \ (< L) & & = 9.623502 \\ :: \quad 20.61 \quad (LN) & & = 1.314078 \\ \hline : \sin. 41° \ 47' & & = 9.823637 \end{array}$$

Angle $LMN = 180° - 41° \ 47' = 138° \ 13'$ (see note).
Angle $LMN = 180° - (138° \ 13' + 24° \ 51') = 16° \ 56'$.

NOTE. — When the side MN, whose length only is given, is longer than the closing line LN, the angle M must be acute; if shorter, the angle M may be acute or obtuse, depending upon the length of the side LM, the bearing of which only is known. Hence, when this last relation obtains, it is necessary, in the application of this case, to remove ambiguity, that enough be known concerning the length of the side, whose bearing only is given, to indicate whether the angle M is greater or less than a right angle.

In the example, LM is known to be shorter than NM, and hence angle M is obtuse. The ambiguity is not removed by employing the method given in Article 215.

The bearing of NM, S. 75° 09′ W−16° 56′=S. 58° 13′ W.

To find the length of LM:

$$
\begin{array}{llll}
 & \sin.\ 24^\circ\ 51' & \text{Ar. co.} & = 0.376498 \\
: & \sin.\ 16^\circ\ 56' & & = 9.464279 \\
:: & 13.00 & & = \underline{1.113943} \\
: & 9.01\ (LM) & & = 0.954720
\end{array}
$$

The student may verify by the method in Article 215.

Example. As an exercise, from any of the preceding problems strike out from two sides that do not adjoin the bearing of one and the distance of another, and compute them.

SECTION VIII.

PLOTTING A COMPASS OR TRANSIT SURVEY.

218. In addition to the drawing-instruments explained in chain surveying, the draughtsman will now find very convenient an instrument for measuring angles, or,

A Protractor. It is made of metal* or paper, usually in the form of a semi-circle, the arc of which is divided into 180 equal parts, or degrees, subdivided and numbered both ways.

To draw a line making a given angle with another at a certain point. Bring the diameter of the protractor to coincide with the given line, its centre with the point, and the arch lying in the direction of the desired line; then with a sharp pencil or fine needle prick off the required number of degrees; joining the point thus fixed and the given point completes the problem.

Some plain scales are graduated to degrees on three edges so

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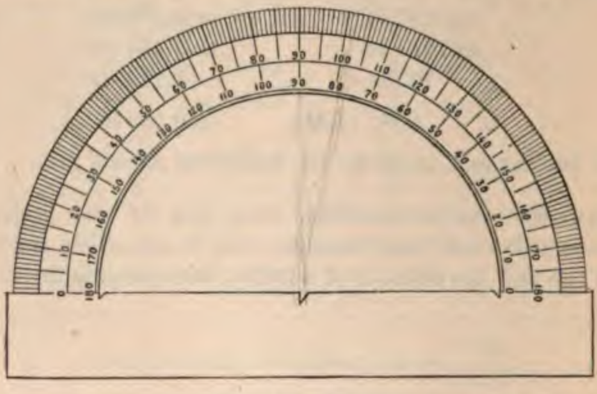
* For more accurate work there is attached a movable arm or ruler, extending beyond the circumference and carrying a vernier.

12-inch protractors,—complete circle,—made of heavy paper, on which are printed the divisions to quarter-degrees, are quite reliable.

as to be used like a protractor, but are objectionable on account of the obliquity of the divisions and their varying lengths.



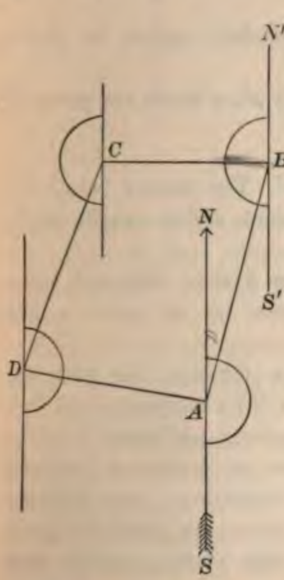
219. Illustration. To plot a survey the record of which is as follows:

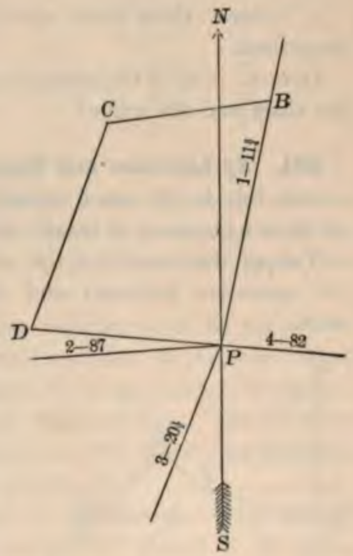
(1)	N. $11\frac{3}{4}°$	E.	13.19	chains;
(2)	S. 87°	W.	8.50	"
(3)	S. $20\frac{1}{2}°$	W.	11.75	"
(4)	S. 82°	E.	10.03	"

With a Protractor. *First Method.* Represent the meridian by drawing on the paper a line so situated that there will be sufficient room on either or both sides of it, as the case may be, to complete the drawing. Fix upon a point in this line to indicate a corner of the tract, usually "the place of beginning." In this particular example the first corner is the easterly boundary, and as it runs northerly, we will draw our meridian near the lower right-hand side of the paper, as at A. Prick off the angle $11\frac{3}{4}°$ from the north end of the protractor-arch to the right, and draw the line 13.19 chains (AB) to any convenient scale, say 2 chains to an inch, or 6.6 inches. Pass another meridian $N'S'$ through B; and since the bearing is south-westerly, we prick off the degrees, 87 from the south point, and draw the line 8.50 (BC) to the same scale. In a similar man-

ner draw the line CD, and finally DA, which should end at A. If it does not end precisely at A, an error in plotting, or inaccuracy in the survey, would thereby be indicated.

An error in plotting a line by this method would affect the *position*, but not the *direction* of the following lines.





220. *Another Method.* By laying off the angles from one point, or from one position of a protractor having a complete circle. With the protractor at any convenient point, P, in the meridian NS, prick off the degrees shown by the bearings, and indicate each, and the side to which it belongs, as per figure. Then, by instruments used for drawing parallel lines, transfer them to their proper places, and make the lengths correspond to the scale adopted. The point P, from which all the angles were set off, may or may not be one of the corners of the field. The figure shows that it saves one transfer if so taken.

EXAMPLES.

1. Plot a triangle, given two sides and the included angle.

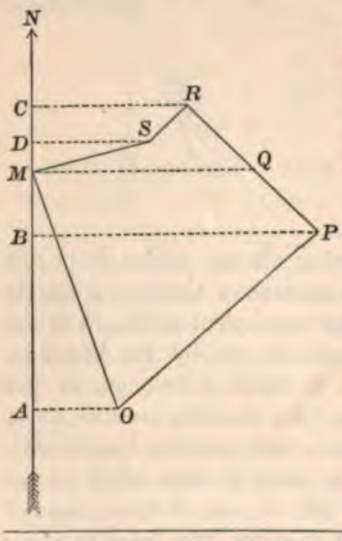
2. Given two angles and the included side, to plot the triangle.

3. Given three sides and two included angles, to plot a trapezium.

QUERY. Can a trapezium be plotted when there are given all the sides and one angle?

221. By Latitudes and Departures. The survey being *balanced*, this is the most accurate method, and is equally applicable to a compass or transit survey.

Taking the record of the survey in Article 208, and, using the corrected latitudes and departures, let us make a plot of it.



Draw through the first station* (M) a meridian, and find, by algebraic additions, from the columns of corrected latitudes and departures, the distance each corner is north or south from this station, called total latitude, and east or west from the meridian called total departure. These distances may be ascertained mentally as we proceed with the drawing, but to avoid error it is best to tabulate them, using three columns, as follows: + indicates distance north or east, and −, south or west, from the references.

* Any station will answer, but the one through which the meridian is supposed to pass in calculating the area is preferable.

STATIONS.	TOTAL LATITUDES FROM STATION *M*.	TOTAL DEPARTURES FROM MERIDIAN *NS*.
1	0	0
2	− 12.27	+ 4.68
3	− 3.21	+ 14.84
4	+ 0.20	+ 11.58
5	+ 3.54	+ 8.23
6	+ 1.97	+ 6.25
1	0	0

The total latitude of the last station is the latitude of the last line with its sign changed. The same is true regarding the total departure of last station. A check is thus had on the work.

From *M* lay off on the meridian negatively, or to the south, 12.27 chains according to the scale adopted, to *A*; from *A* set off perpendicularly to the east, with the same scale, 4.68 chains, to *O*; connect *M* and *O*, showing the first line. Set off from *M*, again to the south, 3.21 chains, to *B*; thence perpendicularly to the right, or east, 14.68 chains, to *P*.

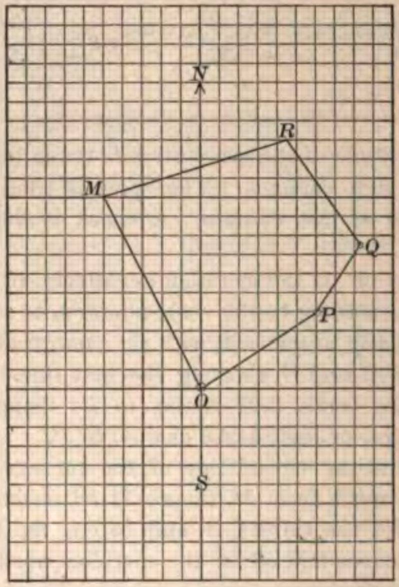
OP represents the second line of the survey. Next lay off 20 links to the north from *M*, thence 11.58 chains to the east, and join *PQ* for the third line, and so on, the last line, *SM*, requiring a distance laid off on the meridian north = 1.97; and a perpendicular thereto, = 6.25 east, when drawn closes the survey, thus proving the correctness of the work.

A variation of the method just given is to draw two lines, one representing the meridian, the other an east and west line. On the first lay off, as before, the latitudes of the sides, and on the second the corresponding departures; then, by means of dividers, obtain the intersection of co-ordinates, and joining these points shows the plot.

For plots of ordinary farm surveys the method given above, being equally accurate and more expeditious, is recommended;

for plots of extraordinary size, extending over a large drawing-board or made on a large table, the *variation* noted should be adopted.

222. Using Cross-Section Paper* and the latitudes and departures, a tolerably accurate plot may be made with great facility. The vertical and horizontal lines of the paper may



represent respectively meridians, and east and west lines. Assume any convenient point O as the beginning of the survey, and suppose the latitude of the first line $= 4.00$ chains N., the departure $= 6.00$ chains E. Count from O northward four spaces, thence eastward six spaces, to P; join OP, thus delineating the first side. Suppose the latitude and departure of the

* Note-books may be procured having the alternate pages ruled in small squares, like cross-section paper.

second side = respectively 3.50 chains N. and 2.25 chains E.; count off, as before (estimating the fractions of chain), three and a half spaces north and two and a quarter east; connect the points P and Q for the second side, and so on to the place of beginning.

SECTION IX.

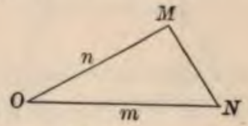
On Determining Areas.

A. PARTICULAR FORMS AND CASES.

Triangles.*

223. *First Method.* Measure two sides, as m and n, and the included angle O. Then the

$$\text{area} = A = \frac{m \times n \sin O}{2}.$$



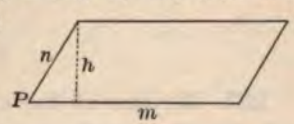
224. *Second Method.* Measure two angles, as O and N, and the included side m. Then

$$A = \frac{m^2 \sin N \sin O}{2 \sin(N + O)}.$$

Parallelograms.

225. Measure two adjacent sides, m and n, and their included angle, P. Then h denoting the altitude,

$$A = mh = m \times n \sin P.$$



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* For other methods than those found in this section of surveying triangles, quadrilaterals, and other polygons, see Chain Surveying, Articles 63 to 70.

EXAMPLES.

1. Two sides of a triangle measure 756 feet and 1024 feet, and their included angle 42° 45′; determine the area in acres.

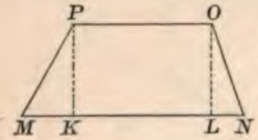
2. Two angles of a triangle are 59° 29′ and 65° 18′, and their included side 932 feet. How many acres does it contain? Plot.

3. Two sides of a triangle measure 15.24 chains and 13.18 chains, and the angle opposite the first 54° 25′. Find the area.

4. Two adjacent sides of a parallelogram are 856 feet and 1252 feet, and their included angle 75° 48′. Compute the area.

Trapezoids.

226. Measure three sides, say PM, MN, and NO, and one of the included angles, as N. From the data thus obtained compute the altitude, $OL = PK$, and the parallel side, PO. Then



$$A = \frac{MN + PO}{2} \times PK.$$

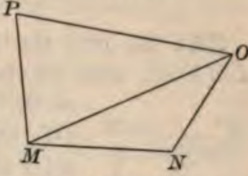
Or, instead of measuring the inclined sides, if it is equally convenient measure the parallel sides, and one of the other sides and an angle as before; then

$$A = \frac{MN + PO}{2} \times NO \sin N.$$

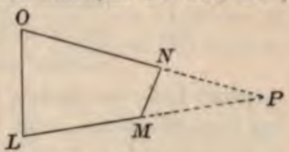
Trapeziums.

227. Measure all the sides and one angle. With the data calculate the length of a diagonal dividing the tract into two triangles, in one of which two sides and the included angle will be given, and in the other three sides, whence the area may be found.

228. Or, measure three sides, PM, MN, and ON, and the included angles N and PMN. Draw MO, calculate the area of the triangle MNO, the diagonal MO, and the angle OMN. Subtract OMN from PMN; then, having two sides and the included angle in the triangle PMO, its area may be computed, which added to the area of MNO gives the required content.



229. *Otherwise.* Measure two opposite sides, as OL and MN, and three angles, as O, L, and M. Conceive the sides ON and LM to be prolonged to meet in some point, P. From the data calculate the areas of the triangles POL and PMN. The difference will give the area sought.*



EXAMPLES.

1. Given in a trapezoid (see figure, Article 226) $PM = 33$ rods, $MN = 68$ rods, $NO = 30$ rods, and the angle $N = 70°$; to find the area, and make a plot.

2. Given in a trapezium $PMNO$ (see figure, Article 228) $PM = 7$ chains, $MN = 7.50$ chains, $NO = 6$ chains, the angle $N = 120°$, and $M = 108°$; to find the area, and make a plot.

3. Given in a trapezium $LMNO$ (see last figure) $LO = 8$ chains, $MN = 5$ chains, and the angles L, M, and N respectively 87°, 70°, and 80°; to find the area, and make a plot.

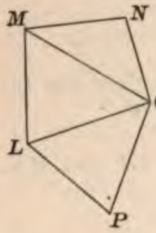
4. Given in a trapezium the angle M a right angle, the sides MN, NO, OP, and PM respectively 20, 12, 30, and 15 rods; also a perpendicular to MN from N extending to $PO = 10$ rods; to find the area.

QUERY. Could the area be found without NO?

* If practicable, observe all the angles, and thereby obtain a check on the measurements.

Polygons.

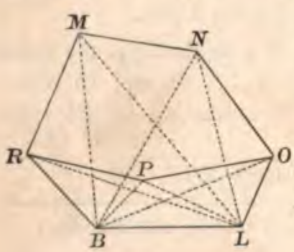
230. To find the area of an irregular pentagonal field *LMNOP*, when all the corners can be seen from one corner, as *O*. Measure the sides *ON*, *OP*, the diagonals *OL*, *OM*, and the three angles at *O*. Then two sides and the included angle of each triangle thus formed will be given, whence their areas may be calculated and, by addition, the area of the required polygon may be obtained. In like manner, a survey of any small irregular polygonal lot, in which all the corners are visible from one corner, may be effected. If there are n sides, measure from one corner two sides and $n-3$ diagonals, observing from the same point the $n-2$ angles which are formed by these diagonals and the two sides. Then, as above, the tract will be divided into $n-2$ triangles, the area of each may be calculated, and the sum of these areas taken for the area of the polygon.



231. Or, measure from some point within or without the field radial lines to all the corners, and observe at the same point the angle which these lines make with each other.

There will thus be given two sides and the included angle of a series of triangles, whence the bounding lines and area may be computed.

232. *Otherwise.* Measure a *base line* within or without the tract, or use a portion or all of one side as a base line, and observe from each extremity of this line the angles formed by it and a visual line through each corner of the tract. There will thus be known two angles and the included side of a series of triangles, whence the bounding lines and area may be calculated.



The marginal figure represents the

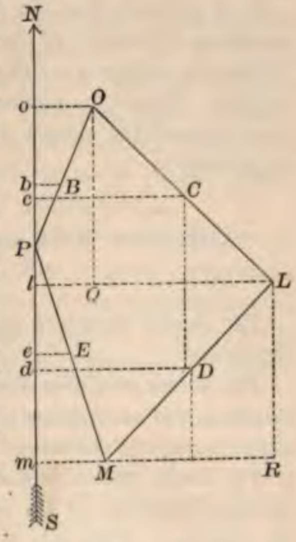
case where the base line BL is taken outside the tract. It will be noticed that it is possible by this method to survey a farm without entering upon it.

B. GENERAL METHOD.

233. The methods given in the last three articles are, however, quite limited in their application, since it rarely happens in a tract of considerable magnitude that all the corners are visible from any one corner, or from any point within or without the field.

The following method of determining the area by means of latitudes and departures is applicable to all right-lined figures, and is the most general and accurate.

Let $POLM$ represent a tract of land, the area of which is desired. Measure all the sides and angles, interior or deflection, with a single bearing, if the transit is used, or take all the bearings with a compass. Distribute the angular error, if any made by transit (see note, Article 207). Obtain the latitudes and departures, and balance the survey.



Let NS represent a meridian passing through P, the most westerly station of the tract, and Bb, Cc, and Dd meridian distances. Now, if perpendiculars be dropped from the angles O, L, and M to the meridian, it will readily appear that the area of $POLMP =$ area $oOLMmo$ minus the sum of the areas of the triangles PoO and PmM, or $POLMP =$ trapezoids $(OoLl + LlmM)$ − triangles $(PoO + PmM)$

$$= Cc \times OQ + Dd \times LR - Bb \times Po - Ee \times Pm.$$

The computation, then, involves the latitudes and departures, and meridian distances; the former having been already explained, we shall now indicate how the latter may be obtained, or rather how the double meridian distances are found, since in order to lessen fractions the double lengths are used.

The double meridian distance, or D.M.D., of the side

$$PO = 2\,Bb = Oo, \text{ its departure.}$$

$$\begin{aligned}\text{The D.M.D. of } OL &= 2\,Cc = Oo + Ll = Oo + Ql + QL \\ &= 2\,Bb + Oo + QL.\end{aligned}$$

$$\text{The D.M.D. of } LM = 2\,Dd = 2\,Cc + QL - MR.$$

$$\begin{aligned}\text{The D.M.D. of } MP &= 2\,Ee = 2\,Dd - MR - Mm \\ &= Mm = \text{its departure.}\end{aligned}$$

It is evident that, in a corresponding manner, the double meridian distances of the bounding lines of a tract may be found, no matter what the number of sides or magnitude of the angles. Hence, considering east departures plus (+), and west departures minus (−), the above deductions may be expressed in

A General Rule for Obtaining Double Meridian Distances.

The double meridian distance of the first side is equal to its departure.

The double meridian distance of the second side is equal to the double meridian distance of the first side, plus its departure, plus the departure of the second side.

The double meridian distance of any side is equal to the double meridian distance of the preceding side, plus its departure, plus the departure of the side itself.

The double meridian distance of the last side deduced by the

*foregoing rule should equal its departure, and will serve as a check on this part of the work.**

234. Continuing now the work of computing areas and referring to the last figure, we may form the following table:

STATIONS.	LINES.	N. LAT.	S. LAT.	E. DEP.	W. DEP.	D.M.D.	NORTH DOUBLE AREAS.	SOUTH DOUBLE AREAS.
1	*PO*	*Po*	. .	*Oo*	. .	2 *Bb*	$2\ Bb \times Po$	. . . .
2	*OL*	. .	*OQ*	*QL*	. .	2 *Cc*	. . . .	$2\ Cc \times OQ$
3	*LM*	. .	*LR*	. .	*MR*	2 *Dd*	. . . .	$2\ Dd \times LR$
4	*MP*	*Pm*	. .	. .	*Mm*	2 *Ee*	$2\ Ee \times Pm$	. . . .

The double meridian distances are placed in the column headed D.M.D. In the column headed North Double Areas are placed $2\,Bb \times Po$ and $2\,Ee \times Pm$, the product of the first and fourth double meridian distances and their corresponding latitudes. In the south double area column we find $2\,Cc \times OQ$ and $2\,Dd \times LR$, or the product of the double meridian distances of the second and third sides, and their respective latitudes. In other words, the column in which each of the products of double meridian distance and latitude is to be placed is indicated by the latitude employed in the multiplication.

Now, twice the area of the triangles *POo* and *PMm*, or the subtractive portion of the figure *oOLMmo*, is given in the north double area column, and twice the area of the trapezoids *OoLl* and *LlMm*, which include the triangles named, is given in the column of south double areas. Half the difference, therefore,

* The position of the meridian (NS) may be assumed to pass through any other corner, or even through a point outside the survey. A slight modification of the rule just given would make it applicable to any of these cases. For convenience, it is generally assumed to pass through the most westerly station. When a survey is made with the transit, and the area only required, it is most convenient to consider one of the sides of the tract the meridian.

between these sums is the area *POLMP* required. The reasoning being general, and independent of the number of sides, we have for finding the area of any rectilineal figure, the bearings and distances of all the sides being known, the following

Rule.

1. *Prepare a table as exhibited below.*

2. *Find, and place in their proper columns, the latitudes and departures of the several sides of the tract.*

3. *Balance the survey (if necessary).*

4. *Find the double meridian distances, with reference to a meridian passing through the most westerly* * *station, and place them in the D.M.D. column.*

5. *Multiply each double meridian distance by its corresponding corrected latitude, and place the product in the column of double areas indicated by the latitude.*

6. *One-half the difference of the sums of the columns of double areas will be the required area.*

Let us now take the field notes given in Article 198, and compute the area of the tract.

The student will perceive that the meridian is assumed to pass through the most westerly station, that the double meridian distances are found as directed in 233, that each is multiplied by its corresponding latitude, and the resulting double area product placed in the column of the same name as the latitude.

Lastly, the difference of the two columns of double areas is taken, the remainder divided by two, giving the number of square chains in the tract, and the result divided by $10 = 12{,}032$ acres, which is the area sought.

On account of the meridian passing through the most westerly station, and because the field is to the left,† the areas of

* For convenience simply, see note, preceding article.

† In the last figure, if the bearings are taken or recited in the order *PM*, *ML*, *LO*, etc., the tract is considered on the left; if this order is reversed, the tract is on the right.

Stations.	Bearings.	Distances.	Latitudes.		Departures.		Corrections.		Corrected Latitudes.		Corrected Departures.		D.M.D.	North Double Areas.	South Double Areas.
			N.	*S.*	*E.*	*W.*	*Lat.*	*Dep.*	*N.*	*S.*	*E.*	*W.*			
1	S. 20° 53′ E.	13.11	. . .	12.25	4.67	. . .	2	1	. . .	12.27	4.68	. . .	4.68	. . . .	57.4236
2	N. 48° 10′ E.	13.62	9.08	. . .	10.15	. . .	2	1	9.06	. . .	10.16	. . .	19.52	176.8512	. . . .
3	N. 43° 40′ W.	4.73	3.42	. . .	. . .	3.26	1	. .	3.41	. . .	. . .	3.26	26.42	90.0922	. . . .
4	N. 45° 08′ W.	4.75	3.35	. . .	. . .	3.36	1	1	3.34	. . .	. . .	3.35	19.81	66.1664	. . . .
5	S. 51° 30′ W.	2.53	. . .	1.57	. . .	1.98	. .	. .	. . .	1.57	. . .	1.98	14.48	. . . .	22.7336
6	S. 72° 30′ W.	6.56	. . .	1.96	. . .	6.26	1	1	. . .	1.97	. . .	6.25	6.25	. . . .	12.3125
			15.85	15.78	14.82	14.86	7	4	15.81	15.81	14.84	14.84		333.1098	92.4697
			15.78			14.82								92.4697	

Error 7 links. Error 4 links.

2)240.6401

10)120.32005

12.032 acres.

the trapezoids are north, and those of the triangles south. If we had assumed the meridian to pass through the most easterly corner, the areas of the trapezoids then formed would be south, and those of the triangles north.

If the bearings of the lines were reversed, or the survey made with the field to the right, the reverse of the preceding statement would be true.

In either case, however, one-half the difference of the sums of the double areas will give the contents.

As an exercise the student may obtain an expression for the area of *POLMP*, last figure, assuming the meridian to pass through *L*, and passing round by *MP*, etc., that is, keeping the field to the right. He may also, with the meridian through *P*, and keeping the field to the left, obtain an expression for the area.

As a further exercise he may verify the result in the last example solved, taking the meridian through the most easterly station.

Calculate the areas from the following notes; also make a plot of each:

1.

(1)	N.	9° W.	15.50	chains;
(2)	N.	31° E.	25.40	"
(3)	S.	69° E.	10.00	"
(4)	S.	$10\frac{1}{2}$° E.	19.70	"
(5)	S.	$10\frac{3}{4}$° W.	14.60	"
(6)	N.	89° W.	21.00	"

2.

Stations.	Lines.	Dists.	Azimuth with *LM*.
L	*LM*	22.45	0°
M	*MN*	1.30	22°
N	*NO*	66.30	90°
O	*OP*	23.85	180°
P	*PL*	67.10	270°
L	*LM*	. . .	360° or 0°

3.

(1) N. $11\frac{3}{4}°$ E. 13.19 chains;
(2) S. 87° W. 8.50 "
(3) S. $20\frac{1}{2}°$ W. 11.75 "
(4) S. 82° E. 10.03 "

Ans. $11\frac{175}{1000}$ acres.

If in Article 76 we substitute respectively for abscissa and ordinate of a corner of a tract, departure and latitude of the side ending at said corner, the rule there given may be applied to surveys made with an angular instrument.

To illustrate, take the example given in the last article:

Corrected Latitudes.		Corrected Departures.		Total Latitudes.	Total Departures.	Differ. between Alternate Departs.	Double Areas.
N.	*S.*	*E.*	*W.*				
. .	12.27	4.68	. .	0.00	. .	. .	. . . . .
9.06	. .	10.16	. .	−12.27	4.68	−14.84	182.0868
3.41	. .	. .	3.26	− 3.21	14.84	− 6.90	22.1490
3.34	. .	. .	3.35	.20	11.58	6.61	1.3220
. .	1.37	. .	1.98	3.54	8.23	5.33	18.8682
. .	1.97	. .	6.25	1.97	6.25	8.23	16.2131
							2)240.6391
							10)120.32 sq. ch.
							12.032 acres.

In this case the axes were taken through the most westerly station, thereby making the total departures all plus, but giving both plus and minus total latitudes. On account of the signs the double areas are all plus. The axis of ordinates passing through the most westerly station makes the total latitude of that station zero, and consequently there is one less multiplication to be performed. The same would be the case if the Y axis passed through the most easterly corner.

Since the difference of the *alternate total* departures is equal to the sum of the *adjacent* departures, the rule just given may be stated as follows:

Multiply the total latitude of each station by the sum of the departures of the adjacent sides, and take half the sum of these products for the area.

To illustrate, take the following example:

STATIONS.	BEARINGS.	DISTANCES.	N.	S.	E.	W.	TOTAL LATITUDES.	ADJACENT DEPARTS.	DOUBLE AREAS.
1	N. 25° E.	433	393	. .	183	. .	000	. .	. . . . .
2	N. 76° 55′ E.	191	43	. .	186	. .	393	369	145017
3	S. 6° 41′ W.	539	. .	535	. .	62	436	124	54064
4	S. 25° W.	40	. .	36	. .	17	− 99	− 79	7821
5	N. 65° W.	320	135	. .	. .	290	−135	−307	41445
									2)248347
									43560)124173.5 sq. ft.
									2.852 acres.

The student may verify the preceding example by this method.

2. Given the bearings and distances of the sides of a field, as follows, to find the area by each of the two preceding methods. Ascertain, also, the error of the survey.

(1) N. $6\frac{1}{2}$° W. 9.38 chains;
(2) N. $65\frac{1}{2}$° E. 8.25 "
(3) S. 39° E. 6.51 "
(4) S. 2° W. 4.45 "
(5) S. 46° W. 5.00 "
(6) N. 88° W. 6.79 "

3. Given the boundaries of a tract of land with the corresponding weights, as follows, to determine the area by double

meridian distances, using the weights in balancing the survey as indicated in 3°, Article 208. Determine, also, the error of the survey.

(1)	S. 79° 10′ W.,	dist.	27.00	chains,	weight,	1;
(2)	S. $\frac{1}{2}$° W.,	"	34.08	"	"	3;
(3)	N. 89$\frac{1}{2}$° E.,	"	10.47	"	"	1$\frac{1}{2}$;
(4)	N. 1° 55′ E.,	"	15.30	"	"	2;
(5)	S. 80$\frac{3}{4}$° E.,	"	7.15	"	"	2;
(6)	S. 58$\frac{1}{2}$° E.,	"	11.50	"	"	2$\frac{1}{2}$;
(7)	N. 39° E.,	"	9.20	"	"	1;
(8)	N. 16$\frac{1}{4}$° W.,	"	24.63	"	"	1.

4. The distances and interior angles of a farm, together with the bearing of one line, are given below. The angles were measured very accurately. It is required to calculate the area, by either of the preceding methods, balancing the survey by (2°) Second Case, Article 208. Also make a plot.

Angle	*L*,	90°;	side	*LM*,	28.00	chains.
"	*M*,	148$\frac{1}{2}$°;	"	*MN*,	25.20	"
"	*N*,	81$\frac{1}{2}$°;	"	*NO*,	14.70	"
"	*O*,	220°;	"	*OP*,	12.48	"
"	*P*,	90°;	"	*PQ*,	27.96	"
"	*Q*,	90°;	"	*QR*,	15.16	"
"	*R*,	270°;	"	*RS*,	11.90	"
"	*S*,	90°;	"	*SL*,	21.60	"

Bearing of *LM*, N. 10° E.

5. The notes of a survey are given below; it is required to determine the area by double meridian distances after correcting the latitudes and departures by a combination of 2° and 3°, Article 208. See also note in same article.

The interior angles were observed.

Angle	*L*,	91° 44′;	side	*LM*,	17.16	chains;	weight,	2.
"	*M*,	168° 20′;	"	*MN*,	9.48	"	"	1.
"	*N*,	104° 49′;	"	*NO*,	8.39	"	"	1$\frac{1}{2}$.
"	*O*,	179° 30′;	"	*OP*,	15.28	"	"	2.

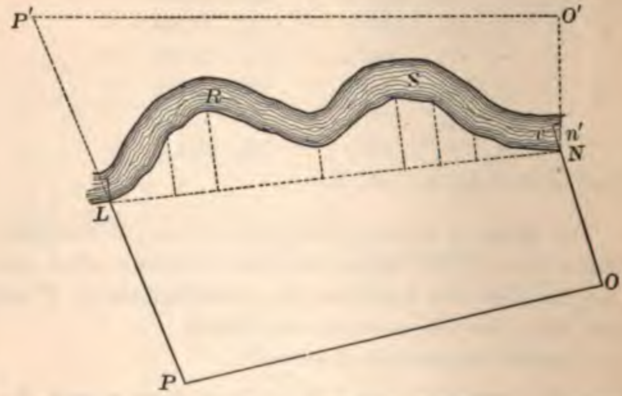
Angle P,	$90^\circ\ 19'$;	side PQ,	16.05 chains;	weight, $2\frac{1}{2}$.
" Q,	$90^\circ\ 05'$;	" QR,	15.68 "	" 3.
" R,	$283^\circ\ 49'$;	" RS,	11.40 "	" 1.
" S,	$71^\circ\ 24'$;	" SL,	13.80 "	" 1.

6. Select a tract of land, some of the sides being much more difficult than the others to align and measure, survey it, weight the sides, balance the latitudes and departures according to the weights, and calculate the area.

7. Let one party of students survey a tract of uneven or hilly land of considerable magnitude, by means of transit and stadia and rectangular co-ordinates; another party at the same time, or the same party subsequently, survey the same tract in the usual way. Compare results.

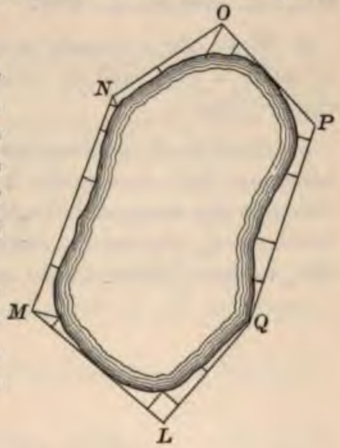
C. WHEN OFFSETS ARE TAKEN.

235. Let the annexed figure represent the case. The property lines are NO, OP, PL, and the centre of the creek * RS. Obtain sufficient data to compute the area of the rectilinear



* When a non-navigable stream forms a boundary of a tract of land, the middle of it is considered the property line, unless otherwise specified. In navigable rivers and tidal waters, the boundary is low-water mark.

figure *LNOP*, and take offsets from the line *LN* to the middle of the stream, as directed in Offsets and Tie-Lines, Article 73; and in Traversing, Article 164. Calculate the area of *LNOP* by one of the preceding methods; to this add* the sum of the areas of the trapezoids, and triangles formed by the offsets from the line *LN* to the middle of the creek. If the width of the stream is considerable, and especially if great accuracy is required, the surveyor must not ignore the small triangles † formed at *L* and *N*.



236. To Find the Area of a Pond or Small Lake, traverse, or take the bearings of the sides *LM*, *MN*, *NO*, etc., and measure them; also take offsets, at proper points, from these lines to the edge of the water.

Calculate the area included between the right lines, and subtract therefrom the area found by the offsets; the remainder will be the area required.

EXERCISES.

1. Let one party survey a field with compass and chain, taking bearings and distances of all the sides; another party survey the same field, using transit and chain, and observing

---

* If the base line *LN* is without the tract, as in *LNO′P′*, the area included between the middle of the stream and *LN* must be *subtracted* from that of *LNO′P′*.

† Other things being equal, the areas of these small triangles depend upon the obliquity of *PL* and *ON*. There will be none formed when *PL* and *NO* are perpendicular to the base *LN*. In the case presented, the area of the triangle at *L* is to be added, and that of *Nn′v* subtracted from the sum of the areas of the trapezoids, to obtain the correct content between *LN* and the middle of the creek.

the interior or deflection angles; a third party, using the chain only. Each party should use proof lines, make record, plot, and calculate the area. Compare results.

2. With a transit, survey a field, a part of which is bounded by a creek, lake, or some crooked line requiring offsets to be taken; make a plot, and compute the area.

3. Triangulate a portion of a river or small lake; make a plot, and compute the area.

4. Make the necessary measurements to write a description, to make a plot, and to compute the area of a portion of a crooked road.

5. Observe all the bearings and measure all the sides of a polygonal field, except the bearing and distance of one side. Compute the area, and length and bearing of omitted side. Subsequently observe the bearing and distance, and note, if any disagreement, how much the area is affected thereby.

CHAPTER III.

DECLINATION OF THE MAGNETIC NEEDLE, OR VARIATION OF THE COMPASS.

237. It has been already remarked (Article 82) that the magnetic and geographic meridian do not in general coincide. The angle included by the vertical planes containing these lines, or the angle which the direction of the needle makes with the geographic meridian, is the declination of the needle, sometimes called the variation of the compass. It is different at different places, and is a variable quantity at any place.

The declination is termed *east* or *west*, according as the north end of the needle points to the east or west of the geographic, or true meridian.

The magnetic declinations of a few places for the year 1885 are given below:

Eastport, Me.,	19° 10′ W.	Sitka, Alaska,	28° 50′ E.
Albany, N.Y.,	10° 11′ W.	Milledgeville, Ga.,	2° 32′ E.
Pittsburg, Pa.,*	2° 52′ W.	New Orleans, La.,	6° 11′ E.
Omaha, Neb.,	10° 06′ E.	City of Mexico, Mex.,	7° 24′ E.
San Francisco, Cal.,	16° 34′ E.		

238. Irregular Changes. The magnetic needle is subject to disturbances during a thunder storm, or an exhibition of aurora, solar changes, and sometimes it is considerably agitated without any apparent cause, but probably on account of magnetic or electric disturbances more or less remote.

The changes, however, which especially concern the surveyor, are the diurnal and secular.

* At this place, September, 1887, the magnetic declination = 3° 01′ W.

239. The Diurnal Variation. It has been ascertained, by repeated observations at various places, that the magnetic needle is subject to daily changes; that at a time varying from two to three hours after sunrise the north end of the needle attains its maximum deviation to the east, or, as it is called, its eastern elongation; from this time it is deflected westward, attaining its western elongation between 1 and 2 o'clock P.M., whence it retrogrades towards the east. There is sometimes an interruption of the motion at night, but generally a small reversed movement is exhibited, the magnetic meridian being crossed a second time between 7 and 9 P.M. The times at which these limits are reached vary with the seasons: during the north declination of the sun the averages for eastern and western elongations, respectively, are about 7.30 A.M. and 1.15 P.M.; for the remainder of the year, about 8.45 A.M. and 1.45 P.M.

The average daily direction or mean magnetic meridian is reached in summer about 10.15 A.M., and in winter about 10.45 A.M., at Philadelphia, and generally within half an hour of these times at other places.

The angular range between these limits is not constant, but, as may be seen by the table subjoined, it is considerably greater in summer than in winter, amounting at Philadelphia to 10′ 30″ in August, and only 6′ in November, or a yearly average of 8′, while at Key West, Florida, the average for the year is about 5′ 30″; in higher magnetic latitudes the average being more than in the lower. It is least in years of minimum sun spots (as in 1878, for instance), and greatest in years of maximum sun spots (as in 1870), the ratio being about as 7 to 13 of the average amount of these years respectively. The daily variation is at times interrupted, at others enfeebled, and frequently in the winter there are days on which it cannot be recognized. On account of the daily movement of the needle, its variable range during the year, and disturbances from atmospheric phenomena, it is well, when taking the bearing of any important line, to record the date, time of day, and condition of the atmosphere, using the subjoined table as far as practicable.

240. For reducing the direction of the needle observed at other hours to the mean magnetic meridian, the following table (taken from instructions to United States Deputy Surveyors), is furnished. It gives to the nearest minute the variations of the needle from its average position during the day, for each hour in the day, for the four seasons of the year.

TABLE FOR REDUCING THE OBSERVED DECLINATION TO THE MEAN DECLINATION OF THE DAY.

	A.M.	A.M.	A.M.	A.M.	A.M.	A.M.	M.	P.M.	P.M.	P.M.	P.M.	P.M.	P.M.
Hour	6	7	8	9	10	11	12	1	2	3	4	5	6
Spring	3′	4′	4′	3′	1′	1′	4′	5′	5′	4′	3′	2′	1′
Summer	4′	5′	5′	4′	1′	2′	4′	6′	5′	4′	3′	2′	1′
Autumn	2′	3′	3′	2′	0′	2′	3′	4′	3′	2′	1′	1′	0′
Winter	1′	1′	2′	2′	1′	0′	2′	3′	3′	2′	1′	1′	0′

241. The Secular Variation. Observations extending through many years, at various places, indicate a continual change taking place in the declination of the needle; that these changes are not continuous in direction nor uniform in intensity; that in this country the movement which, at the end of the last century, was *eastward* is now *westward* at all places east of the Rocky Mountains, and that a period of 250 or 300 years may elapse before the needle will again resume the position it now occupies.*

242. The Line of no Declination,† or **Agonic Line,** is the locus of all points on the earth where the direction of the needle is

* The explanation of the secular change must ultimately be referred to forces of a periodic character, acting for centuries with great regularity. So far no approach has yet been made towards the discovery of the cause of the motion. . . . The study of the variation of the declination so far would seem to indicate a secular change cycle for stations in the United States, extending over, or varying between, the limits of about 220 or 360 years. The data, however, are very uncertain. (U. S. C. & G. S., 1879.)

† Sometimes called the *Line of no Variation.*

coincident with the geographic meridian. At all places on the American continent situated to the east of this line the declination is *west*, and at all places to the west of it, the declination is *east*.

The line of no declination has been moving westward during the present century. From a chart published by Professor Loomis, in the *American Journal of Science*, 1840, it appears that the lines of equal declination, or isogonic lines, crossed the United States in a N.N.W. direction; the deflection towards the west being greatest in Maine. The line of no declination at that time entered North Carolina about midway between Newbern and Wilmington, passed through the middle of Virginia, and into Lake Erie at a point nearly equidistant from Erie, Pa., and Cleveland, Ohio.

In 1885 the Agonic Line entered the United States a little to the east of Beach Inlet, S.C., thence through Greensboro, N.C., Christiansburg, Va., Point Pleasant, W.Va., St. Clairsville, Ohio, a short distance west of Detroit, and a few miles east of Fort Mackinac, Mich.

In the year 1700 the declination at Philadelphia, Pa., was $8\frac{1}{2}°$ west. During the next century it diminished, reaching a minimum in 1800 of $1\frac{1}{2}°$ west, since which time it has been increasing, and is now, January, 1887, at the Philadelphia State House, lat. 39° 56′ 54″, long. 75° 09′, 6° 50′, with an annual increase of 5′.

243. Mr. Charles A. Schott, late chief of the computing division of the U. S. C. & G. S., tabulated the declinations observed at various stations, and deduced from them formulas by which the magnetic declination at various places may be computed.*

The places are arranged geographically as far as practicable, and are given by latitude and longitude (west of Greenwich). The epoch to which the formulas refer is 1850, or $m = t - 1850$.

* U. S. C. & G. S., 1882. App. 12.

Formulas Expressing the Magnetic Declination at Various Places in the United States, and for any Time within the Limits of Observation.

Name of Station and Location.	Latitude.	Longitude.	Expression for Magnetic Declination.
Portland, Me.	43° 38.8′	70° 16.6′	$D = +10^\circ.72 + 2^\circ.68 \sin(1.33\,m + 24^\circ.1)$
Burlington, Vt.	44° 28.2′	73° 12.3′	$D = +10.81 + 3.65 \sin(1.30\,m - 20.5)$ $+ 0.18 \sin(7.0\,m + 132)$
Rutland, Vt.	43° 36.5′	72° 55.5′	$D = +10.03 + 3.82 \sin(1.5\,m - 24.3)$
Portsmouth, N.H.	43° 04.8′	70° 43.0′	$D = +10.63 + 3.17 \sin(1.44\,m - 4.7)$
Newburyport, Mass.	42° 48.4′	70° 49.0′	$D = +10.07 + 3.10 \sin(1.4\,m + 1.9)$
Salem, Mass.	42° 31.9′	70° 52.5′	$D = +\ 9.80 + 3.61 \sin(1.50\,m - 1.0)$
Boston, Mass.	42° 21.5′	71° 03.8′	$D = +\ 9.52 + 2.93 \sin(1.30\,m + 5.0)$
Cambridge, Mass.	42° 22.9′	71° 07.7′	$D = +\ 9.58 + 2.69 \sin(1.3\,m + 7.0)$ $+ 0.18 \sin(3.2\,m + 44)$
Nantucket, Mass.	41° 17.0′	70° 06.0′	$D = +\ 9.29 + 2.78 \sin(1.35\,m + 5.5)$
Providence, R.I.	41° 49.5′	71° 24.1′	$D = +\ 9.10 + 2.99 \sin(1.45\,m - 3.4)$ $+ 0.19 \sin(7.2\,m + 116)$
Hartford, Conn.	41° 45.9′	72° 40.4′	$D = +\ 8.06 + 2.90 \sin(1.25\,m - 26.4)$
New Haven, Conn.	41° 18.5′	72° 55.7′	$D = +\ 7.78 + 3.11 \sin(1.40\,m - 22.1)$
Albany, N.Y.	42° 39.2′	73° 45.8′	$D = +\ 8.17 + 3.02 \sin(1.44\,m - 8.3)$
Oxford, N.Y.	42° 26.5′	75° 40.5′	$D = +\ 6.19 + 3.24 \sin(1.35\,m - 18.9)$
Buffalo, N.Y.	42° 52.8′	78° 53.5′	$D = +\ 3.66 + 3.47 \sin(1.4\,m - 27.8)$
Toronto, Can.	43° 39.4′	79° 23.4′	$D = +\ 3.60 + 2.82 \sin(1.4\,m - 44.7)$ $+ 0.09 \sin(9.3\,m + 136)$ $+ 0.08 \sin(19\,m + 247)$
Erie, Pa.	42° 07.8′	80° 05.4′	$D = +\ 2.26 + 2.71 \sin(1.55\,m - 29.7)$
Marietta, Ohio	39° 25.0′	81° 28.0′	$D = +\ 0.02 + 2.89 \sin(1.4\,m - 40.5)$
Cleveland, Ohio	41° 30.3′	81° 42.0′	$D = +\ 0.10 + 2.07 \sin(1.40\,m - 6.2)$
Detroit, Mich.	42° 20.0′	83° 03.0′	$D = -\ 0.97 + 2.21 \sin(1.50\,m - 15.3)$
Sault de St. Marie, Mich.	46° 29.9′	84° 20.1′	$D = +\ 1.54 + 2.70 \sin(1.45\,m - 58.5)$
Cincinnati, Ohio	39° 08.6′	84° 25.3′	$D = -\ 2.40 + 2.62 \sin(1.42\,m - 39.8)$
St. Louis, Mo.	38° 38.0′	90° 12.2′	$D = -\ 7.15 + 2.33 \sin(1.4\,m - 20.1)$
New York, N.Y.	40° 42.7′	74° 00.4′	$D = +\ 6.40 + 2.29 \sin(1.6\,m - 5.5)$ $+ 0.14 \sin(6.3\,m + 6.4)$
Hatborough, Pa.	40° 12.0′	75° 07.0′	$D = +\ 5.23 + 3.28 \sin(1.54\,m - 13.2)$ $+ 0.22 \sin(4.1\,m + 157)$
Philadelphia, Pa.	39° 56.9′	75° 09.0′	$D = +\ 5.38 + 3.29 \sin(1.55\,m - 23.9)$ $+ 0.39 \sin(4.0\,m + 161)$
Harrisburg, Pa.	40° 15.9′	76° 52.9′	$D = +\ 2.93 + 2.98 \sin(1.50\,m + 0.2)$
Baltimore, Md.	39° 17.8′	76° 37.0′	$D = +\ 3.20 + 2.57 \sin(1.45\,m - 21.2)$
Washington, D.C.	+38° 53.3′	+77° 00.6′	$D = +\ 2.47 + 2.52 \sin(1.40\,m - 14.6)$
Cape Henry, Va.	+36° 55.5′	+76° 00.5′	$D = +\ 2.54 + 2.41 \sin(1.50\,m - 35.4)$
Charleston, S.C.	32° 46.6′	79° 55.8′	$D = -\ 2.14 + 2.74 \sin(1.35\,m - 1.3)$
Savannah, Ga.	32° 04.9′	81° 05.5′	$D = -\ 2.54 + 2.32 \sin(1.5\,m - 28.6)$
Key West, Fla.	24° 33.5′	81° 48.5′	$D = -\ 3.90 + 2.93 \sin(1.4\,m - 33.5)$
Havana, Cuba	23° 09.3′	82° 21.5′	$D = -\ 4.52 + 2.00 \sin(1.3\,m - 26.7)$

FORMULAS EXPRESSING THE MAGNETIC DECLINATION.— *Continued.*

NAME OF STATION AND LOCATION.	LATITUDE.	LONGITUDE.	EXPRESSION FOR MAGNETIC DECLINATION.
Kingston, Jamaica . . .	17° 55.9′	76° 50.6′	$D = -\overset{\circ}{4}.64 + \overset{\circ}{2}.04 \sin(1.2\,m + 1\overset{\circ}{5}.9)$
Panama, New Granada .	8° 57.1′	79° 32.2′	$D = -6.80 + 1.82 \sin(0.9\,m + 10.4)$
Florence, Ala.	34° 47.2′	87° 41.5′	$D = -4.25 + 2.33 \sin(1.3\,m - 52.8)$
Mobile, Ala.	30° 41.4′	88° 02.5′	$D = -4.40 + 2.69 \sin(1.45\,m - 76.4)$
New Orleans, La. . . .	29° 57.2′	90° 03.9′	$D = -5.61 + 2.57 \sin(1.4\,m - 61.9)$
Vera Cruz, Mexico . . .	19° 11.9′	96° 08.8′	$D = -4.38 + 5.04 \sin(1.10\,m - 65.0)$
Mexico, Mexico	19° 25.9′	99° 06.0′	$D = -4.34 + 4.44 \sin(1.0\,m - 79.2)$
Acapulco, Mexico . . .	16° 50.5′	99° 52.3′	$D = -4.13 + 4.82 \sin(1.0\,m - 81.1)$
San Blas, Mexico . . .	21° 32.6′	105° 15.7′	$D = -6.51 + 2.74 \sin(0.9\,m - 106.3)$
Magdalena Bay, L. Cal. .	24° 38.4′	112° 08.9′	$D = -7.52 + 3.27 \sin(1.25\,m - 140.6)$
San Diego, Cal.	32° 42.1′	117° 14.3′	$D = -12.52 + 1.60 \sin(1.2\,m - 179.8)$
Monterey, Cal.	36° 36.1′	121° 53.6′	$D = -12.90 + 3.28 \sin(1.0\,m - 142.6)$
San Francisco, Cal. . . .	37° 47.5′	122° 27.2′	$D = -13.34 + 3.23 \sin(1.00\,m - 130.3)$
Cape Disappointm't, W.T.	46° 16.7′	124° 02.0′	$D = -20.26 + 2.36 \sin(1.25\,m - 180.0)$
Sitka, Alaska	57° 02.9′	135° 19.7′	$D = -26.77 + 2.33 \sin(1.4\,m - 111.6)$
Unalashka, Alaska . . .	53° 52.6′	166° 31.5′	$D = -18.34 + 1.45 \sin(1.4\,m - 67.8)$
Tyrone, Pa.	40° 40.0′	78° 15.5′	$D = +3.46 + 0.0550(t - 1875.5)$
Pittsburg, Pa.	40° 27.6′	80° 00.8′	$D = +2.36 + 0.0566(t - 1878.7)$
Chicago, Ill.	41° 50.0′	87° 36.7′	$D = -6.03 + 0.0281(t - 1850) + 0.00082(t - 1850)^2$
Grand Haven, Mich. . .	43° 05.2′	86° 12.6′	$D = -4.95 + 0.0380(t - 1850) + 0.00120(t - 1850)^2$
Madison, Wis.	43° 04.6′	89° 24.2′	$D = -6.43 + 0.0655(t - 1880.3)$
Duluth, Minn.; and Superior City, Wis. . . .	46° 45.5′	92° 04.5′	$D = -10.17 + 0.0868(t - 1875.8)$
Rio Janeiro, Brazil . . .	−22° 54.8′	43° 09.5′	$D = +0.282 + 0.1395(t - 1850) + 0.00545(t - 1850)^2$
San Antonio, Tex. . . .	+29° 25.4′	98° 29.3′	$D = -10.14 + 0.0204(t - 1850) + .000024(t - 1850)^2$
Omaha, Neb.; and Council Bluffs, Iowa.	41° 15.7′	95° 56.5′	$D = -11.66 + 0.0439(t - 1850)$
Denver, Col.	39° 45.3′	104° 59.5′	$D = -14.79 + 0.0258(t - 1872.9)$
Salt Lake City, Utah. . .	40° 46.1′	111° 53.8′	$D = -15.51 - 0.0930(t - 1850) + 0.00180(t - 1850)^2$

To illustrate the use of the table: Suppose it is desired to ascertain the declination of the needle at Harrisburg for the last of September, 1877, or $t = 1877.75$.

Take from the table the expression for the declination at Harrisburg; that is:

$$D = +2.93 + 2.98 \sin(1.50\,m + 0.2).$$

Find $m = 1877.75 - 1850 = 27.75$;

$$1.50\,m + 0.2 = 41.625 + 0.2 = 41.825,$$

and $2.98 \times \text{natural sin } 41.825 = 2.98 \times .66686 = 1.987.$

$\therefore D = 2.93 + 1.987 = 4.917 = 4^\circ\ 55'$ *west* (the result being *plus*). The observed declination for the same time was $4^\circ\ 53'\ 5''$. The difference between the computed and observed declination is seen to be very small.

In running old lines it may be necessary to determine the declination at a time anterior to 1850; then m will be *negative*. Suppose the declination at Washington, D.C., for the year 1841 is desired. The tabular expression is:

$$D = 2.47 + 2.52 \sin(1.4\,m - 14.6),$$
$$m = 1841 - 1850 = -9,$$
$$(1.4\,m - 14.6) = -27.2,$$
$$2.52 \sin(-27.2) = -1.15.$$

$\therefore D = 2.47 - 1.15 = 1.32$ *west* (the resulting sign being *plus*), which agrees practically with the observed declination.

244. The following table is taken from U. S. C. & G. S. Report, 1882, App. 12, Mr. Schott's paper on Secular Variation. It exhibits the computed epoch of greatest easterly deflection reached in the secular motion; *i.e.*, the date when last reached, or the date (in parenthesis) when it is next expected to be in that position; the amount in degrees and fractions, and direction (+ west, − east) at this, the nearest stationary epoch; and the computed annual changes in the declination of the magnetic needle for the years 1870, 1880, and 1885, a plus sign indicating north end of needle moving westward, a minus sign indicating north end of needle moving eastward.

Location.	Nearest Stationary Epoch of Easterly Digression.	Amount at Easterly Digression.	Annual Change.		
			In 1870.	In 1880.	In 1885.
Paris, France	1581	−10.6°	−7.0′	−6.1′	−9.5′
Halifax, Nova Scotia .	1728	+12.4°	+1.8′	+1.0′	+0.5′
Quebec, Canada	1809	+12.1°	+4.2′	+1.6′	+0.5′
Montreal, Canada . . .	1816	+ 7.6°	+5.1′	+3.1′	+2.8′
Eastport, Me.	1760	+12.5°	+3.3′	+2.7′	+2.3′
Portland, Me.	1764	+ 8.0°	+2.4′	+1.6′	+1.2′
Burlington, Vt.	1810	+ 7.2°	+5.0′	+6.0′	+5.8′
Rutland, Vt.	1806	+ 6.2°	+6.0′	+5.6′	+5.3′
Portsmouth, N.H.	1791	+ 7.5°	+4.4′	+3.7′	+3.3′
Newburyport, Mass. . .	1784	+ 7.0°	+3.9′	+3.3′	+2.9′
Salem, Mass.	1791	+ 6.2°	+5.0′	+4.1′	+3.5′
Boston, Mass.	1777	+ 6.6°	+3.4′	+2.9′	+2.5′
Cambridge, Mass.	1783	+ 6.9°	+2.9′	+2.1′	+1.8′
Nantucket, Mass.	1779	+ 6.5°	+3.3′	+2.7′	+2.4′
Providence, R.I.	1780	+ 6.1°	+3.8′	. . . .	. . . .
Hartford, Conn.	1799	+ 5.2°	+3.8′	+3.7′	+3.6′
New Haven, Conn. . . .	1802	+ 4.7°	+4.6′	+4.3′	+4.1′
Albany, N.Y.	1793	+ 5.2°	+4.3′	+3.7′	+3.4′
Oxford, N.Y.	1797	+ 3.0°	+4.5′	+4.3′	+4.0′
Buffalo, N.Y.	1806	+ 0.2°	+5.1′	+5.0′	+4.8′
Toronto, Canada	. . . .	. . . .	+4.8′	+4.5′	+2.3′
Erie, Pa.	1811	− 0.5°	+4.4′	+4.2′	+4.0′
Marietta, O.	1815	− 2.9°	+4.2′	+4.2′	+4.2′
Cleveland, O.	1790	− 2.0°	+2.8′	+2.5′	+2.2′
Detroit, Mich.	1800	− 3.2°	+3.4′	+3.0′	+2.8′
Sault de St. Marie, Mich.	1828	− 1.2°	+3.6′	+4.0′	+4.1′
Cincinnati, O.	1815	− 5.0°	+3.8′	+3.9′	+3.8′
St. Louis, Mo.	1800	− 9.5°	+3.4′	+3.2′	+3.0′
New York, N.Y.	1797	+ 4.0°	+2.4′	+2.5′	+2.6′
Hatborough, Pa.	1797	+ 1.8°	+4.6′	+4.5′	. . . .
Philadelphia, Pa.	1800	+ 1.9°	+4.9′	+4.9′	+5.3′
Baltimore, Md.	1802	+ 0.6°	+3.9′	+3.6′	+3.2′
Harrisburg, Pa.	1790	0.0°	+4.1′	+3.3′	+2.8′
Washington, D.C.	1796	0.0°	+3.5′	+3.2′	+3.0′

Location.	Nearest Stationary Epoch of Easterly Digression.	Amount at Easterly Digression.	Annual Change.		
			In 1870.	In 1880.	In 1885.
Cape Henry, Va.	1814	+ 0.1°	+3.8′	+3.7′	+3.6′
Charleston, S.C.	1784	− 4.9°	+3.5′	+3.0′	+2.7′
Savannah, Ga.	1809	− 4.9°	+3.6′	+3.5′	+3.3′
Key West, Fla.	1810	− 6.8°	+4.3′	+4.2′	+4.1′
Havana, Cuba	1801	− 6.5°	+2.7′	+2.7′	+2.6′
Kingston, Jamaica . . .	1762	− 6.7°	+2.0′	+1.6′	+1.4′
Panama, New Granada	1739	− 8.6°	+1.5′	+1.4′	+1.3′
Florence, Ala.	1821	− 6.6°	+2.8′	+3.1′	+3.2′
Mobile, Ala.	1841	− 7.1°	+2.8′	+3.4′	+3.7′
New Orleans, La.	1830	− 8.2°	+3.1′	+3.5′	+3.7′
Vera Cruz, Mexico . .	1827	− 9.4°	+4.2′	+4.9′	+5.2′
Mexico, Mexico	1839	− 8.8°	+2.4′	+3.0′	+3.3′
Acapulco, Mexico . . .	1841	− 9.0°	+2.4′	+3.2′	+3.5′
San Blas, Mexico . . .	1868	− 9.3°	+0.1′	+0.5′	+0.7′
Magdalena Bay, L.Cal.	(1890)	−10.8°	−1.8′	−1.0′	−0.5′
San Diego, Cal.	(1925)	−14.1°	−1.8′	−1.6′	−1.5′
Monterey, Cal.	(1903)	−16.2°	−1.8′	−1.3′	−1.0′
San Francisco, Cal. . .	(1890)	−16.6°	−1.0′	−0.5′	−0.3′
C. Disappointm't, W.T.	(1922)	−22.6°	−2.8′	−2.5′	−2.2′
Sitka, Alaska	1865	−29.1°	+0.4′	+1.2′	+1.6′
Unalashka, Alaska . . .	1834	−19.8°	+1.6′	+1.9′	+2.0′
Tyrone, Pa.	....	....	....	+3.3′	....
Pittsburg, Pa.	....	....	....	+3.4′	....
Chicago, Ill.	1833	− 6.3°	....	+4.6′	+5.1′
Grand Haven, Mich. . .	1834	− 5.3°	....	+6.6′	+7.3′
Madison, Wis.	....	....	....	+3.9′	....
Duluth, Wis. . . . . . } Superior City, Wis. . }	....	....	....	+5.2′	....
Rio Janeiro, Brazil . .	....	....	....	+10.3′	+10.7′
San Antonio, Tex. . . .	....	....	....	+2.1′	+2.2′
Omaha, Neb. . . . . . } Council Bluffs, Ia. . . }	....	....	....	+2.6′	....
Denver, Col.	....	....	....	+1.6′	....
Salt Lake City, Utah .	1876	−16.7°	....	+0.9′	+2.0′

The variability of the change will be noticed. For example, take New York, Philadelphia, and Harrisburg, places comparatively near together.

At New York the change in 1870 was only one-half that at Philadelphia; but, both increasing, this ratio was maintained throughout the 15 years. At Harrisburg, on the contrary, the annual change in 1870 was nearly six-sevenths that at Philadelphia, but the change constantly increasing at the latter place while diminishing rapidly at the former, the annual variation at Harrisburg in 1885 was only a little more than one-half that at Philadelphia.*

245. Effects of the Secular Change. It is evident that if a surveyor should ignore this change, in attempting to establish the corners and to trace the boundary lines of a farm from their description in an old deed, it would be possible for him to return to his place of beginning, but probably none of his other corners would coincide with the true corners.

A line in the vicinity of Philadelphia, which 12 years ago had a bearing N. 19° E., would now bear N. 20° E., and in the same locality a bearing which at that time was recorded N. 19° W. would now be N. 18° W. A variation which, if not corrected, would indicate the end of a line 15 chains long over 26 links from its true position.

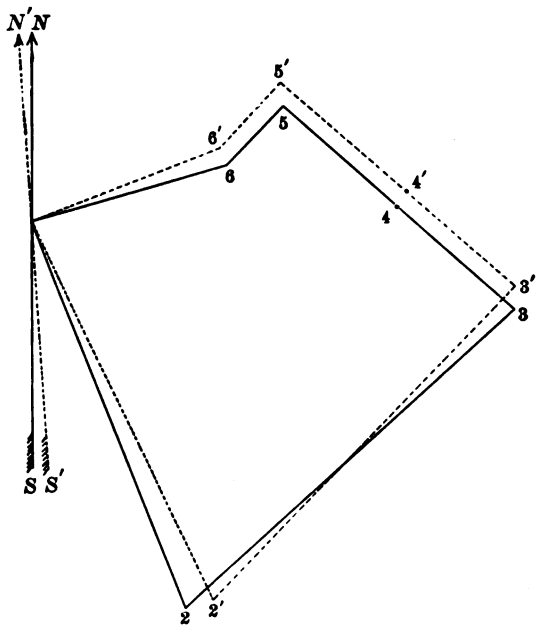
Take, for example, the notes given in Article 208, page 161, and suppose an interval has elapsed sufficient to make the variation two degrees. The accompanying figure shows the true lines and corners; also those corresponding to a survey made without taking the variation into account.

The bearings and distances are as follows:

(1)	S. 20° 53′ E.	13.11 chains;
(2)	N. 48° 10′ E.	13.62 "
(3)	N. 43° 40′ W.	4.73 "

* For extended investigations on magnetic declination, see U. S. C. & G. S. Reports, 1879, 1881, and 1882.

(4) N. 45° 08′ W. 4.75 chains;
(5) S. 51° 30′ W. 2.53 "
(6) S. 72° 30′ W. 6.56 "

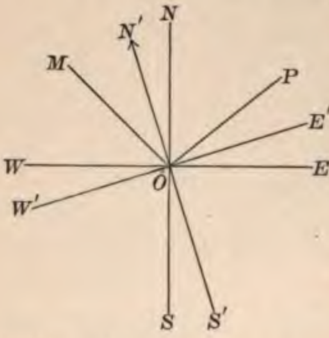


To allow for a variation of two degrees, we should have the following bearings:

(1) S. 18° 53′ E.;
(2) N. 50° 10′ E.;
(3) N. 41° 40′ W.;
(4) N. 43° 08′ W.;
(5) S. 53° 30′ W.;
(6) S. 74° 30′ W.

246. *To deduce a general rule for obtaining the magnetic bearings of old lines when the variation is known.*

Let NS represent the direction of the magnetic meridian in the vicinity of a survey made several years ago; $N'S'$, its direction several years later, at the time of re-survey, and that the north end of the needle points 2° farther west. It is evident that at the time of the re-survey, the line NS will bear N. 2° E., and OP, which according to the old survey bears N. 48° E., will have its bearing increased 2° or N. 50° E.; but the line OM, the bearing of which was N. 42° W., will now bear N. 40° W. A line recorded as east will be traced by a course S. 88° E., and so on.



Hence the rule: Increase by the change the bearings which are northeasterly or southwesterly, and diminish by the same amount the bearings which are northwesterly or southeasterly. The foregoing rule is directly applicable *now* in the United States, except on the Pacific coast, because the variation is *west*. That is, the north end of the needle is moving west, thereby increasing the readings of bearings in the N. E. and S. W. quarters, and diminishing the readings of those in the N. W. and S. E. quarters. When it becomes *east*, the words "increase" and "diminish" should be interchanged to make it correct. If a vernier compass is used, the variation may be set off and the lines traced by the old bearings.

247. Change Determined by Old Lines. If the bearing and date of survey of a line are known, and its extremities visible from each other, setting the instrument on one end and sighting the other will give, by comparison with the recorded bearing, the variation.

NOTE. — Care must be taken by the surveyor, when called upon to run out old lines, the corners not being definitely marked, that the time of the former survey be known; the date of the deed does not indicate that of the survey. The description of the lines may have been copied, as they frequently are, from an older deed.

The variation to be applied to correct magnetic bearings is frequently determined in this way.

If the boundaries of a tract of land are to be traced, whether the date of the previous survey be known or not, the surveyor seeks to find, if possible, two consecutive marked corners; then, taking the bearing of these and comparing with the record, he obtains the change sought.

This change, properly applied to each side, should indicate its direction.

It frequently, and in large tracts generally, happens that though the corners at the end of a line may be established, they cannot be observed from each other. In such case run a line as nearly as possible from one corner towards the other by the bearing given in the deed, or make first an allowance which may seem proper from the data at hand; measure from the end of the line thus run the distance to the true corner, and by the 57.3 rule, Article 177; or, by the tangent method, same article, find the angle to be added or subtracted, as the case may require, to correct the bearing with which to run the line. The difference between the bearing given in the deed and the corrected bearing will be the change in the declination since the survey recorded in the deed.

EXAMPLES.

1. A line, said to have been surveyed in 1860, recorded N. 18° 30′ E., 24.40 chains, was run in 1885 with a bearing N. 19° 45′ E.,—the variation being about 3′ to the west per year in its locality, — and the corner was 7 links to the right (farther easterly) of the end of the line run. The corrected magnetic bearing and variation are required.

$$1^\circ\ 15' + \frac{57.3 \times 7}{24.40} = 1^\circ\ 15' + 10' = 1^\circ\ 25' = \text{variation}.$$

Adding the variation to the bearing of the line run, since the true corner was farther to the east, there results N. 19° 55′ E. as the corrected magnetic bearing of the line.

2. If in Example 1 the corner had been found 7 links to the *left*, what would be the correct bearing of the line?

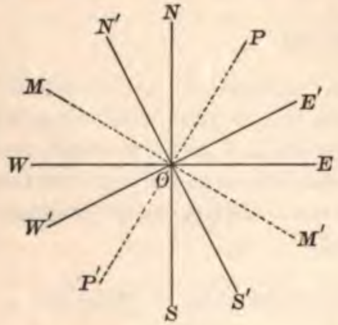
3. A line which in 1862 ran S. 34° 15′ W. 18.56 chains, in 1886 bore S. 35° 35′ W. What was the average change in the declination per year?

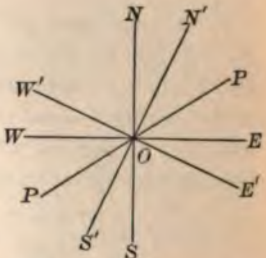
4. Give the corrected magnetic bearing for 1886 of a line in the same locality as that in Example 3, which in 1868 ran due east.

5. In 1876 a line had a bearing S. 89° 45′ W. 16.80 chains; in 1886, running by the same bearing, the true corner was 20 links to the north. Give the average annual change, and correct the bearing.

6. If a line 60.00 chains in length were surveyed in the early part of the day, where the needle deviates 5 minutes east of the mean magnetic meridian, and the same line surveyed soon after mid-day, the needle then pointing 5 minutes *west* of the mean magnetic meridian, how far apart would the lines be at their ends, and what the area included between them?

248. To Obtain the True Bearing of a Line, that is, the bearing with respect to the geographical meridian, when the





declination is *west*. Assume NS and $N'S'$ (left-hand figure) to represent respectively the true and magnetic meridian. Then it is evident that the bearing of any line between the north and

east, or south and west, as OP or OP', will be *less* referred to NS than when referred to $N'S'$ by the amount of the angle $NON' = SOS' =$ the declination.

A line running between north and west, as OM, or south and east, as OM', will evidently have its bearing increased by the amount of the change.

The reverse is true where the declination is *east*, as may be perceived by reference to the right-hand figure.

Hence, to get the true bearing from the magnetic for all places east of the line of no declination, *i.e.* where the declination is *west*, *subtract* the declination from a bearing which is northeasterly or southwesterly, and *add* the declination to a bearing which is northwesterly or southeasterly. Where the declination is *east*, as at all places west of the line of no declination, *add* the declination to a bearing which is northeasterly or southwesterly, and *subtract* the declination from a bearing which is northwesterly or southeasterly. Where the declination is *west*, a bearing that reads north, when reduced to the true bearing, will evidently be west of north the amount of the declination; if the declination is 3°, the bearing will be N. 3° W., and supposing the same declination, a line running due east magnetically will be truly N. 87° E.

The reverse of the last paragraph is true where the declination is *east*.

REMARK. If, when applying the rule, a negative result is obtained, care must be exercised in the interpretation of it. For example, if the declination is 3° West, and the needle indicates the bearing of a line N. 1° E., there results, by the rule, —2°. This shows simply that the true bearing is to the *west* of north, or N. 2° W. If the bearing is S. 89° E., adding the declination, as the rule requires, gives evidently the reading N. 88° E.

Reduce to their true bearings the following, the declination being 2° 55′ W.:

N. 2° 15′ E., East, S. 45° E., South; S. 87° 30′ W., N. 88° 15′ W., North.

Also the following, the declination being 3° 40′ E.:

N. 88° E., East, S. 2° E., South; S. 88° 30′ W., N. 40° W., North.

249. To Ascertain the Declination.* If a geographical meridian were traced on the earth convenient to the operations of the surveyor, he would have the means always at hand by which to determine the declination. He could simply set up his instrument at a point on the meridian, take the bearing of another point in it, and the reading would be the declination. So the problem resolves itself into the determination of a geographic or true meridian.

250. By Polaris. If there was a celestial object precisely at the point where the prolongation of the earth's axis pierces the celestial sphere, the direction of the meridian could be obtained by simply sighting to the object. This, however, is not the case, but Polaris, or Alpha Ursæ Minoris, is a star whose polar distance is, January, 1887, 1° 17′ 38″,† and which apparently revolves about the north pole in 23 hours 56 minutes. It therefore culminates twice daily, and twice it attains its greatest distance directly east and west of the pole, called respectively its eastern and western elongation. If, therefore, the Pole Star could be observed at the instant of its culmination, the line of sight would be in the meridian plane; but since in general the local time of transit is not precisely known, and since the star is then moving at right angles to the plane of the meridian respecting which its motion is at that time a maximum, and consequently a small difference in time would introduce a considerable error in arc, this method is not as reliable as that by means of Polaris at its eastern or western elongation, as then the star for a few minutes appears to move in the direction of the vertical wire, or compass-slit, thus affording a favorable

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* For other methods, see Chapter II. Section I., Solar Attachment; and Chapter VI., Art. Solar Compass.

† Its polar distance is diminishing at the rate of 20″ (19.06″) per year. This diminution will continue until the star is within half a degree of the pole, when it will recede.

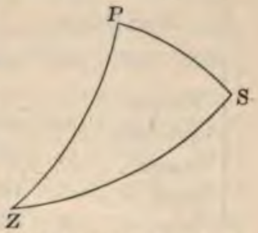
In 1890 its polar distance will be 1° 16′ 42″.

In 1900 its polar distance will be 1° 13′ 33″.

In 1910 its polar distance will be 1° 10′ 26″.

opportunity for observing it, and the precise time of observation need not be known.

Conceive a spherical triangle, the vertices of which are, Z, the zenith of the observer; P, the north pole; and S, Polaris. This triangle, when the star is at an elongation, will be right-angled at the star. In this right-angled spherical triangle are known the co-latitude of the observer's station, and the co-declination or polar distance of the star, to find the azimuth * and hour angle.† Using natural functions, the formula for the hour angle is $\cos P = \tan PS \cot PZ$, and for the azimuth,



$$\sin Z = \frac{\sin PS}{\sin PZ} = \frac{\sin PS}{\cos \text{lat}}. \ddagger$$

It may be well to remark, though it has only a theoretical significance, that these formulas are not applicable to *all* north latitudes. In other words, there will be no hour angle shown by the first formula, nor azimuthal angle by the second, on that parallel of latitude which agrees in arc distance with Polaris from the equator, and for any point between that parallel and the pole the formulas fail.

This remark is in general applicable to any circumpolar star.

QUERIES. Is Polaris a longer time passing from eastern to western elongation, than from western to eastern, to an observer whose latitude is 40°? What is the difference in time to an observer whose latitude is 60°? 80°? Where would this difference be a mimimum? Where a maximum?

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* The azimuth of a star is the angle between the meridian plane and the vertical plane through the star.

† The angle SPZ included between the meridian plane PZ and the plane PS passing through the star.

‡ The azimuth of Polaris at elongation varies with the latitude and with the year, as may be seen by the table on page 217

251. Table of mean local time astronomical (from noon) of the elongations and culminations of Polaris for 1885, latitude 40°, and longitude 6 hours west of Greenwich.

First Day of	E. E.	U. C.	W. E.	L. C.
	h. m.	*h. m.*	*h. m.*	*h. m.*
January	0 35.3	6 29.9	12 24.6	18 28.0
February	22 29.0	4 27.6	10 22.2	16 25.6
March	20 38.5	2 37.1	8 31.8	14 35.1
April	18 36.4	0 35.0	6 29.7	12 33.1
May.........	16 38.6	22 33.3	4 31.8	10 35.2
June	14 37.0	20 31.7	2 30.3	8 33.7
July.........	12 39.5	18 34.2	0 32.8	6 36.2
August	10 38.1	16 32.8	22 27.5	4 34.8
September	8 36.6	14 31.3	20 26.0	2 33.3
October.......	6 38.9	12 33.6	18 28.2	0 35.5
November	4 37.0	10 31.7	16 26.4	22 29.7
December	2 38.9	8 33.5	14 28.2	20 31.6

To correct the tabular times so as to apply to any year *subsequent* to 1885, *add* 0.35 minutes for every year. For any year *previous* to that date, *subtract* 0.35 minutes for every year.

For days not given in the table, interpolate, or allow 3.94 minutes for each day, the times varying by this amount.

To allow for difference of latitude between the limits of 30° and 50°, *add* 0.14 for every degree south of 40°; *subtract* 0.18 for every degree north of 40°.

To refer the tabular times to any year in a quadriennium, observe —

For the first year after a leap year the table is perfect; for the second year after a leap year *add* 1 minute; for the third year after a leap year, *add* 2 minutes; for a leap year, and before March 1, *add* 3 minutes; and for the remainder of the year *subtract* 1 minute.

It will be noticed that there occur two eastern elongations on Jan. 9, and two western elongations on July 9.

AZIMUTH (FROM THE NORTH) OF POLARIS, WHEN AT ELONGATION, BETWEEN THE YEARS 1887–1895, FOR DIFFERENT LATITUDES BETWEEN + 25° AND + 50°.

Lat.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.
+ 25°	1°25.7′	1°25.3′	1°25.0′	1°24.6′	1°24.3′	1°23.9′	1°23.6′	1°23.2′	1°22.9′
26	26.4	26.0	25.7	25.3	25.0	24.6	24.3	23.9	23.6
27	27.1	26.8	26.4	26.0	25.7	25.4	25.1	24.7	24.3
28	27.9	27.6	27.2	26.8	26.5	26.2	25.8	25.4	25.1
29	28.8	28.4	28.0	27.6	27.3	27.0	26.6	26.3	25.9
30	29.6	29.3	28.9	28.5	28.2	27.8	27.5	27.1	26.8
31	30.5	30.2	29.8	29.4	29.1	28.8	28.4	28.0	27.6
32	31.5	31.2	30.8	30.4	30.1	29.7	29.3	29.0	28.6
33	32.6	32.2	31.8	31.4	31.1	30.7	30.3	30.0	29.6
34	33.6	33.3	32.9	32.5	32.1	31.8	31.4	31.0	30.6
35	34.8	34.4	34.0	33.6	33.2	32.9	32.5	32.1	31.7
36	36.0	35.6	35.2	34.9	34.4	34.0	33.6	33.2	32.9
37	37.2	36.8	36.4	36.0	35.6	35.2	34.8	34.5	34.1
38	38.5	38.1	37.7	37.3	36.9	36.5	36.1	35.7	35.3
39	39.9	39.5	39.1	38.7	38.3	37.9	37.5	37.1	36.7
40	41.4	41.0	40.5	40.1	39.7	39.3	38.9	38.5	38.1
41	42.9	42.5	42.0	41.6	41.2	40.8	40.4	40.0	39.6
42	44.5	44.1	43.6	43.2	42.8	42.4	42.0	41.5	41.1
43	46.1	45.7	45.3	44.9	44.4	44.0	43.6	43.2	42.7
44	47.9	47.5	47.1	46.6	46.2	45.8	45.3	44.9	44.4
45	49.8	49.4	48.9	48.5	48.1	47.6	47.1	46.7	46.2
46	51.8	51.3	50.9	50.4	50.0	49.5	49.0	48.6	48.2
47	53.8	53.4	52.9	52.5	52.0	51.5	51.0	50.6	50.2
48	56.0	55.6	55.1	54.6	54.2	53.7	53.5	52.8	52.3
49	58.3	57.9	57.4	56.9	56.5	56.0	55.5	55.0	54.5
+ 50°	2°00.8′	2°00.3′	1°59.8′	1°59.3′	1°58.8′	1°58.4′	1°57.9′	1°57.4′	1°56.9′

252. To Establish a True Meridian with a Transit.* See that the instrument is in good adjustment. Allow sufficient time before an elongation of the star to "set up" the transit in a desirable position.† See that it is planted firmly, levelled carefully, and that the cross-wires are illuminated ‡ and properly focused. For convenience, set the vernier at zero, and unclamp the lower plate.

Observe the star a few minutes before its elongation, and keep the vertical wire on it by clamping the lower plate and using the slow-motion screws attached to it. When it has attained its greatest elongation, it will appear for a few moments to coincide with the vertical wire, and then retrograde. Unclamp the vernier plate, and turn off with it the amount of the azimuth § corresponding to the time and place as given in the table of the preceding article. The telescope will then point in the direction of the true meridian, and a mark should be set at as long range as practicable. If preferred, a stake may be set in line of sight at elongation, leaving the turning off of azimuth, and setting mark in meridian until the next day. It would be a little more accurate to take the mean of several observations — direct and reverse — at eastern and western elongations.

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* See Solar Attachment, Chapter II. Section I.; also Solar Compass, Chapter VI.

† Twenty to thirty minutes usually, depending upon the observer.

‡ Perforated silvered reflectors, for this purpose, can be obtained of instrument makers. Or, cover with white paper a board 12 or 15 inches square, make a perforation through it of 2 or 3 inches' diameter, and nail on a piece of board to hold a candle. This reflector may be attached to a staff, that it can slide up and down, and adjusted to the height of the telescope. It should be placed about a foot from the object-glass, so that the reflection from the paper will render the cross-wires visible, and at such a height that the star can be observed through the opening.

§ The meridian will lie to the west or east of the direction of the telescope when elongation was observed, according as the elongation was east or west. The azimuth must be turned off accordingly. Since the direction of the line from the observer's station to the star at elongation is known, the declination may be ascertained even before the meridian is established.

253. The Direction of the Meridian may be found, though less accurately, by means of a compass-sight and plumb-line.

Take a smooth plank about 3 feet in length, and fix it firmly level, and nearly east and west, on supports about 2 feet high. Attach a compass-sight to a board 6 or 8 inches square. At 15 or 20 feet north of the plank suspend a plumb-line by artificial supports, from some projecting point on a building or at the end of a staff projecting from a high window.

At fifteen or twenty minutes before the time of elongation of the star let an assistant hold a light in such position that the plumb-line may be distinctly seen through the compass-sight when placed on the plank. Move the sight until the plumb-line covers the star. Continue to keep the star and line in that relative position until the star begins to retrograde. The direction of the line of sight then corresponds to that observed by the transit as indicated in the preceding article; and applying the azimuth therein directed, the meridian may be set out.*

254. To Obtain approximately the Meridian. In old works on surveying it is stated that the north star (Polaris) is very nearly the meridian when it and Alioth † are in the same vertical plane or line. Others add the time that must elapse after one is vertically above the other before the north star makes its transit, and then by sighting the north star at that instant the meridian may be found.

This interval is, January, 1887, nearly half an hour. Other stars are now used, being more suitable. Zeta, or Mizar, the star next to Alioth in the tail of the Great Bear, comes to the meridian now almost simultaneously with Polaris and at a convenient time in the autumn and early winter to make the obser-

* If possible, a night should be chosen when there is no wind. The slightest disturbance in the air causes considerable vibration of the plumb-line. Using a heavy "bob," and allowing it to vibrate in a vessel of water, will tend to the accuracy of the result.

† Alioth, or Epsilon: the star in the tail of the Great Bear nearest the quadrilateral.

vation. Delta Cassiopeiæ, which is on the same side of the pole as Polaris, makes its transit also about the same time with it, and may be used in the spring and early summer when it is not practicable to make use of Zeta. To make either of these observations, use a transit, or a plumb-line and compass-sight, as explained in the preceding articles; watch the movements of the stars until they coincide with the plumb-line. The direction of the line of sight then will indicate quite closely the meridian.*

* The vertical plane including Zeta and Polaris is slowly moving eastward at about the rate of two minutes in six years. At the present time (1887) Polaris is on the meridian about two minutes before Zeta of the Great Bear, but in six years their respective upper and lower transits will coincide. The vertical plane, including Delta Cassiopeiæ and Polaris, is moving westward at about the same rate. Polaris now comes to the meridian about one minute before this star.

CHAPTER IV.

LAYING OUT AND DIVIDING LAND.

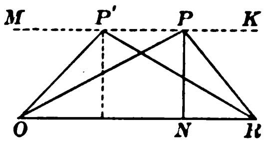
SECTION I.

Laying out Land.

A. TRIANGLES.

255. *To lay out a given quantity of land in the form of a triangle when the length of the base is given.*

Denote the given area in square chains or square rods by A,* the length of the base (referred to the same unit) by b, and the unknown altitude by x. Then $\frac{bx}{2}=A$, or $x=\frac{2A}{b}$. Measure the base, and at any point in it erect a perpendicular equal to $\frac{2A}{b}$. Join the extremity of the perpendicular with the extremities of the base, and a triangle fulfilling the conditions of the question will be exhibited.



256. *When the area is given and the base and altitude in a given ratio.*

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* Why not let A denote the number of acres?

Note. The locus of the vertices of the triangles answering the conditions is a line parallel to the given base and at a distance therefrom $=\frac{2A}{b}$.

Designate, as before, the area by A, the base and altitude respectively by x and y, and $\frac{x}{y}=\frac{m}{n}$ the ratio; then

$$y=\sqrt{\frac{2An}{m}},$$

$$x=\sqrt{\frac{2Am}{n}}.$$

Or, let $mx=$ base and $nx=$ altitude; then

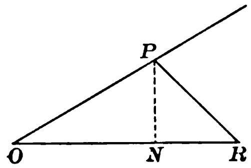
$$mnx^2=2A;$$

whence $$x=\sqrt{\frac{2A}{mn}},$$

and $$mx=\sqrt{\frac{2Am}{n}},$$

$$nx=\sqrt{\frac{2An}{m}}.$$

257. *Given area, base, and one side, to make a given angle with the base.*



Denote the base and area as above; then, since

$$PN=OP\sin O,$$

and $$A=\frac{b\times OP\sin O}{2},$$

$$OP=\frac{2A}{b\sin O}.$$

EXAMPLES.

1. Lay out an isosceles triangle to contain 6 acres, making the base $\frac{5}{8}$ the altitude. Locate the altitude and find its length.

2. Lay out a right triangle containing 4 acres, having the base $\frac{4}{3}$ the altitude.

3. It is required to lay out 2 acres in the form of a triangle, the base to be 7.50 chains. Find the length of a side of this triangle which shall make an angle of 40° with the base.

258. *To lay out an equilateral triangle to contain a given area.*

Let x = the side, and A = the area; then, since

$$\frac{x^2}{4}\sqrt{3} = A,$$

$$x = 2\sqrt{\frac{A}{\sqrt{3}}} = \sqrt{\frac{A}{.433}}.$$

259. *Given the area and the two sides, to lay out the triangle.*

Denote the given sides by b and c, the area by A, and the unknown angle by a; then, since

$$\frac{bc}{2}\sin a = A,$$

$$\sin a = \frac{2A}{bc}.$$

EXAMPLES.

1. Find the side of an equilateral triangle containing one acre.

2. What is the altitude of the triangle in Example 1? How far is it from the foot of the perpendicular to the centre of the figure? How far from either angle to the centre?

3. Lay out a triangle containing 2 acres, two sides to be 8 chains and 6 chains. What must be the included angle?

B. QUADRILATERALS.

SQUARES.

260. *To lay out a given quantity of land in the form of a square.*

Denote the required area in square chains or square rods by A, and one of the sides by x; then $x = \sqrt{A}$.

Measure a distance equal to the $\sqrt{A}$; at each extremity of this line erect a perpendicular of the same length; connect the extremities of the perpendiculars; the figure will be a square.

RECTANGLES.

261. *To lay out a given quantity of land in the form of a rectangle, one side being given.*

Denote, as before, the area by A, the given side by b, and by x the unknown side; then

$$x = \frac{A}{b}.$$

262. *Given the area, and the length to the breadth in a given ratio.*

Denote the area as above; the length and breadth respectively by x and y; m and n their ratio, so that

$$\frac{x}{y} = \frac{m}{n}.$$

Then, since $xy = A$, there results, by substitution,

$$x = \sqrt{\frac{Am}{n}},$$

$$y = \sqrt{\frac{An}{m}}.$$

Or, let $mx =$ the length, and $nx =$ the breadth; then

$$mnx^2 = A;$$

whence $$mx = \sqrt{\frac{Am}{n}},$$

and $$nx = \sqrt{\frac{An}{m}}.$$

263. *Given the area and the sum of the length and breadth.*

Denote the sum of the sides by S; the other notation as above; then

$$xy = A,$$

and $$x + y = S;$$

whence $$x = \frac{S + \sqrt{S^2 - 4A}}{2},$$

and $$y = \frac{S - \sqrt{S^2 - 4A}}{2}.$$

264. *Given the area and the difference of the length and breadth.*

Denote the difference of the sides by d; the other notation as before; then

$$xy = A,$$

$$x - y = d;$$

whence $$x = \frac{\sqrt{d^2 + 4A} + d}{2};$$

and $$y = \frac{\sqrt{d^2 + 4A} - d}{2}.$$

EXAMPLES.

1. How many rods in each side of a square lot which contains 1 acre? How many chains? How many yards?

2. Lay out 6 acres in the form of a rectangle, the length of one side to be 10 chains. Find the adjacent side.

3. Find the sides of a rectangle which shall contain 15 acres, and the length $\frac{5}{3}$ the breadth.

4. It is required to lay out a rectangle containing 12 acres, so that the sum of two adjacent sides shall equal 26 chains. What must be the length and breadth?

5. Find the sides of a rectangle which shall contain 640 square rods, and the difference of whose sides is 10 rods.

PARALLELOGRAMS.

265. *To lay out a given quantity of land in the form of a parallelogram, the base being given.*

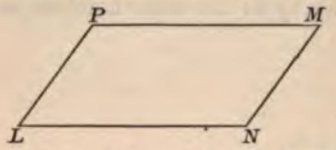
Denote the area and base, as above, and the altitude by x; then

$$x = \frac{A}{b}.$$

From any point in the base erect a perpendicular equal to $A \div b$, and through the extremity of the perpendicular run a line parallel and equal to the base: a parallelogram will thus be formed, fulfilling the conditions of the question.

266. *Given the area, one side, and adjacent angle.*

Denote the area by A, the base by b, the given angle by α, and by x the side adjacent; then



$$bx \sin \alpha = A;$$

whence $$x = \frac{A}{b \sin \alpha}.$$

Turn off at L and N, the given angle, measure the distances LP and NM, equal x, and connect M and P for the desired figure.

267. *Given the area and two adjacent sides, to find the included angle.*

Denote the sides by b and c, their included angle by α, and the area as above; then

$$bc \sin \alpha = A;$$

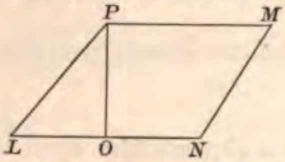
whence $$\sin \alpha = \frac{A}{bc}.$$

QUERIES. What will the figure become when $bc = A$? When $b = c$? May the product of bc be less than A? Can an expression for the sine be obtained for each case?

EXAMPLES.

1. It is required to lay out a parallelogram to contain 200 square rods, having a base of 20 rods. What must be the altitude?

2. If in Example 1 it is required that the perpendicular shall be erected at the middle of the base, and terminate at the angle P, as per figure, what length must be given LP, and what the magnitude of the angle L?



3. It is required to lay out a parallelogram to contain 48 square chains, one side to be 8 chains, and the adjacent angle 70°. What must be the length of the adjacent side?

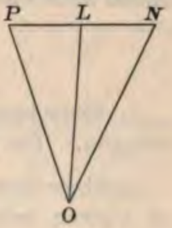
4. It is required to lay out a parallelogram to contain 2.4 acres, the base and adjacent side to be respectively 6 and 5 chains. Determine the altitude and tell how to lay out the land.

5. It is required to lay out a rhombus to contain 32 square chains, each side to be 6 chains. Compute the altitude, and state how to set out the tract; that is, to establish every corner.

C. POLYGONS.

268. *To lay out a given quantity of land in the form of a regular polygon of any number of sides.*

Denote the area by A, the number of the sides by n, and the length of one of the sides, as PN in the figure, by x. and ON, the radius of the circumscribed circle, by y; then



$$n \times OL \times LN = A.$$

But the angle $LON = \frac{180°}{n}$, $OL = \frac{x}{2} \cot \frac{180°}{n}$,

and $$LN = \frac{x}{2}.$$

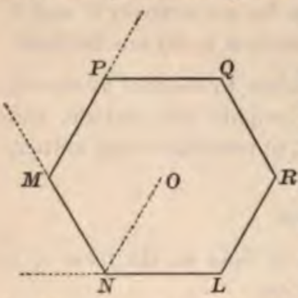
$$\therefore n \times \frac{x^2}{4} \times \cot \frac{180°}{n} = A.$$

Whence $$x = 2\sqrt{\frac{A \tan \frac{180°}{n}}{n}},$$

and $$y = \frac{x}{2 \sin \frac{180°}{n}} = \frac{x}{2} \operatorname{cosec} \frac{180°}{n}.$$

To lay out the tract, find by the above formula the length of one side, as LN, and stake it out. Then with an instrument for measuring angles (transit) set up at one end, as N, sight L, plunge the telescope, deflect $\frac{360°}{n}$ to M. Measure $NM = NL$. Remove the instrument to M, deflect from the prolongation of MN, as before, $\frac{360°}{n}$, measure MP, and so continue around, locating PQ,

and finally returning to L. The figure will be the polygon required.



In a small polygon, if the centre is fixed, it will be better to set up on it and measure therefrom a distance y to N, turn off an angle (the instrument still at the centre) $=\frac{360°}{n}$, and measure the same distance to M, again turning off an angle equal to the last, measure the same distance to P, and so on. A stake planted at each extremity of the radial lines will indicate the angular points of the tract.*

EXAMPLES.

1. Show how to lay out 1210 square yards in the form of an octagon. The same for a pentagon; decagon.

2. Show how by Article 57 the length of a side of a polygon of a given area and any number of sides, within the limits of the table, may be found.

D. CIRCLES AND ELLIPSES.

CIRCLES.

269. *To lay out a given quantity of land in the form of a circle.*

Denote the area by A, and the radius by x; then, since $\pi x^2 = A$,

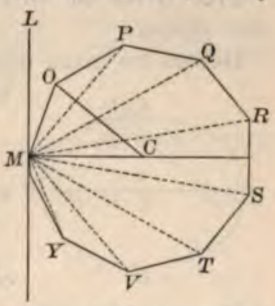
$$x = \sqrt{\frac{A}{\pi}}.$$

* A small lot, when great accuracy is not required, may be laid out by fastening one end of a tape at O, and with a length ON mark out a circumference by means of a pin. Then, beginning at any point in the circumference, measure off the distance x, and continue round the curve, driving a stake at the extremity of each side.

When great accuracy is not required, and small circles generally may be laid out by fastening one end of a tape at the centre, and with a common marking-pin held firmly and perpendicularly along it at x distance, describe and mark out the circumference.

270. Or, fix the extremities of two diameters run out perpendicular to each other, connect these with chords, and the versed sine of 45° to the known radius will give at once the perpendicular distance from the centre of each chord to the circumference. If necessary, the points thus located may be connected and others found in a similar manner. Or the perpendicular distance from any given point in a chord, of known length, to the circumference may be found by simple geometrical truths deduced from the right triangle.

271. If the circle is too large to be laid out as above, it may be accomplished by means of deflection angles as follows: With the known radius find the angle at the centre C, which is subtended by a chord OM of any length, say 100 feet; then with the instrument at M, deflect from the tangent ML to O an angle $LMO =$ one-half the central angle OCM, and measure the distance $MO = 100$ feet. O is a point in the curve.* Again deflect an angle $OMP =$ one-half the central angle, and measure $OP = 100$ feet to locate P, another point in curve,* and so on to locate the others. If there is a fractional part of the deflection angle at the closing point, the corresponding fractional part of 100 feet may be used.



* The angle formed by the tangent and chord drawn to the point of contact is measured by one-half the intercepted arc. An inscribed angle has the same measure.

Ellipses.

272. *To lay out a given quantity of land in the form of an ellipse, the greater and lesser diameters to be in a given ratio.*

Denote the area by A, the greater and less diameter (axes) respectively by mx and nx, in which m and n express the given ratio; then

$$\frac{\pi}{4} mx^2 = A;^*$$

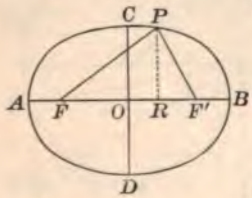
whence
$$x = \sqrt{\frac{4A}{\pi mn}},$$

$$mx = 2\sqrt{\frac{Am}{\pi n}},$$

and
$$nx = 2\sqrt{\frac{An}{\pi m}}.$$

273. *Given the area and one of the diameters, to find the other diameter.*

Denote the given diameter by d, the unknown by x, and the area as before; then, since



$$\frac{\pi}{4} dx = A,$$

we have
$$x = \frac{4A}{\pi d}.$$

An ellipse of small size may be laid out as follows:

Measure AB equal to the greater diameter (transverse axis), and from the centre O lay off $OF = OF'$, each equal to the square root of the difference of the squares of the semi-diameters OA, OC. Fix the ends of a steel wire or ribbon of the length AB at F and F', and with a continuous motion of a marking-pin P, held perpendicularly, keeping the wire taut, the required curve will be traced.

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* See any work on General Geometry or Conic Sections for the area of an ellipse.

Or, having found the axis as above, P being any point in the curve, and PR perpendicular to AB at R, by setting off any number of points on AB, we may find from the proportion

$$\overline{PR}^2 : RB \times AR = \overline{OC}^2 : \overline{OA}^2,$$

the corresponding values of PR.

EXAMPLES.

1. Find the radius of a circle containing 1 acre.

2. Find the radius of a sector containing 20 square rods, the angle at the centre being 72°.

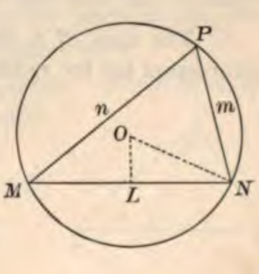
3. The area of an ellipse is 1 acre, its diameters in the ratio of 3 : 2; find their length.

4. An ellipse contains 80 square rods, its greater diameter 12 rods; find the lesser diameter.

5. The greater diameter of an elliptical plot of ground enclosed by a wall 1 foot thick is 240 links, and the lesser 160 links, inside measurements. What is the area of the plot, and how much land is occupied by the wall?

274. Let it be required to lay out a circle circumscribing a triangle, the sides of which are m, n, and p.

Let O be the centre of the circle, R the radius, OL a perpendicular to MN, $p = MN$, and the other sides as indicated in the figure.



Now $NL = \frac{p}{2}$, and angle $NOL = P$.

$$\therefore \frac{p}{2} = R \sin P,$$

or

$$R = \frac{\frac{p}{2}}{\sin P} = \frac{p}{2 \sin P}.$$

To find an expression for R in terms of the three sides, substitute for $\sin P$ its value

$$2 \sin \tfrac{1}{2} P \cos \tfrac{1}{2} P = 2 \frac{\sqrt{\tfrac{1}{2}s(\tfrac{1}{2}s - m)(\tfrac{1}{2}s - n)(\tfrac{1}{2}s - p)}}{mn};$$

whence $$R = \frac{mnp}{4\sqrt{\tfrac{1}{2}s(\tfrac{1}{2}s - m)(\tfrac{1}{2}s - n)(\tfrac{1}{2}s - p)}},$$

in which s represents the sum of the sides of the triangle.

ADDITIONAL EXAMPLES.

1. Circumscribe a circle about a triangle the sides of which are 10, 15, and 20 chains.

2. Find an expression for the radius with which to inscribe a circle in a triangle the sides of which are m, n, and p.

Ans. Twice the area of the triangle, divided by the sum of the sides.

3. Describe a circle in a triangle the sides of which are 30, 40, and 50 rods.

5. A circular walk, 6 feet wide, is to be made inside of a square which contains $\frac{1}{2}$ an acre; required the area of the walk.

5. The area of a square is 1 acre, and a circular walk is required to be made in it, touching each side at a point, of such a width that it will take up $\frac{1}{8}$ the area of the square. Find the width of the walk and the length of its centre line.

6. The area of a circular sector of $d°$ is m rods; find an expression for the radius. If $d = 60$ and $m = 300$, find R.

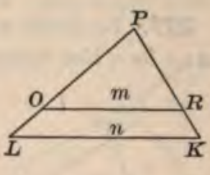
SECTION II.

Dividing Land.

A. TRIANGLES.

275. *To divide a given triangle into two parts in the ratio of $m : n$ by a line parallel to one side.*

To solve the problem fully, and furnish a check on the work, requires the location of the point O or R, and the length of OR. Denote OR by x, OP by y, and by p and k the sides respectively opposite the angles P and K; then



$$p^2 : x^2 = m + n : m,$$

or $$x = p\sqrt{\frac{m}{m+n}}.$$

Again, $$k^2 : y^2 = m + n : m;$$

whence $$y = k\sqrt{\frac{m}{m+n}}.$$

If the triangle is to be equally divided, then $m = n$, and there results

$$x = \frac{p}{2}\sqrt{2}, \quad \text{and} \quad y = \frac{k}{2}\sqrt{2}.$$

Queries. Is it necessary that LK be known to find either PO or PR? Must LK be given to find OR?

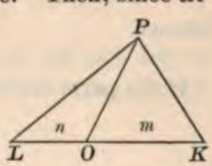
EXAMPLES.

1. Find a general expression for the distance RK (last figure).

2. Show how to divide the triangle LKP into four equivalent parts by lines parallel to the base.

276. *To divide a given triangle into two parts in the ratio of $m : n$ by a line from a vertex to the opposite side.*

Let PO be the line, $x = LO$, and p as above. Then, since triangles having the same altitude are to each other as their bases, we have



$$p : x = m + n : n;$$

whence $$x = \frac{pn'}{m+n}.$$

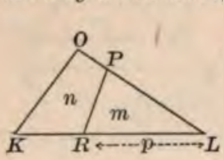
EXAMPLES.

1. Locate O on the supposition that the triangle is to be divided into two equivalent parts.

2. Find where the lines from P will meet the base dividing the triangle into three equivalent parts.

3. The same for any number n parts.

277. *To divide a given triangle into two parts in the ratio of $m : n$ by a line through a given point in one of the sides.*



Denoting PL by x, and the other sides in the usual manner, we have

$$m + n : m = ko : px;$$

whence $$x = \frac{mko}{p(m+n)}.$$

If the parts are to be equivalent, $m = n$, and there results

$$x = \frac{ko}{2p}.$$

EXAMPLE.

Show how the given triangle LKO may be divided into three equivalent parts by lines radiating from a given point R.

NOTE. The lines may or may not fall on the same side. Examine both cases.

278. The same conditions as in the last case, except the triangle is to be isosceles.

Using the same notation and figure as in that case, we have the following equality of ratios:

$$m + n : m = ko : x^2;$$

whence $$x = \sqrt{\frac{mko}{m+n}}.$$

If the parts are to be equivalent, $m = n$, and we have

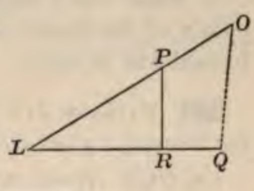
$$x = \sqrt{\frac{ko}{2}} = \tfrac{1}{2}\sqrt{2\,ko}.$$

EXAMPLE.

Show how to cut off a given area, in the form of an isosceles triangle, from the corner of a field, only the angle being given.

279. *The bearings of two sides of a field being given, to cut off a triangle having a given area by a line running in a given direction and intersecting the given sides.*

a. Suppose the division line is to make a right angle with either side. Let LO and LQ be the sides, the bearings of which are known, and PR the division line perpendicular to LQ. The angle L becomes known through the bearings of the sides which include it, and there follows



$$p \tan L = PR = l.$$

But $$\tfrac{1}{2}pl = \text{area} = A;$$

hence $$\tfrac{1}{2}p^2 \tan L = A,$$

and $$p = \sqrt{\frac{2A}{\tan L}}.$$

b. Suppose the angle at R is *oblique*. Denote LR by x, and LP by y, and find from the bearings the angles at P and R. Then from the two equations,

$$\tfrac{1}{2}xy \sin L = A$$

and $$\frac{x}{y} = \frac{\sin P}{\sin R}$$

may be deduced $$x = \sqrt{\frac{2A \sin P}{\sin L \sin R}}$$

and $$y = \sqrt{\frac{2A \sin R}{\sin L \sin P}}.$$

QUERY. Is it necessary that the bearings of LO and LQ be given if the field is triangular and the length of the sides given?

EXAMPLES.

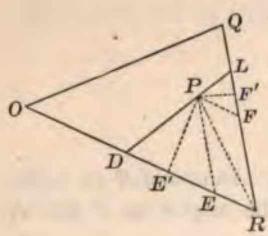
1. The bearing of LO (last figure) is N. 50° E., and LQ S. 82° E. It is required to find the lengths of LR and PR perpendicular thereto, so that 3 acres may be contained in the triangle PLR.

2. Suppose $LO = 10$, $LQ = 8$, and $OQ = 6$ chains. Find the position and length of the division line PR, which, with an angle $PRL = 84°$, will cut off a triangle PRL containing 2.5 acres.

3. Show that if three lines be drawn connecting the middle points of the three sides of a triangle, the four triangles thus formed will be equal.

280. *To divide in a given ratio a given triangle by a line passing through a given point within it.*

Let OQR represent the given triangle, and P the point within; DL the required division line, and $DRL : LDOQ = m : n$.



The point P may be located by co-ordinates as PF and PE, lines parallel respectively to OR and QR; or by its bearing and distance from one of the corners, as R; or by perpendicular distances PF', PE' from the sides. The distances PF and PE may be calculated if the direction and distance PR be known. Denote PF by d, PE by b, DR by x, and RL by y; then

$$x : y = d : y - b, \qquad \text{or } xy = bx + dy;$$

and
$$xy : qo = m : m + n, \text{ or } xy = \frac{mqo}{m+n};$$

hence
$$bx + dy = \frac{mqo}{m+n}.$$

Or, substituting the value of $y = \dfrac{mqo}{(m+n)x}$ from equation above, we obtain

$$bx + \frac{dmqo}{(m+n)x} = \frac{mqo}{m+n};$$

whence, by reducing and completing the square, there results

$$x = \frac{mqo \pm \sqrt{m^2q^2o^2 - 4\,bdmqo(m+n)}}{2\,b(m+n)},$$

$$y = \frac{2\,bmqo}{mqo \pm \sqrt{m^2q^2o^2 - 4\,bdmqo(m+n)}}.$$

If the question were to cut off from a corner of a tract of land a given area, by a line passing through a given point within, we might proceed more simply, as follows:

Denote the area to be cut off by A, and the other notation as above; then

$$xy \sin R = 2\,A,$$

and $$x : y = d : y - b;$$

whence there results

$$x = \frac{A \pm \sqrt{A^2 - 2\,Abd \sin R}}{b \sin R},$$

$$y = \frac{2\,Ab}{A \pm \sqrt{A^2 - 2\,Abd \sin R}}.$$

In each of the two preceding problems there are in general two division lines, as indicated by the double sign, fulfilling the conditions of the question. The student will point out when, if ever, one of these results will not practically answer the first case. Would either result answer practically the second? When, if ever, would the result be imaginary? Why?

If P were located by its distance PR, and the angle PRL or PRD, the lines PF and PE could be calculated, as before remarked, and the solution above given made applicable; or we may proceed as follows:

Denote PR by d, $d \sin PRD$ by b, $d \sin PRL$ by c, and the other notation as above; then

$$xy \sin DRL = 2\,A,$$

$$bx + cy = 2\,A.$$

Substituting the value of y from the first equation in the second, and reducing, there results,

$$x^2 - \frac{2\,Ax}{b} = -\frac{2\,cA}{b \sin R};$$

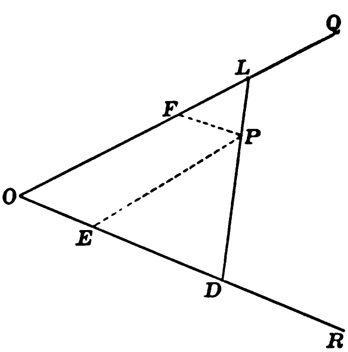
whence
$$x=\frac{A}{b}\pm\sqrt{\frac{A^2}{b^2}-\frac{2cA}{b\sin R}},$$

$$y=\frac{2A}{\left[\frac{A}{b}\pm\sqrt{\frac{A^2}{b^2}-\frac{2cA}{b\sin R}}\right]\sin R};$$

or,
$$y=\frac{2Ab}{A\sin R\pm\sqrt{A^2\sin^2 R-2bcA\sin R}}.$$

EXAMPLES.

1. Given the three sides of a triangular tract of land (see last figure), $QR=17$, $OQ=19$, and $OR=22$ chains, to divide it into two equivalent parts by a line passing through a point P, within the field. PF and $PE=$ respectively 4 and 9.50 chains. The location and length of the division line are required.



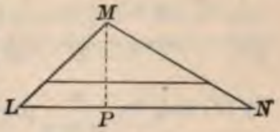
2. It is required to cut off from the angle O, which is 60°, a triangular field to contain 10 acres, by a line DL passing through a point P. The distances PF and PE being 4 and 12 chains respectively, the location and length of the division line are required.

3. Given the angle $ORQ=56°$ (see last figure but one), $PRL=20°$, and $PR=12$ chains. It is required to cut off a

triangle DRL, containing 8 acres, by a line DL passing through the point P. The location and length of the division line are required.

4. Divide a triangular piece of land into three equal parts by lines radiating from a point within.

SUGGESTION. The locus of the vertices of all triangles having the base LN and one-third the area of LMN is a line parallel to LN and at $\frac{1}{3}$ the height PM. Similarly for any other side. Find point of intersection.



5. Apply the principle employed in Example 4 to divide a triangle into three parts, in the ratio of 1, 2, and 3, by lines radiating from a point within.

6. Given two sides of a triangle 6 and 8 chains; it is required to locate a division line which shall cut off from the vertex an isosceles triangle whose area shall be to the area of the given triangle as 3 : 4.

7. Given the sides of a triangle 8, 10, and 12 chains; it is required to divide it into a triangle and a trapezium, the ratio of the former to the latter as 2 : 3, by a line extending from the middle of the longest side to some point on the medium side.

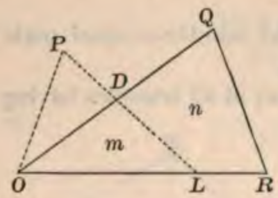
The location of this point and the length of the division line are required.

8. Divide the triangle given in Example 7 into three equivalent parts by lines radiating from the middle of the longest side. Locate the extremities of the division lines.

9. An angle QOP of a field $= 42° \ 30'$; it is required to cut off from some point D, in the line OP, by a line DL, making an angle $LDO = 78° \ 30'$, a triangle containing 2 acres. Locate the division line, and determine its length.

10. The sides of a triangle are 16, 18, and 24 chains; it is required to divide it into two parts in the ratio of 2 : 3 by a line perpendicular to the longest side. Locate the division line, and determine its length.

281. *To divide a given triangle in a given ratio by a line passing through a given point without it.*



Let ORQ represent the triangle, P the point given by the angle POQ and distance OP, DL the line which shall divide the triangle, so that

$$ODL : DLRQ = m : n.$$

Denote OP by b, OL by x, OD by y, the angle DOL by O, the angle POD by O', and the $\frac{m}{m+n}$ part of the area by A; then

$$\tfrac{1}{2}xy \sin O = A; \qquad (1)$$

also
$$\tfrac{1}{2}by \sin O' = \text{area } POD, \qquad (2)$$

and
$$\tfrac{1}{2}bx \sin (O + O') = \text{area } POL. \qquad (3)$$

$$\therefore \tfrac{1}{2}bx \sin (O + O) - \tfrac{1}{2}by \sin O' = A. \qquad (4)$$

Substituting in the last equation the value of y taken from (1) and reducing, there results,

$$bx \sin (O + O') - \frac{2\,Ab \sin O'}{x \sin O} = 2\,A;$$

or,
$$x^2 - \frac{2\,Ax}{b \sin (O + O')} = \frac{2\,A \sin O'}{\sin O \sin (O + O')};$$

whence
$$x = \frac{A}{b \sin (O + O')} \pm \sqrt{\frac{2\,A \sin O'}{\sin O \sin (O + O')} + \frac{A^2}{b^2 \sin^2 (O + O')}}.$$

y may be found by substituting the value thus obtained for x, and thence the length of the division line DL.

EXAMPLES.

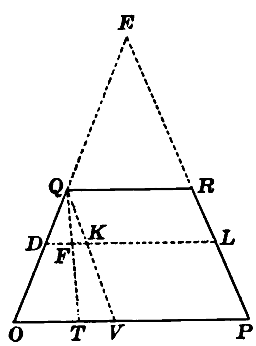
Given, in the triangle OQR, $OR = 18.40$ chains, $RQ = 10.20$ chains, $QO = 20.60$ chains, $OP = 9.50$ chains, and the angle

$POQ = 28° 30'$, to divide the triangle into two parts so that $OLD : DLRQ = 3 : 4$. The position and length of the division line DL are required.

B. QUADRILATERALS.

TRAPEZOIDS.

282. *Given the parallel sides of a trapezoid and the perpendicular distance between them, to divide it by a line parallel to these sides into two parts having a given ratio.*



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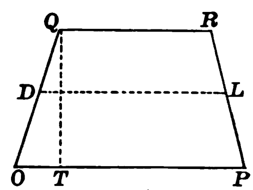

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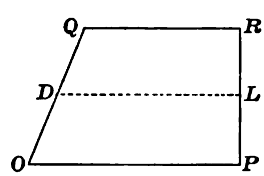


(e)

Let $OPQR$ (Fig. c) be the trapezoid, the sides OP, OQ, and the perpendicular distance QT between the bases being given. It is required to divide it by a line DL, so that $OPLD : DLRQ = m : n$; that is, practically to locate and determine the length of the division line DL.

Denote the lower base by b, the upper base by b', the perpendicular distance between the bases by h, the perpendicular distance between the upper base and division line by x, the length of the division line by y, and the area $OPQR$ by A. Draw QV parallel to RP; then the similar triangles give

$$OV : DK = QT : QF.$$

Or, $$OP - QR : DL - QR = QT : QF.$$

Or, substituting proper values,

$$b - b' : y - b' = h : x;$$

whence $$x = \frac{(y - b')\,h}{b - b'}. \tag{1}$$

But the area of $DLQR = (y + b')\dfrac{x}{2} = \dfrac{An}{m + n}$.

Representing for convenience the right-hand member of the last equation by A', we may write

$$xy + b'x = 2\,A',$$

and $$y = \frac{2\,A'}{x} - b'. \tag{2}$$

Substituting the value of x from (1) in (2) and reducing, there results

$$y = \sqrt{\frac{2\,A'(b - b')}{h} + b'^2},$$

and $$x = \frac{-\,b'h \pm \sqrt{2\,A'h(b - b') + b'^2h^2}}{b - b'}.$$

Restoring the value of A', we obtain,

$$y = \sqrt{\frac{2\,An}{h\,(m + n)}(b - b') + b'^2},$$

$$x = \frac{-\,b'h \pm \sqrt{\dfrac{n}{m + n}\,2\,Ah(b - b') + b'^2h^2}}{b - b'}.$$

The student may indicate how he would trace out on the field the division line thus found.

283. If instead of the perpendicular distance there be given one of the sloping sides, as OQ (Fig. c).

Denote OQ by d, OD by x, and the other notation as above. Produce the sides until they meet in some point E; then

$$OPE : QRE = b^2 : b'^2,$$

$$DLE : QRE = y^2 : b'^2;$$

or, by division, $OPQR : QRE = b^2 - b'^2 : b'^2,$

and $DLQR : QRE = y^2 - b'^2 : b'^2;$

whence $OPQR : DLQR = b^2 - b'^2 : y^2 - b'^2.$

By division $OPLD : DLQR = b^2 - y^2 : y^2 - b'^2;$

inserting values, $m : n = b^2 - y^2 : y^2 - b'^2;$

whence $y = \sqrt{\frac{b^2n + b'^2m}{m+n}}.$

The similar triangles OVQ and QDK give

$$b - b' : y - b' = d : d - x.$$

$$\therefore\ x = \frac{d(b-y)}{b-b'};$$

or, $x = \frac{d}{b-b'}\left[b - \sqrt{\frac{b^2n + b'^2m}{m+n}}\right].$

In Figure d, the unknown sides are symmetrical with respect to a line joining the centres of the parallel sides; in Figure e, PR is perpendicular to the parallel sides. The student will show what modification, if any, may be made in the formulas of the two preceding cases for either of these.

EXAMPLES.

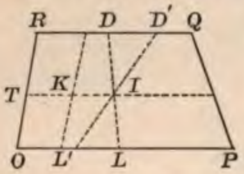
1. Given $OP = 20$ chains, $QR = 15$ chains, $QT = 18$ chains, to find the length of the division line DL, so that $QRLD$ shall contain two-thirds as much land as $OPLD$.

2. In Figure d, whose sides are equally inclined to the bases, $OP = 24$ chains, $QR = 16$ chains, and the perpendicular distance $QT = 20$ chains; it is required to locate the extremities of the division line DL, and determine its length, so that it shall divide the tract into two equivalent parts.

3. In Figure e, suppose $QR : OP : PR = 3 : 4 : 5$, and that the area $= 1750$ rods; locate and find the length of the division line DL that shall divide the tract, making $OPLD : QRLD = 3 : 4$.

284. *To divide a given trapezoid into two parts having a given ratio, by a line intersecting the parallel sides.*

Let $OPQR$ represent the trapezoid, and let it be required to divide it into two equal parts. It is evident if the bases be bisected, and a line, as DL, be drawn connecting the points of division, it will be the division line required.



Similarly, if the ratio is $m : n$; denote OP by b, and RQ by b'; then take $OL = \dfrac{mb}{m+n}$, $RD = \dfrac{mb'}{m+n}$, and join DL for the line required.

The student will give the reason.

If the division line is to pass through a given point D', obtain DL as above directed, then measure from D to D', and lay off this distance from L to L'. Join $D'L'$ for the division line required. Why?

To divide a trapezoid by a line perpendicular to the bases, or parallel to one of the non-parallel sides, divide the line joining the middle points of the non-parallel sides into two parts in the given ratio, and through the point of division run the required line. If $m : n$ is the ratio, and the bases b and b', the distance TK in the last figure $= \dfrac{m(b+b')}{2(m+n)}$.

The student will give the reason.

EXAMPLES.

1. Divide a given trapezoid into three equivalent parts by lines intersecting the parallel sides.

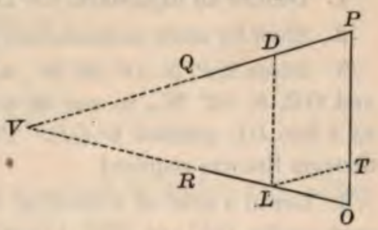
2. Divide a given trapezoid into three parts in the ratio of $m : n : p$, by lines intersecting the parallel sides.

3. The bases of a trapezoid are, $OP = 20$ chains, and $QR = 15$ chains. It is required to divide it into two parts in the ratio of $2:3$. $OL' = 8.50$ chains; locate D'.

4. Show that I, being the centre of the line connecting the middle of the bases of a trapezoid, is the point through which, if any straight line be drawn meeting the parallel sides, it will divide the trapezoid into two equivalent parts.

285. *Given one side and the adjacent angles of a tract of land, to cut off a trapezoid of a given area by a line parallel to the given side.*

Let PO be the given base, P and O the known angles indicating the direction of the sides PQ and OR. Denote the area $OPLD$, to be cut off by A; the given side OP by s, PD by y, OL by x, DL by z, and suppose



$$(O + P) < 180°.$$

Produce OR and PQ until they meet in V.

Then area OPV − area $LDV = A$;

or, $$\frac{s^2 \sin O \sin P}{\sin V} - \frac{z^2 \sin O \sin P}{\sin V} = 2A;$$

whence $$z = \sqrt{s^2 - \frac{2A \sin(O+P)}{\sin O \sin P}}.$$

When $(O + P) > 180°$, the produced lines meet in a point on the other side of OP, the sin $(O + P)$ is also negative, and therefore the fraction under the radical becomes positive. Draw LT parallel to VP; then in the triangle LOT, by sine proportion, $\sin L \,(= \sin V) : \sin T \,(= \sin P) = s - z : x$;

whence $$x = \frac{(s-z)\sin P}{\sin V}.$$

Similarly, $$y = \frac{(s-z)\sin O}{\sin V}.$$

REMARK. When great accuracy is not required, and especially if the tract is small and the sides nearly parallel, an approximate perpendicular distance between the bases OP and DL may be obtained by dividing the area to be cut off by the given side OP; then measure the perpendicular and a line through its extremity parallel to the base for an approximate division line. Calculate the area thus cut off, divide the difference between it and the required area by the approximate division line for a new perpendicular, and thence obtain more nearly the division line sought.

EXAMPLES.

1. Deduce an expression for DL by another method.

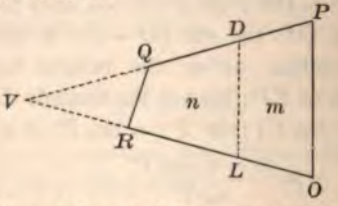
2. Show by other methods how OL or PD may be determined.

3. Given OP, N. 16° 30′ W., 8.40 chains; PQ, S. 62° 15′ W; and OR, S. 82° W., to cut off a trapezoid containing 4 acres, by a line DL parallel to OP. The position and length of the division line are required.

4. Given a side of a tract of land 20 chains, and the adjacent angles 105° and 130°, to cut off 36 acres by a line parallel to the given side. Required the position and length of the division line.

TRAPEZIUMS.

286. *Given the area of a trapezium, one of its sides and adjacent angles, to divide it by a line parallel to the given side into two parts having the ratio m : n.*



Produce the sides PQ and OR to meet in V. Let $OP = s$, $OZ = x$, $PD = y$, $DL = z$.

Calculate the area of

$$OPV = A' = \frac{s'^2 \sin O \sin P}{2 \sin V};$$

then $$A' - A = \text{area } DLV,$$

and the formula $$\frac{z^2 \sin O \sin P}{\sin V} = 2(A' - A)$$

gives $$z = \sqrt{\frac{2 \sin V (A' - A)}{\sin O \sin P}}.$$

Having found z, x and y may be deduced as in the foregoing case.

$$x = \frac{(s - z) \sin P}{\sin V},$$

$$y = \frac{(s - z) \sin O}{\sin V}.$$

REMARK. This problem may be solved by Article 285, taking for the given area to be cut off $\frac{m}{m+n}$.

EXAMPLE.

The boundaries of a trapezium are as follows:

(1) N. 2° E. 8.00 chains;
(2) N. $58\frac{1}{2}$° E. 13.85 "
(3) S. $31\frac{1}{2}$° E. 14.80 "
(4) S. $82\frac{1}{2}$° W. 20.00 "

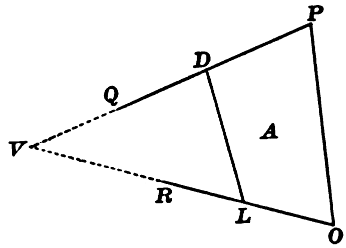
It is required to divide it into two equivalent parts by a line parallel to the third side. Locate it, and determine its length.

287. *Given the bearings of three adjacent sides of a tract of land and the length of the middle one, to cut off a trapezium having a given area, by a line running in a given direction.*

Produce the sides PQ and OR till they meet at V. As before, denote OP by s, OL by x, PD by y, and LD by z. Obtain the angles from the bearings, calculate the area of

$$POV = A' = \frac{s^2 \sin O \sin P}{2 \sin V},$$

and find area $\quad DLV = A' - A.$



Whence the division line $DL = z$ may be found from the formula

$$\frac{z^2 \sin D \sin L}{2 \sin V} = A' - A,$$

or

$$z = \sqrt{\frac{2 \sin V (A' - A)}{\sin D \sin L}}.$$

By the sine proportion

$$VO = \frac{s \sin P}{\sin V},$$

and

$$VL = \frac{z \sin D}{\sin V};$$

whence

$$VO - VL = LO = x = \frac{s \sin P - z \sin D}{\sin V},$$

and

$$y = \frac{s \sin O - z \sin L}{\sin V}.$$

REMARK. If $(O + P) > 180°$, $A' - A$ in the equation for z will become $A' + A$, and in the formulas for x and y the signs in the numerators will be interchanged, or

$$x = \frac{z \sin D - s \sin P}{\sin V},$$

and

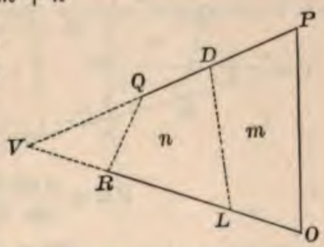
$$y = \frac{z \sin L - s \sin O}{\sin V}.$$

EXAMPLE.

Given LO, S. 76° E.; OP, N. 8° W. 12.40 chains; PD, S. 72° W.; it is required to cut off 7 acres by a line bearing N. 23° W. The length of the division line and the distances OL and DP are to be computed.

288. *Given a trapezium, to divide it into two parts having a given ratio, by a line extending from a given point in one of the sides.*

Let $OPQR$ represent the trapezium the area of which is A, m and n the given ratio. Prolong the sides PQ and OR till they meet in V. Let $OR = v$, the division line $DL = z$, RL the given distance to the point $L = d$, and $QD = y$. Calculate the area of $QRV = A'$, and add it to $\frac{n}{m+n}A$, thereby obtaining area of DLV.



Find by the sine proportion VR, and add it to RL, thus obtaining VL.

Putting $VD = x$, and $VL = b$,

$$bx \sin V = 2\left(\frac{n}{m+n}A + A'\right).$$

Whence $x = VD$ may be found.

Finally, with the two sides VD and VL and the included angle V, compute the angle L, and the direction and length of the division line DL; y may be calculated by a preceding method to check the work.

EXAMPLES.

1. Given in a trapezium $MNOP$ (no figure):

	MN,	13.00 chains;
	NO,	7.30 "
	OP,	10.40 "
	PM,	11.10 "
and diagonal	PN,	13.70 "

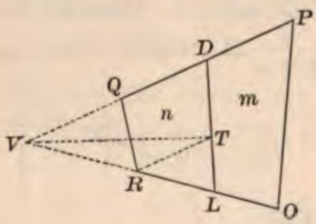
It is required to divide it into two equivalent parts by a line running from a point in the side MN, 6 chains from M. Find

the length of the division line and locate the other extremity of it.

2. Divide the tract described in Example 1 into two parts, in the ratio of 3 : 4, by a line DL running from some point in MN, and falling perpendicularly upon PO. The part $PMDL$ is to be the greater. Locate the line required, and determine its length.

289. *Given a trapezium, to divide it into two parts having a given ratio, by a line passing through a given point within the tract.*

Let $OPQR$ represent the given trapezium T, the point within it, given by its bearing and distance from some angle, as R. Produce the sides OR and PQ to meet in V. Denote the ratio by m and n, the area $OPQR$ by A, QR by v, DL by z, VL by x, and VO by y. Find by the sine proportion



$$VR = \frac{v \sin Q}{\sin V}, \quad VQ = \frac{v \sin R}{\sin V},$$

and thence the area $VQR = A'$. Then in the triangle VRT, having two sides and the included angle, compute VT, which call b, and the angle $TVR = \alpha$. Putting $V - \alpha = \beta$, and $\frac{n}{m+n} A + A' = A''$, the following equations may be written:

$$xy \sin V = 2A'', \tag{1}$$

and
$$bx \sin \alpha + by \sin \beta = 2A''. \tag{2}$$

Substituting in (2) the value of y from (1), and reducing, there results,

$$x = \frac{A''}{b \sin \alpha} \pm \sqrt{\frac{A''^2}{b^2 \sin^2 \alpha} - \frac{2A'' \sin \beta}{\sin \alpha \sin V}},$$

and $$RL = x - VR = \frac{A''}{b \sin \alpha} \pm \sqrt{\frac{A''^2}{b^2 \sin^2 \alpha} - \frac{2A'' \sin \beta}{\sin \alpha \sin V}} - \frac{v \sin Q}{\sin V}$$

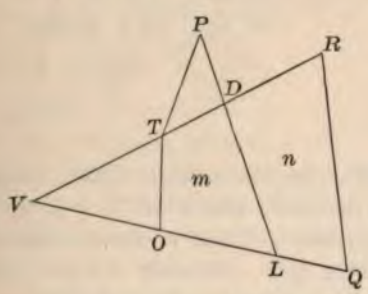
EXAMPLE.

Given the boundaries of a trapezium as follows:

(1)	N.	$16\frac{1}{4}°$ W.	24.63	chains;
(2)	S.	79° W.	27.00	"
(3)	S.	$\frac{1}{4}°$ W.	34.28	"
(4)	N.	65° E.	37.20	"

To divide it into two equivalent parts by a line extending from the first to the third side, and passing through a point 20 chains distant from the first and second corners. Locate the line and find its length.

290. *Given a trapezium, to divide it into two parts having a given ratio, by a line passing through a given point without the tract.*



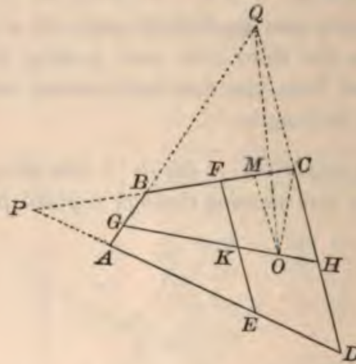
Let $OQRT$ represent the trapezium given by the bearings and distances of its sides, P the point without, located by its bearing and distance from T, the ratio $m:n$. Extend the sides RT and QO until they meet in V. Then the problem may be solved in a similar manner to that in Article 281.

291. *Given a trapezium, to divide it into four equivalent parts, by two lines intersecting opposite sides, one of the division lines being parallel to one of the given sides of the tract.*

Let $ABCD$ represent the given trapezium, FE the division line parallel to DC, and GH the other division line. It is re-

quired to locate both division lines. Prolong the sides AD and BC to meet in P; also DC and AB to Q. Find AE and EF by methods already given.

Now, any line cutting the parallel sides of a trapezoid and dividing it into two equivalent parts must pass through a point O (the middle of the middle line between the bases). See Article 284. Hence MO becomes known $= \frac{1}{4}(CD + EF)$, and



also $MC = \frac{1}{2}FC$. In the triangle OMC, compute the angle MCO and the line OC; add $\angle MCO$ to $\angle MCQ$, and having previously calculated QC, find in the triangle QCO the angle CQO and the side QO. Subtract $\angle CQO$ from $\angle CQB$ and obtain $\angle OQB$. Then putting the side $QO = a$, $QH = x$, and $QG = y$, we may write the following equations:

$$xy \sin HQG = 2 \text{ area } HQG,$$

$$ax \sin CQO + ay \sin OQG = 2 \text{ area } HQG.$$

From these equations obtain y. Subtract it from AQ, found by sine proportion, and the distance from the corner A to the extremity of the division line GH at G will be the result.

Then in the triangle QGH find QH; whence the length and bearing of GH may be computed.

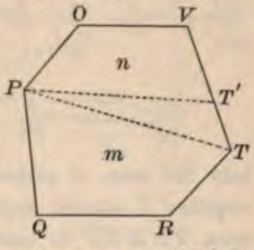
EXAMPLE.

It is required to divide the farm described in 288 (Example 1) into four equivalent parts by two lines intersecting opposite sides; one of the division lines is to be parallel to the first side. Locate the division lines, and determine their lengths.

C. POLYGONS.

292. *Given a polygon, to divide it into two parts having a given ratio, or to cut off a given area, by a line through a given point.*

Let $OPQRTV$ represent the polygon given by its bearings and distances, or angles and sides, and suppose the line be required to run from P, either an angle or any given point in a side. Calculate the area of the polygon, and take the $\frac{m}{m+n}$ part of it as the area to be cut off to the right of the line extending from P.



Run a *trial line** from P as PT, calculate the area of $PQRT$, and determine whether the area thus cut off is too small or too large, and how much. Suppose it is too small; then the extremity T of the division line PT must be moved towards V to some point T'. To find this point, denote TT' by x, the angle $T'TP$ by T, the distance PT by b, and the area of the triangle PTT' by a; then

$$\tfrac{1}{2}\, bx \sin T = a,$$

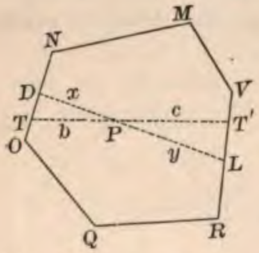
from which we find $$x = \frac{2a}{b \sin T}.$$

* The bearing and distance of PT may be calculated from the data given — without a trial line — as in supplying omissions. If, however, this is done, the surveyor should not omit to measure the division line to verify his work. In fact, it is the best practice, no matter what method is adopted to obtain the division line, to always test the computation by measurement.

This distance measured from T to T' will locate T', a point which connected with P will give the division line sought.

293. *Given a polygon, to divide it into two parts in a given ratio, or to cut off a given area, by a line through a given point within the tract.*

Let the marginal figure represent the tract, P the given point.



If the area to be cut off is not directly given, calculate the contents of the tract, and then by the ratio determine the quantity to be cut off, and denote it by A. Run a trial line TT' through P, dividing the polygon as nearly as may be judged in the required manner. Measure $TP = b$, $PT' = c$, and the angles T and T'. Calculate the area of either part of the polygon, and thus ascertain whether T should approach or recede from O. Suppose the area $TNMVT'$ is calculated and found too small by a quantity a, and that DL represents the division line. Put $DP = x$, $PL = y$, the angle $PTD = T$, $PT'L = T'$, and the angle at the point $P = P$, which is required, since that will indicate the direction of the division line.

Then $$\tfrac{1}{2}\, cy \sin P - \tfrac{1}{2}\, bx \sin P = a \qquad (1)$$

$$x = \frac{b \sin T}{\sin(T + P)}, \qquad (2)$$

$$y = \frac{c \sin T'}{\sin(T' + P)}. \qquad (3)$$

Substituting the values of x and y from (2) and (3) in (1), there results

$$\frac{c^2 \sin T' \sin P}{\sin(T' + P)} - \frac{b^2 \sin T \sin P}{\sin(T + P)} = 2a. \qquad (4)$$

Expanding the denominators, dividing each fraction, numerator and denominator, by its numerator, and writing for $\frac{\cos}{\sin}$ the cot, there results

$$\frac{c^2}{\cot P + \cot T'} - \frac{b^2}{\cot P + \cot T} = 2a. \tag{5}$$

Putting $\cot P = p$, $\cot T = t$, and $\cot T' = t'$, we may write more simply:

$$\frac{c^2}{p+t'} - \frac{b^2}{p+t} = 2a,$$

or

$$p^2 + t + t' - \frac{c^2 - b^2}{2a} = \frac{tc^2 - b^2t'}{2a} - tt';$$

whence

$$p = -\frac{1}{2}\left(t + t' - \frac{c^2 - b^2}{2a}\right) \pm \sqrt{\frac{c^2t - b^2t'}{2a} - tt' + \left[\frac{1}{2}\left(t + t' - \frac{c^2 - b^2}{2a}\right)\right]^2}.$$

Restoring values, we have

$$\cot P = -\frac{1}{2}\left(\cot T + \cot T' - \frac{c^2 - b^2}{2a}\right)$$

$$\pm \sqrt{\frac{c^2 \cot T - b^2 \cot T'}{2a} - \cot T \cot T' + \frac{1}{4}\left(\cot T + \cot T' - \frac{c^2 - b^2}{2a}\right)^2}.$$

The problem may be simplified when it is practicable to run the trial line at right angles to one of the sides of the polygon. In the tract given, suppose TT' to be run perpendicularly to RV; then $\cot T' = 0$, and Equation (5) may be written

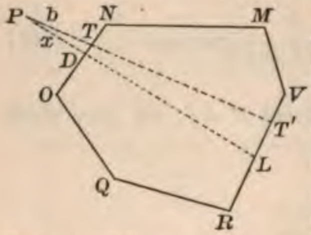
$$\frac{c^2}{\cot P} - \frac{b^2}{\cot P + \cot T} = 2a,$$

and

$$\cot P = -\frac{1}{2}\left(\cot T - \frac{c^2 - b^2}{2a}\right) \pm \sqrt{\frac{c^2 \cot T}{2a} + \frac{1}{4}\left(\cot T - \frac{c^2 - b^2}{2a}\right)^2}.$$

294. *Given a polygon, to cut off a given area by a line passing through a given point without the tract.*

Let the marginal figure represent the case.



As in the preceding article, run a trial line PT' from P, and suppose it is made perpendicular to RV. Calculate, as before, the content of $TNMVT'$, and ascertain the amount to be added to make the required area. Denote, as before, this area by a, $PT=b$, $PT'=c$, $PD=x$, $PL=y$; the angles at P, T, etc., by P, T, etc., and DL the division line; then

$$\tfrac{1}{2}cy\sin P-\tfrac{1}{2}bx\sin P=a, \qquad (1)$$

$$x=\frac{b\sin T}{\sin(P+T)}, \qquad (2)$$

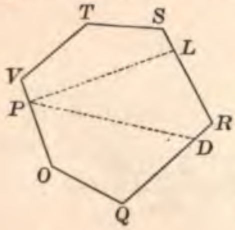
$$y=\frac{c}{\cos P}. \qquad (3)$$

Substituting the values of x and y from (2) and (3) in (1), and reducing as in the preceding problem, there results

$$\cot P=-\frac{1}{4a}\left(2a\cot T+b^2-c^2\right)$$
$$\pm\sqrt{\frac{c^2\cot T}{2a}+\left[\frac{1}{4a}\left(2a\cot T+b^2-c^2\right)\right]^2}.$$

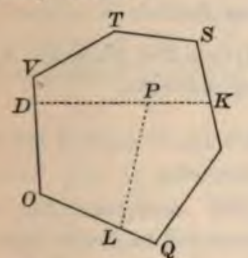
The student may verify the value found.

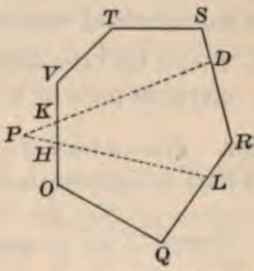
295. *Given a polygon, to divide it into three parts having a given ratio, by lines radiating from a given point.*



a. Let the figure represent the polygon, and suppose the point is in one side at P. Calculate the area of the whole tract and ascertain how much each division is to contain; then, by Article 292, cut off the required areas $PVTSL$ and $POQD$, and the problem is solved.

b. If the point is within the tract,* cut off, by Article 293, one required portion *DVTSKD* by a line *DK* through *P*, and by the preceding article divide the remainder by the line *PL* as required. If *PL* cuts off a quadrilateral on either side, Article 288 may be used.





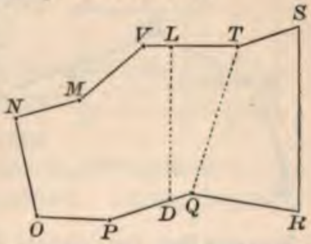
c. If the point is without,* proceed, as in Article 294, to cut off the required portion *KVTSDK* and *HOQLH*; the remainder *HLRDKH* will be the third portion.

It is evident that this principle may be extended to any number of parts.

296. *To cut off from a given polygon a given area by a line running in a given direction.*

Let the figure represent a tract which it is required to divide into two equivalent parts by a line *DL* parallel to *RS*.

Join *QT*, calculate its length and bearing, and also the content of *QRST*. Subtract said content from one-half the area of the whole tract, thereby obtaining the area *DLTQ*. Then, by Article 287, the length and position of the division line may be determined. It is evident that this principle may be extended to any number of subdivisions.



* Calculate the area, and ascertain, by the ratio, how much each division is to contain.

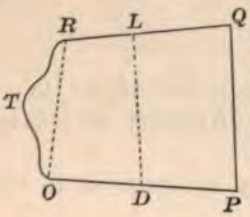
EXAMPLES.

1. The student may indicate how he would divide $DQRSTL$ into two equivalent parts by a line perpendicular to DL.

2. Show how to divide $DPONMVL$ into two equivalent parts by a line extending from the middle of DL.

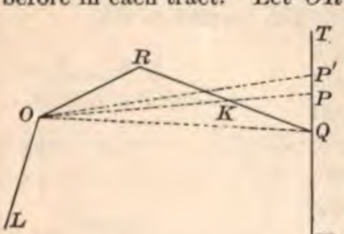
3. Divide the farm described in Article 234, Example 4, into two equivalent parts by a line running due east.

297. *From a tract of land of which one or more of the boundary lines is irregular, to cut off a given area.*



Let $OPQRT$ represent the tract which it is required to divide into two equal parts by a line DL parallel to PQ. Survey the land, taking offsets along RO, and calculate the area. Then the problem may be solved by Article 285.

298. To Straighten Boundary Lines. It is sometimes required to substitute a straight line for an irregular or crooked one between farms, and to leave the same quantity of land as before in each tract. Let ORQ be the line which it is required



to straighten by a line extending from O, the bearings and distances OR, RQ, and the bearing of QT being known. Run a trial line OP, noting the distances RK, OK, KP, and PQ, and calculate the areas of the triangle ROK and PQK. If it happens that the triangle ROK is equivalent to PQK, then OP will represent the line sought. If, as is generally the case, their areas are not equal, take the difference, and suppose in this case PQK the less. The problem, then, is simply this: Given one side, OP, of a triangle, and the direction of another,

PT, to cut off a given area by a line OP', to find the distance PP'. The solution is given in Article 257.

Otherwise, with the given bearings and distances calculate the area of the triangle ORQ and the length and bearing of the closing line OQ. Then, as before, having one side of a triangle, the direction of another, and the area, find QP' and the bearing and distance of OP'. The work should be verified by actual measurement of angle and distance.

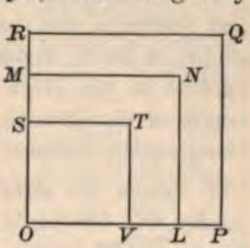
EXAMPLE.

Given OR, N. 59° 30′ E. 10.60 chains; RQ, S. 70° 15′ E. 19.32 chains; QT, N. 12° W., to find QP' and the bearing and distance of a line OP' which will straighten the boundary.

MISCELLANEOUS EXAMPLES.

1. It is required to lay out a lot to contain one acre, and having an equal frontage on two streets which intersect at an angle of 84° 40′. Locate the corners of the property.

2. From a square tract of land $OPQR$, which originally contained 160 acres, the southwest quarter was sold. It is required to find the uniform width of a strip $MNLVTS$ which shall contain 40 acres. How many rods of fencing will the tract require?



3. A rectangular tract of land 16.20 chains long, and 8.60 chains wide, valued at $200 per acre, is to be divided among three persons so that the first shall have $1,000 worth of it; the second, $900; and the third, the remainder. Locate the points of division on the long side.

4. The bearings of two sides of a triangle are OM, N 60° E., and ON, S. 40° E. It is required to cut off from the corner O an isosceles triangle containing 16 acres. Locate and find the length of the division line.

5. There is a farm in the form of a trapezium the area of which is given as 87.78 acres. The description of its boundaries is very much effaced; all that is legible is as follows:

Beginning at the northwest corner, thence (1) S. 76° E. (distance effaced); (2) S. 10° E., distance 25 chains; (3) S. 62° W. (distance effaced); (4) N. 6° W. (distance effaced).

It is required to perfect the description.

Suggestion. Prolong the second and fourth sides until they meet, and calculate the area of the triangle exterior to the tract. Add it to the given area, whence the length of the first side may be readily computed; the second and fourth sides may be found easily by either of two methods.

6. Required the length of a chord which will cut off one-third part of a circle whose radius is 100 feet.

Suggestion. Let 2θ denote the central angle, and r the radius, for convenience. Then $\frac{\pi r^2 \theta}{180} - \frac{r^2}{2} \sin 2\theta = \frac{\pi r^2}{3}$. Whence θ may be obtained, and hence the chord. The angle will be the same, no matter what the radius may be.

7. A trapezoidal field, the two parallel sides of which are 16 and 10 chains, and the perpendicular distance between them, 12 chains, is to be divided into two equivalent parts by a line parallel to the given sides. It is required to determine the length of the division line and locate its extremities, the sides being equally inclined to the bases.

8. Given the sides of a triangle OR, 280 yards; RQ, 200 yards; OQ, 300 yards; the distance from O to a point P outside the tract, 220 yards; and the angle POQ, 20°. It is required to run the centre line of a straight road through P and across the field, so as to divide the tract into two equal parts. Locate the points where the road will cross the triangle.

9. Given the sides of an irregular pentagon, and the perpendicular distance to each from a point within. Show how to divide the tracts into their equivalent parts. Also into three parts, having the ratio $m : n : p$.

10. Given in a trapezoid *MNOP* (no figure), $PM = 38.50$ chains; *MN*, one of the parallel sides, 64.80 chains; *NO*, 41 chains; the angle *M*, 85° 30′; and *N*, 75° 40′. It is required to divide the tract into two parts in the ratio of 2 : 3, by a line *DL* parallel to the parallel sides. The part *MNDL* is to be the greater. Find the length and location of the division line.

11. Given one side of a triangular field, 120 yards; the angle opposite, 20°; and the ratio of the other two sides, 7 : 10. Find the area.

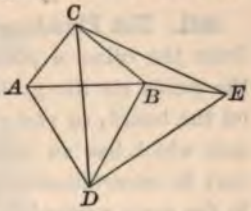
12. Show that the area of a trapezium is equal to one-half the product of its diagonals, by the sine of the angle of their intersection.

13. From a point within a triangular field, the sides of which were equal, I measured the distances to the three angles, and found them 12.5, 10, and 7.5 chains respectively; required the area.

Ans. 12 A. 1 R. 23 P.

The above problem is given in Gummere's Surveying, and by some surveyors it is considered difficult. The following is an outline of a solution; the student will supply what is wanting:

With the given distances form the triangle *ABC*. On *AB* describe an equilateral triangle *ABD*; join *CD* by a right line, and on it describe an equilateral triangle *CDE*. *CDE* is the triangle in question, and *B* the point within. For *BC* and *BD* are evidently two of the measured distances, and *BE*, it will be perceived, is the other, through the similarity and equality of the triangles *ADC* and *BDE*. To find the area of *CDE*, compute the angle *BAC*, whence the angle *CAD* becomes known; now with the two sides *AC*, *AD*, and the included angle *CAD*, *CD* is easily determined, and hence the required area of the triangle *CDE*.



CHAPTER V.

PLANE-TABLE SURVEYING.

299. The Plane-Table, as its name indicates, is a table or board which, being covered with paper, and having certain appliances for levelling and sighting, enables the surveyor to determine points and lines, and to delineate them on the paper in their relative position.

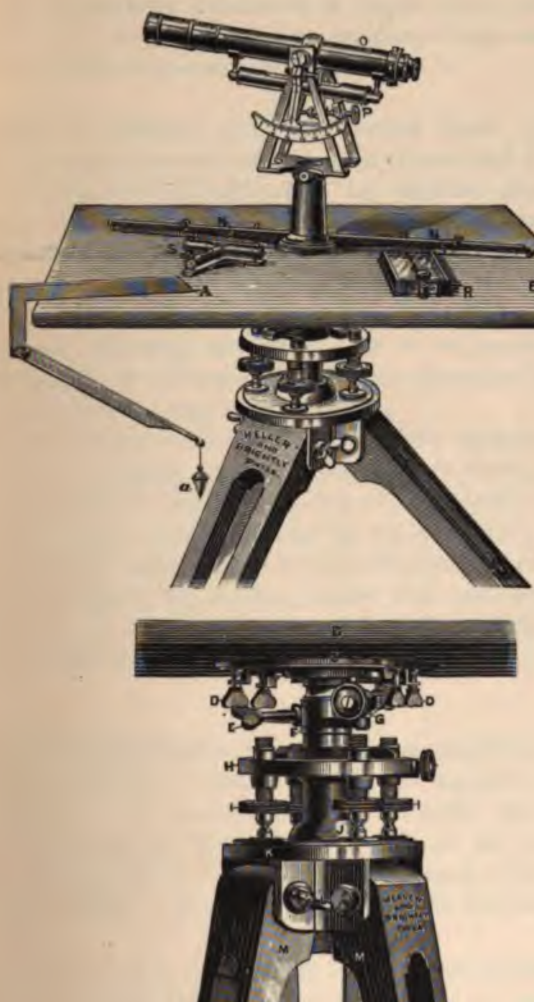
It is used in "filling in" the details of topographical work, and generally for the location of points where great accuracy is not required, on account of the rapidity with which surveys by it may be effected.

300. The Board, which is rectangular in shape, usually 24 by 30 inches, is made of pieces of well-seasoned wood joined advantageously together to prevent warping, and is furnished with rollers or clamps, by means of which the paper is kept securely stretched upon it.

301. The Plumbing-Arm, which is pointed at one end, and from the other a plummet is suspended, is used to determine the point on the ground immediately under its representative on the board, or *vice versa*. The lower part of it moves upon an axis which has an index at its extremity, by means of which it may be ascertained when the bob and point upon the table are in the same vertical line.

302. The Tripod and its Head are similar to those of the ordinary transit, though heavier.

A metallic plate, screwed fast to the table and having a solid conical spindle projecting from its centre, affords the means of attaching the head to the table.



PLANE-TABLE,

AS MADE BY HELLER & BRIGHTLY, PHILADELPHIA, PA.

The tripod-head admits of a slight lateral motion to the board, and is provided with levelling-clamp and tangent-screws similar to the common transit.

303. Of Alidades there are several kinds. One of the best, however, for ordinary purposes is indicated in the figure. It consists of a brass ruler or straight edge about 22 inches long and two inches wide, from which rises a column surmounted by a telescope. The power of the telescope at least equals that of the common transit, and it is provided with stadia wires, has an attached level, vertical arc, with the necessary adjusting movements. It is set on the column so that the line of collimation is in or near the same vertical plane with the bevelled edge of the ruler.

A parallel ruler allowing a very slight deviation from this plane is sometimes used, and the work is thereby facilitated. A small level is placed on the top of the column, which serves to indicate any unequal settling of the instrument. Two spirit levels at right angles to each other are placed upon the table to indicate when by the levelling-screws it is made horizontal; or, the levels are attached to the ruler of the alidade, one in the longitudinal direction of the ruler, the other perpendicular to it.

304. The Declinator is simply a box containing a magnetic needle which has a range of 12 or 15 degrees on each side of the zero. It is used in *orienting* the table; that is, to place a given point on the table over that on the ground which it represents, and to cause a line of the paper to lie in the same vertical plane, or parallel thereto, with its counterpart on the ground.

Before the table is removed from its first position, or at the time of drawing the first line of the survey, the declinator may be placed upon it, and the needle allowed to rest at zero; then a pencil drawn alongside the box will trace a north and south line, since the sides of the box are made parallel to the line of

zeros.* When the table is oriented at any other station, the declinator will give the same reading if placed along the same line.

ADJUSTMENTS.

305. From the nature of the service in some sections of the country, the plane-table is often necessarily subjected to rough usage, and there is a constant liability to a disturbance of the adjustments; still, in careful hands, a well-made instrument may be used under very unfavorable conditions for a long time without being perceptibly affected. One should not fail, however, to make occasional examinations, and while at work, if any difficulty be encountered which cannot otherwise be accounted for, it should lead directly to a scrutiny of the adjustments.

306. The Fiducial Edge of the Ruler. This should be a true, straight edge. Place the ruler upon a smooth surface, and draw a line along the edge, marking also the lines at the ends of the ruler. Reverse the ruler, and place the opposite ends upon the marked points, and again draw the line. If the two lines coincide, no adjustment is necessary; if not, the edge must be made true.

There is one deviation from a straight line which, by a very rare possibility, the edge of the ruler might assume, and yet not be shown by the above test; it is when a part is convex and a part similarly situated at the other end concave in exactly the same degree and proportion. In this case, on reversal, a line drawn along the edge of the ruler would be coincident with the other, though not a true right line; this can be tested by an exact straight edge.

307. The Level Attached to the Ruler. Place the instrument in the middle of the table, and bring the bubble to the centre by means of the levelling-screws of the table; draw lines

* Any other bearing which may be read will answer the purpose.

along the edge and ends of the ruler upon the board to show its exact position, then reverse 180°. If the bubble remain central, it is in adjustment; if not, correct it one-half by means of the levelling-screws of the table, and the other half by the adjusting-screws attached to the level. This should be repeated until the bubble keeps its central position, whichever way the ruler may be placed upon the table. This presupposes the plane of the board to be true. If two levels are on the rulers, they are examined and adjusted in a like manner.

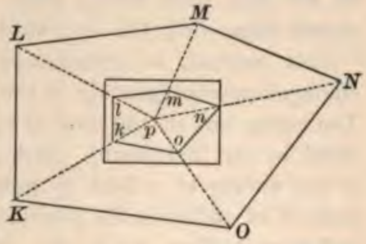
Great care should be exercised in manipulation, lest the table be disturbed.

308. Cause the line of sight to revolve in a vertical plane, make the bubble of the level attached to the telescope read zero when the line of sight is horizontal, and test the vernier arc for index error, each as in the transit.

Methods Employed in Plane-Table Surveying.

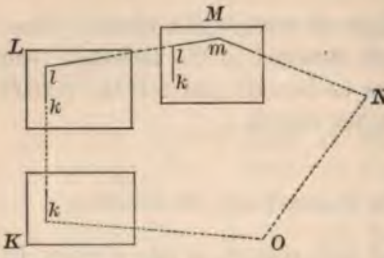
309. Points may be located with respect to one another by either of four methods. In actual practice, however, a combination of some of them is frequently employed.

310. By Radiation. Suppose it is required to make a plot of a field $KLMNO$, all the corners of which can be seen from a point P within it. Place the instrument at P, level and clamp it. Find a point p on the paper, directly over P on the ground, and, keeping the bevelled edge of the ruler on p, point the telescope to any corner of the tract, as K. By means of the stadia wires, or chain, obtain the distance PK, and lay it off to any desired scale in the direction of the point sighted,



thus plotting *pk*. In a similar manner, locate the other corners. Join by straight lines the points thus determined; and the resulting figure *klmno* will represent the tract surveyed. It is obvious that the position of objects such as buildings, trees, etc., if visible, may be determined by this method, and that it is immaterial whether the instrument be set up in the field or at one of the angles, providing all the stations can be seen from the point selected.

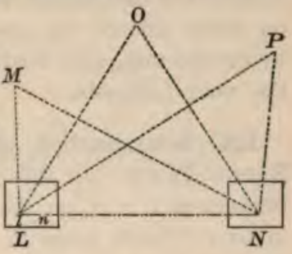
311. By Progression. This method requires the instrument to be set up at every station of the tract to be surveyed. Let *KLMNO* represent, as before, the field, and suppose the instrument is first placed at *K*, and that *k* on the paper designates this point. With the alidade directed towards *L*, draw along it an indefinite line. Obtain by stadia or chain the distance *KL*, and lay it off to a desired scale, thus locating *l*. Remove the instrument to *L*, orient it, and locate *m*. Continue in the same manner to locate *n* and *o*.



When the table is oriented at any station, as *M*, the line *ML* should lie in the vertical plane, with its representative *ml* on the plot, and, having gone round the tract, the last line should close with the first station *k*.

This method, in conjunction with the preceding, may be employed advantageously in the survey of a road, stream, etc. The centre line of the road or bank of the stream may be traversed by the instrument, placing it at each angle or bend, as in the survey of a field by progression, and determine by the method of radiation the position of prominent objects, such as buildings, bridges, trees, etc. If there be added to the above a sketch of the general features of the ground, a complete map will be had of the belt of country traversed.

312. By Intersections. Let it be required to plot the stations M, O, P. Measure carefully the base line LN, and draw to a convenient scale ln on the paper to represent it. At the extremities of this base line orient and point the instrument to the several stations. The intersections of the pairs of lines drawn from the base line to these stations will indicate their position on the plot. Their distances from the base line, if desired, may be obtained by applying the scale used in the construction of ln.



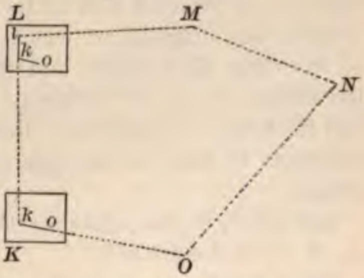
If a field or closed tract of land is to be surveyed, a portion or all of one side may be used as a base line, or a base may be chosen outside the tract.

This method is obviously well adapted to the mapping of harbors, shore lines, and generally to inaccessible points.

Of course in this, as in all triangulations, well-conditioned triangles give more satisfactory results; that is to say, avoid, if possible, angles less than 30° or greater than 150°.

313. By Resection. This method requires the measurement of one line and the accessibility of all the stations.

Let $KLMNO$ represent the points to be plotted.



Obtain the distance between two of them, as OK, lay it off on the table to a suitable scale, and let ok represent it. Orient the table at k, point the alidade to L, and draw along its fiducial edge an indefinite line. Remove the instrument to L, and orient it. Then with the alidade centring on o, point it in the direction of O, and draw a line along its

edge: this line will intersect kL in some point l, which will locate L on the plot. Through l draw a line towards M, remove the instrument to M, and proceed as before. Objects on either side of the lines may be determined by radiation or by intersection, and further details, if desired, sketched in as the work proceeds.

314. Determination of Position by Resection on Three Known Points. In this problem three stations, L, N, O, are plotted, as l, n, o, on the table, and the instrument being set up over a fourth point P, it is required to find the position of this point on the map. This is the three-point problem of which geometrical constructions and analytical solutions are given in Chapter II. Section IV. It may be solved thus: Fasten a sheet of tracing-paper on the board, fix a point p to represent the station at which the instrument is set; with the alidade centring on p, direct the telescope successively to L, O, and N, and draw lines of indefinite length along the ruler's edge towards these stations. Then if the tracing-paper be shifted until the three lines thus drawn coincide with the points l, o, and n, the point p will indicate the position of P.

The position of this point may now be transferred, by pricking, to the map, the tracing-paper removed, and the table oriented.

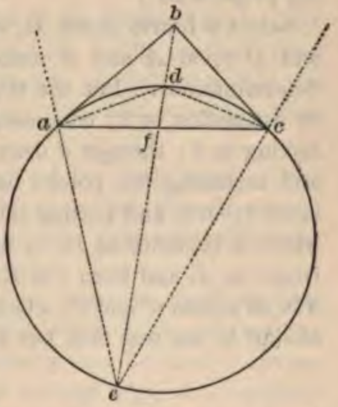
315.* Bessel's Method by Inscribed Quadrilateral. A quadrilateral is constructed with all the angles in the circumference of a circle, one diagonal of which passes through the middle one of the three fixed points and the point sought. On this line the alidade is set, the telescope directed to the middle point, and the table is *in position*. Resection upon the extreme points intersects in this line and determines the position of the point sought.

Let a, b, c, be the points on the sheet representing the signals A, B, C, in the ground.

The table is set up at the point to be determined (d) and

* Articles 315 and 316 are from the U. S. C. & G. S. Report for 1880.

levelled. The alidade is set upon the line *ca*, and *a* directed, by revolving the table, to its corresponding signal *A*, and the table clamped; then, with the alidade centring on *c*, the middle signal *B* is sighted with the telescope, and the line *ce* drawn along the edge of the ruler. The alidade is then set upon the line *ac*, and the telescope directed to the signal *C*, by revolving the table, and the table clamped. Then, with the alidade centring on *a*, the telescope is directed to the middle signal *B*, and the line *ae* is drawn along the edge of the ruler. The point *e* (the intersection of these two lines) will be in the line passing through the middle point and the point sought. Set the alidade upon the line *be*, direct *b* to the signal *B* by revolving the table, and the table will be in position. Clamp the table, centre the alidade upon *a*, direct the telescope to the signal *A*, and draw along the ruler the line *ad*. This will intersect the line *be* at the point sought. Resection upon *C*, centring the alidade on *c* in the same manner as upon *A*, will verify its position.

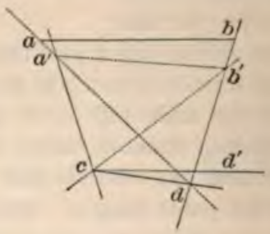


The opposite angles of the quadrilateral *adce* being supplementary, angle *ace* and angle *ade* are subtended by the same chord *ae* and *cae* and *cde* are subtended by the same chord *ce*; consequently, the intersection of *ae* and *ce* at *e* must fall on the line *db*; or, the segments of two intersecting chords in a circle being reciprocally proportional, the triangles *adf* and *cef* are similar, and the triangles *cdf* and *aef* are similar, and *d*, *f*, and *e* must be in a right line passing through *b*.

316. Determination of Position by Resection on Two Known Points. This is called the *two-point problem*, there being given

by their projections a, b, two points A and B, to put the plane-table in position at a third point C. (The capital letters refer to points on the ground, and the small ones to their corresponding projections.)

Select a fourth point D, such that the intersections from C and D upon A and B make sufficiently large angles for good determinations. Put the table approximately in position at D, by estimation or by compass, and draw the lines Aa, Bb, intersecting in d; through d draw a line to C. Then set up at C, and assuming the point c on the line dC at an estimated distance from d, and putting the table in a position parallel to that which is occupied at D, by means of the line cd, draw the lines from c to A, and from c to B. These will intersect the lines dA, dB, at points a' and b', which form with c and d a quadrilateral *similar* to the true one, but erroneous in size and position.



The angles which the lines ab and $a'b'$ make with each other is the error in position. By constructing now through c a line cd', making the same angle with cd as that which ab makes with $a'b'$, and directing this line cd' to D, the table will be brought into position, and the true point c can be found by the intersections of aA and bB.

Instead of transferring the angle of error by construction, we may conveniently proceed as follows, observing that the angle which the line $a'b'$ makes with ab is the error in the position of the table. As the table now stands, $a'b'$ is parallel with AB, but we want to turn it so that ab shall be parallel to the same. If, therefore, we place the alidade on $a'b'$, and set up a mark

in that direction, then place the alidade on *ab*, and turn the table until it again points to the mark, then *ab* will be parallel to *AB*, and the table is in position.

317. Practical Suggestions in using the Plane-Table.* The board should be placed so low as to be readily reached, even at the most remote corner, and yet high enough to enable the observer to take sight with comfort. This will bring it a little below the elbow.

Care must be taken that no part of the body touch or rest against the edge of the board. In using the alidade, steady the standard with the left hand, while the right swings the rear end of the ruler in the proper direction.

Thumb-tacks and rollers for holding down the sheet are both found objectionable, especially in high winds. The edges may be pasted underneath, or spring clamps may be used to advantage. A scale graduated upon the fiducial edge of the alidade is inconvenient, and in some positions impracticable and wasteful of time. A detached triangular boxwood or metal scale is greatly to be preferred. Umbrellas or shades, whilst a great relief to the eyes, are cumbersome and troublesome, and by blowing over on the table may cause damage or derangement. Colored glasses screening the eyes will be better, and by using tinted paper, as manilla, instead of white, still more relief is given, and the sheet can be kept cleaner.

Before leaving the station, and at any intervals not otherwise employed, the "check" shots should be tested to determine any displacement of the board.

Use as hard a pencil, and make as few lines, as possible. In locating points of contours, plot the distance at once along the edge of ruler by detached scale, making only a dot at the point which should receive the number of the contour.

Objects on a straight line may be quickly located by plotting the ends and determining the intermediate points by intersecting shots.

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* From *The Topographer*, by L. M. Haupt, C.E., Philadelphia.

EXERCISES WITH THE PLANE-TABLE.

1. Make a plane-table survey of a field, using one side as a base line.

2. Make a survey embracing 200 or 300 rods of a road or stream, locating prominent objects on either side.

3. Locate several points on the table by *intersections*, and check the work by *resection* from these points.

4. Locate a non-plotted point by resection on three known points — first method; check by Bessel's method.

CHAPTER VI.

THE SURVEY OF THE PUBLIC LANDS OF THE UNITED STATES.

THE SOLAR COMPASS.

318. A description of the Solar Compass, the instrument that is extensively used in the survey of the public lands, its adjustment and use, will be given before describing the method employed by the government in these surveys.

This instrument, so ingeniously contrived for readily determining a true meridian or north and south line, was invented by William A. Burt, of Michigan, and patented by him in 1836.

It has since come into general use in the surveys of United States public lands, the principal lines of which are required to be run with reference to the true meridian.

The arrangement of its sockets and plates is similar to that of the Surveyor's Transit, as shown in Chapter II. Section I., except that the sight-vanes are attached to the under plate or limb, and this revolves around the upper or vernier plate on which the solar apparatus is placed.

The limb is divided to half-degrees, is figured in two rows, as usual, and reads by the two opposite verniers to single minutes.

THE SOLAR APPARATUS.

319. The Solar Apparatus is seen in the place of the needle, and in fact operates as its substitute in the field.

It consists mainly of three arcs of circles, by which can be set off the latitude of a place, the declination of the sun, and the hour of the day.

These arcs, designated in the cut by the letters *a*, *b*, and *c*, are therefore termed the latitude, the declination, and the hour arcs respectively.

320. The Latitude Arc *a* has its centre of motion in two pivots, one of which is seen at *d*; the other is concealed in the cut.

It is moved either up or down within a hollow arc, seen in the cut, by a tangent-screw at *f*, and is securely fastened in any position by a clamp-screw.

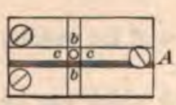
The latitude arc is graduated to quarter-degrees, and reads by its vernier *e* to single minutes; it has a range of about 35 degrees, so as to be adjustable to the latitude of any place in the United States.

321. The Declination Arc *b* is also graduated to quarter-degrees, and has a range of about 28 degrees.

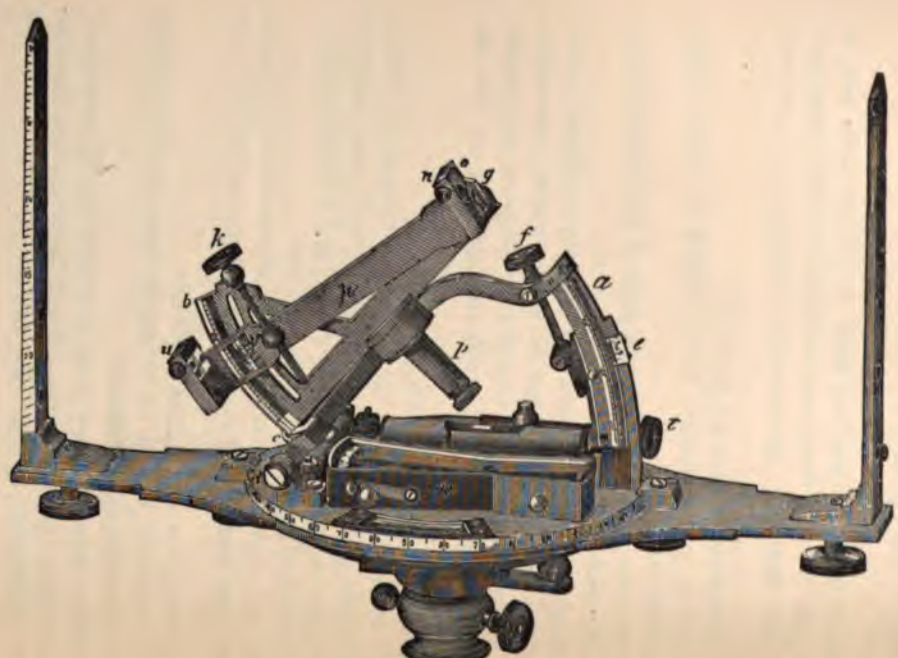
Its vernier *v*, reading to single minutes, is fixed to a movable arm *h*, having its centre of motion at the end of the declination arc at *g*; the arm is moved over the surface of the declination arc, and its vernier set to any reading by turning the head of the tangent-screw *k*. It is also securely clamped in any position by a screw, concealed in the engraving.

322. Solar Lenses and Lines. At each end of the arm *h* is a rectangular block of brass, in which is set a small convex lens, having its focus on the surface of a little silver plate *A* (marginal figure), fastened by screws to the inside of the opposite block.

On the surface of the plate are marked two sets of lines intersecting each other at right angles; of



these *bb* are termed the hour lines, and *cc* the equatorial lines, as having reference respectively to the hour of the day and the position of the sun in relation to the equator. In the cut the equatorial lines are those on the lower block, parallel to the surface of the



SOLAR COMPASS.
A description, etc., of Solar Attachments to Transit is given in Chapter II. pages 79–88.

hour arc c; the hour lines are of course those at right angles to the first.

323. Equatorial Sights. On the top of each of the rectangular blocks is seen a little sighting-piece, termed the equatorial sight, fastened to the block by a small, milled head-screw, so as to be detached at pleasure.

They are used, as will be explained hereafter, in adjusting the different parts of the solar apparatus.

324. The Hour Arc c is supported by the two pivots of the latitude arc already spoken of, and is also connected with that arc by a curved arm, as shown in the figure.

The hour arc has a range of about 120°, is divided to half-degrees, and figured in two series, designating both the hours and the degrees, the middle division being marked 12 and 90 on either side of the graduated lines.

325. The Polar Axis. Through the centre of the hour arc passes a hollow socket p containing the spindle of the declination arc, by means of which this arc can be moved from side to side over the surface of the hour arc, or turned completely round, as may be required.

The hour arc is read by the lower edge of the graduated side of the declination arc.

The axis of the declination arc, or indeed the whole socket p, is appropriately termed the polar axis.

326. The Adjuster. Besides the parts shown in the cut, there is also an arm used in the adjustment of the instrument as described hereafter, but laid aside in the box when that is effected.

The parts just described constitute properly the solar apparatus.

Besides these, however, are seen the needle-box n with its arc and tangent screw t, and the spirit levels, for bringing the whole instrument to a horizontal position.

327. **The Needle-Box** *n* has an arc of about 36 degrees in extent, divided to half-degrees, and figured from the centre or zero mark on either side.

The needle, which is made as in other instruments, except that the arms are of unequal lengths, is raised or lowered by a lever shown in the cut.

The needle-box is attached by a projecting arm to a tangent-screw *t*, by which it is moved about its centre, and its needle set to any variation.

This variation is also read off by the vernier on the end of the projecting arm, reading to three minutes a graduated arc, attached to the plate of the compass.

328. **The Levels** seen with the solar apparatus have ground-glass vials, and are adjustable at their ends like those of other instruments.

The edge of the circular plate on which the solar work is placed is divided and figured at intervals of 10 degrees, and numbered, as shown, from 0 to 90 on each side of the line of sight.

These graduations are used in connection with a little brass pin, seen in the centre of the plate, to obtain approximate bearings of lines, which are not important enough to require a close observation.

329. **Lines of Refraction**. The inside faces of the sights are also graduated and figured, to indicate the amount of refraction to be allowed when the sun is near the horizon.

PRINCIPLES OF THE SOLAR COMPASS.

330. The interval between two equatorial lines *cc*, in figure on page 276, as well as between the hour lines *bb*, is just sufficient to include the circular image of the sun, as formed by the solar lens on the opposite end of the revolving arm *h*, figure on page 277.

When, therefore, the instrument is made perfectly horizontal, the equatorial lines and the opposite lenses being accurately adjusted to each other by a previous operation, and the sun's image brought within the equatorial lines, his position in the heavens, with reference to the horizon, will be defined with precision.

Suppose the observation to be made at the time of one of the equinoxes; the arm *h*, set at zero on the declination arc *b*; and the polar axis *p*, placed exactly parallel to the axis of the earth.

Then the motion of the arm *h*, if revolved on the spindle of the declination arc around the hour circle *c*, will exactly correspond with the motion of the sun in the heavens, on the given day and at the place of observation; so that if the sun's image was brought between the lines *cc* in the morning, it would continue in the same position, passing neither above nor below the lines, as the arm was made to revolve in imitation of the motion of the sun about the earth.

In the morning, as the sun rises from the horizon, the arm *h* will be in a position nearly at right angles to that shown in the cut, the lens being turned towards the sun, and the silver plate on which his image is thrown directly opposite.

As the sun ascends, the arm must be moved around, until when he has reached the meridian, the graduated side of the declination arc will indicate 12 on the hour circle, and the arm *h*, the declination arc *b*, and the latitude arc *a* will be in the same plane.

As the sun declines from the meridian, the arm *h* must be moved in the same direction, until at sunset its position will be the exact reverse of that it occupied in the morning.

331. Allowance for Declination. Let us now suppose the observation made when the sun has passed the equinoctial point, and when his position is affected by declination.

By referring to the almanac, and setting off on the arc his declination for the given day and hour, we are still able to

determine his position with the same certainty as if he remained on the equator.

When the sun's declination is south, that is, from the 22d of September to the 20th of March in each year, the arc b is turned towards the plates of the compass, as shown in the engraving, and the solar lens o, with the silver plate opposite, are made use of in the surveys.

The remainder of the year the arc is turned from the plates, and the other lens and plate employed.

When the solar compass is accurately adjusted, and its plates made perfectly horizontal, the latitude of the place, and the declination of the sun for the given day and hour, being also set off on the respective arcs, *the image of the sun cannot be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth.* The slightest deviation from this position will cause the image to pass above or below the lines, and thus discover the error.

We thus, from the position of the sun in the solar system, obtain a certain direction absolutely unchangeable, from which to run our lines and measure the horizontal angles required.

This simple principle is not only the basis of the construction of the solar compass, but the sole cause of its superiority to the ordinary or magnetic instrument. For in a needle instrument the accuracy of the horizontal angles indicated, and therefore of all the observations made, depends upon "the delicacy of the needle, and the constancy with which it assumes a certain direction, termed the magnetic meridian."

The principal causes of error in the needle, briefly stated, are the dulling of the pivot, the loss of polarity in the needle, the influence of local attraction, and the effect of the sun's rays, producing the diurnal variation.

From all these imperfections the solar instrument is free.

The sights and the graduated limb being adjusted to the solar apparatus, and the latitude of the place and the declination of the sun also set off upon the respective arcs, we are able not only

to run the true meridian, or a due east and west course, but also to set off the horizontal angles with minuteness and accuracy from a direction which never changes, and is unaffected by attraction of any kind.

To Adjust the Solar Compass.

The adjustments of this instrument, with which the surveyor will have to do, are simple and few in number, and will now be given in order.

332. To Adjust the Levels. Proceed precisely as directed in the account of the other instruments we have described, by bringing the bubbles into the centre of the tubes by the levelling-screws of the tripod, and then reversing the instrument upon its spindle, and raising or lowering the ends of the tubes, until the bubbles will remain in the centre during a complete revolution of the instrument.

333. To Adjust the Equatorial Lines and Solar Lenses. First detach the arm h from the declination arc by withdrawing the screws shown in the cut from the ends of the posts of the tangent-screw k, and also the clamp-screw, and the conical pivot with its small screws by which the arm and declination arc are connected.

The arm h being thus removed, attach the adjuster in its place by replacing the conical pivot and screws, and insert the clamp-screw so as to clamp the adjuster at any point on the declination arc.

Now level the instrument, place the arm h on the adjuster, with the same side resting against the surface of the declination arc as before it was detached. Turn the instrument on its spindle so as to bring the solar lens to be adjusted in the direction of the sun, and raise or lower the adjuster on the declination arc, until it can be clamped in such a position as to bring the sun's image as near as may be between the equatorial lines on the opposite silver plate, and bring the image precisely into

position by the tangent of the latitude arc or the levelling-screws of the tripod. Then carefully turn the arm half-way over, until it rests upon the adjuster by the opposite faces of the rectangular blocks, and again observe the position of the sun's image.

If it remains between the lines as before, the lens and plate are in adjustment; if not, loosen the three screws which confine the plate to the block, and move the plate under their heads, until one-half the error in the position of the sun's image is removed.

Again bring the image between the lines, and repeat the operation until it will remain in the same situation, in both positions of the arm, when the adjustment will be completed.

To adjust the other lens and plate, reverse the arm end for end on the adjuster, and proceed precisely as in the former case, until the same result is attained.

In tightening the screws over the silver plate, care must be taken not to move the plate.

This adjustment now being complete, the adjuster should be removed, and the arm h with its attachments replaced as before.

334. To Adjust the Vernier of the Declination Arc. Having levelled the instrument, and turned its lens in the direction of the sun, clamp to the spindle, and set the vernier v of the declination arc at zero, by means of the tangent-screw at k, and clamp to the arc.

See that the spindle moves easily and yet truly in the socket, or polar axis, and raise or lower the latitude arc by turning the tangent-screw f, until the sun's image is brought between the equatorial lines on one of the plates. Clamp the latitude arc by the screw, and bring the image precisely into position by the levelling-screws of the tripod or socket, and without disturbing the instrument, carefully revolve the arm h, until the opposite lens and plate are brought in the direction of the sun, and note if the sun's image comes between the lines as before.

If it does, there is no index error of the declination arc; if not, with the tangent-screw k, move the arm until the sun's image passes over half the error; again bring the image between the lines, and repeat the operation as before, until the image will occupy the same position on both the plates.

We shall now find, however, that the zero marks on the arc and the vernier do not correspond, and to remedy this error, the little flat-head screws above the vernier must be loosened until it can be moved so as to make the zeros coincide, when the operation will be completed.

335. To Adjust the Solar Apparatus to the Compass Sights. First level the instrument, and with the clamp and tangent screws set the main plate at 90° by the verniers and horizontal limb. Then remove the clamp-screw, and raise the latitude arc until the polar axis is by estimation very nearly horizontal, and if necessary, tighten the screws on the pivots of the arc, so as to retain it in this position.

Fix the vernier of the declination arc at zero, and direct the equatorial sights to some distant and well-marked object, and observe the same through the compass sights. If the same object is seen through both, and the verniers read to 90° on the limb, the adjustment is complete; if not, the correction must be made by moving the sights or changing the position of the verniers.

TO USE THE SOLAR COMPASS.

336. Before this instrument can be used at any given place, it is necessary to set off upon its arcs both the declination of the sun as affected by its refraction for the given day and hour, and the latitude of the place where the observation is made.

337. To Set off the Declination. The declination of the sun, given in the ephemeris of the Nautical Almanac from year to year, is calculated for apparent noon at Greenwich, England, or Washington, D.C.

To determine it for any other hour at a place in the United

States, reference must be had, not only to the difference of time arising from the longitude, but also to the change of declination from day to day.

By the use of standard time, which is now quite general throughout the United States, it is very easy to obtain the declination required at any place.

For those using 75th meridian time, a difference of five hours must be allowed for the difference in declination between the place of observation and Greenwich.

The time-piece referred to the 75th meridian as standard indicating 7 A.M. when it is noon at Greenwich.

Where the 90th meridian is used as standard, six hours must be allowed, etc.

To obtain the declination for the other hours of the day, take from the almanac the declination for apparent noon of the given day, and, as the declination is increasing or decreasing, add to or subtract from the declination of the first hour the difference for one hour as given in the ephemeris, which will give, when affected by the refraction, the declination for the succeeding hour; and proceed thus in making a table of the declination for every hour of the day.

338. Refraction. By reason of the increasing density of the atmosphere from its upper regions to the earth's surface, the rays of light from the sun are bent out of their course, so as to make his altitude appear greater than is actually the case.

The amount of refraction varies according to the altitude of the body observed; being 0 when it is in the zenith, about one minute when midway from the horizon to the zenith, and almost 34′ when in the horizon.

339. Effect of Incidental Refraction. It will be seen by referring to the instrument, that the effect of the ordinary refraction upon the position of the sun's image with reference to the equatorial lines, which, in fact, are the only ones to be regarded in running lines with the solar compass, is continually

changing, not only with the change of latitude, but also with that of the sun's declination from hour to hour, and the motion of the revolving arm as it follows the sun in its daily revolution.

If the equatorial lines were always in the same vertical plane with the sun, as would be the case at the equator at the time of the equinoxes, it is evident that refraction would have no effect upon the position of the image between these lines, and therefore would not be of any importance to the surveyor.

But as we proceed further north, and as the sun's declination to the south increases, the refraction also increases, and must now be taken into account.

Again, the angle which the equatorial lines make with the horizon is continually changing as the arm is made to follow the motion of the sun during the course of a day.

Thus, in the morning and evening they are more or less inclined to the horizon, while at noon they are exactly parallel to it.

And thus it follows that the excess of refraction at morning and evening is in some measure balanced by the fact that the position of the sun's image with reference to the equatorial lines is then less affected by it, on account of the greater inclination of the lines to the horizon.

340. Allowance for Refraction. The proper allowance to be made for refraction in setting off the declination of the sun upon the solar compass for any hour of any day of the year is given in the following table:

A TABLE OF MEAN REFRACTIONS IN DECLINATION.

To apply on the declination arc of Solar Attachment of either Compass or Transits.*

HOUR ANGLE.	DECLINATIONS.								
	FOR LATITUDE 30°.								
	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°
0 h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06
	FOR LATITUDE 32° 30′.								
0 h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′03	1′15	1′31	1 53	2 20	3 05	4 25	7 36	
	FOR LATITUDE 35°.								
0 h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 30
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	
	FOR LATITUDE 37° 30′.								
0 h.	18″	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″
2	22	28	35	42	50	1′00	1 12	1 26	1 45
3	29	36	43	52	1′02	1 14	1 29	1 49	2 16
4	43	51	1′01	1′13	1 27	1 49	2 14	2 54	4 05
5	1′11	1′26	1 45	2 10	2 49	3 55	6 15	14 58	

* Computed by Edward W. Arms, C.E., for W. and L. E. Gurley, Troy, N.Y.

Hour Angle.	DECLINATIONS.								
	For Latitude 40°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h.	21″	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″
2	25	32	39	46	52	1′06	1 19	1 35	1 57
3	33	40	48	57	1′08	1 21	1 38	2 02	2 36
4	47	55	1′06	1′19	1 36	1 58	2 30	3 21	4 59
5	1′15	1′31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 42° 30′.								
0 h.	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″
2	28	35	39	50	1′00	1 12	1 26	1 45	2 11
3	36	43	52	1′02	1 13	1 29	1 49	2 17	2 59
4	50	1′00	1′11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′16	1 36	1 58	2 30	3 22	5 00	9 24		
	For Latitude 45°.								
0 h.	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″
2	32	39	46	52	1′06	1 19	1 35	1 57	2 29
3	40	47	56	1′07	1 21	1 38	2 00	2 34	3 29
4	54	1′04	1′16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′23	1 41	2 05	2 41	3 40	5 40	12 02		
	For Latitude 47° 30′.								
0 h.	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″
2	35	42	50	1′00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′27	1 46	2 12	2 52	4 01	6 30	16 19		
	For Latitude 50°.								
0 h.	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″	2′36″
2	38	46	55	1′06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′02	1′14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		

EXPLANATION OF THE TABLE OF REFRACTIONS.*

The table is calculated for latitudes between 30° and 50° at intervals of $2\frac{1}{2}$°, that being as near as is required.

The declination ranges from 0 to 20°, both north and south, the + declinations being north, and − south, and is given for every 5 degrees, that being sufficiently near for all practical purposes.

The hour angle in the first column indicates the distance of the sun from the meridian in hours, the refraction given for 0 hours being that which affects the observed declination of the sun when on the meridian, commonly known as meridional refraction; the refraction for the hours just before and after noon is so nearly that of the meridian, that it may be called and allowed as the same.

When the table is used, it must be borne in mind that when the declination is north or + in the table, the refraction is to be added; when the declination is south or − the refraction must be subtracted.

It will be noticed that the refraction in south or − declination increases very rapidly as the sun nears the horizon, showing that observations should not be taken with the sun when south of the equator, less than one hour from the horizon.

Thus, suppose it be required to obtain the declination for any hour in the day, April 16, 1887, at Pittsburg, Pa., where 75th meridian time is used.

The difference in time is 5 hours, so that the declination given in the ephemeris for apparent noon of that day at Greenwich would be that of 7 A.M. at Pittsburg. Proceed as follows:

Declination at Greenwich, mean noon, April 16, 1887,

N. 10° 6′ 29″

Add 1′ 51″=refract'n for 5 hrs. [lat. Pittsburg 40° 28′].

Or, N. 10° 8′ 20″ =dec. 7 A.M. at Pittsburg.

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* See also Refraction Table, page 92.

To get the declination for 8 o'clock, same day and place, add 53″, the difference for one hour — because the declination is increasing — to the declination taken from the almanac, and this increased by the refraction corresponding to 4 hours from noon will give 10° 8′ 28″ for the required declination.

Again, suppose it be desired to obtain the corrected declination for 8 A.M. Oct. 15, 1887, same place.

The declination being now south, the refraction is to be subtracted, but the hourly difference is to be added because the declination is increasing, as in the first example; thus:

Declination at Greenwich, mean noon, Oct. 15, 1887,

	S. 8° 30′ 20″	
Add	56″	= dec. for 1 hr., and increasing.
	S. 8° 31′ 16″	
Subtract	2′ 23″	= refr. 4 hrs. from noon.
Or,	S. 8° 28′ 53″	= dec. at 8 A.M.;

and so on for any hour in the day, obtaining from the declination at Greenwich, by the proper application of the hourly motion, the declination corresponding to the hour required, and correcting this for refraction due to altitude.

To facilitate operations, the calculation of the declination for the different hours of the day should be made and noted before the surveyor commences his work.

341. To Set off the Latitude. Find the declination of the sun for the given day at noon, at the place of observation as just described, and with the tangent-screw set it off upon the declination arc, and clamp the arm firmly to the arc.

Observe in the almanac the equation of time for the given day, in order to know about the time the sun will reach the meridian.

Then, about fifteen or twenty minutes before this time, set up the instrument, level it carefully, fix the divided surface of the declination arc at 12 on the hour circle, and turn the instru-

ment upon its spindle until the solar lens is brought into the direction of the sun.

Loosen the clamp-screw of the latitude arc, and with the tangent-screw raise or lower this arc until the image of the sun is brought precisely between the equatorial lines, and turn the instrument from time to time so as to keep the image also between the hour lines on the plate.

As the sun ascends, its image will move below the lines, and the arc must be moved to follow it. Continue thus, keeping it between the two sets of lines until its image begins to pass above the equatorial lines, which is also the moment of its passing the meridian.

Now read off the vernier of the arc, and we have the latitude of the place, which is always to be set off on the arc when the compass is used at the given place.

It is the practice of surveyors using the solar compass to set off, in the manner just described, the latitude of the point where the survey begins, and to repeat the observation and correction of the latitude arc every day when the weather is favorable, there being also an hour at mid-day when the sun is so near the meridian as not to give the direction of lines with the certainty required.

342. To Run Lines with the Solar Compass. Having set off in the manner just given the latitude and declination upon their respective arcs, the instrument being also in adjustment, the surveyor is ready to run lines by the sun.

To do this, the instrument is set over the station and carefully levelled, the plates clamped at zero on the horizontal limb, and the sights directed north and south, the direction being given, when unknown, approximately by the needle.

The solar lens is then turned to the sun, and with one hand on the instrument, and the other on the revolving arm, both are moved from side to side, until the sun's image is made to appear on the silver plate; when, by carefully continuing the operation, it may be brought precisely between the equatorial lines.

Allowance being now made for refraction, the line of sights will indicate the true meridian; the observation may now be made, and the flag-man put in position.

When a due east and west line is to be run, the verniers of the horizontal limb are set at 90°, and the sun's image kept between the lines as before.

The solar compass being so constructed that when the sun's image is in position the limb must be clamped at 0 in order to run a true meridian line, it will be evident that the bearing of any line from the meridian may be read by the verniers of the limb precisely as in the ordinary magnetic compass: the bearings of lines are read from the ends of the needle.

343. Use of the Needle. In running lines, the magnetic needle is always kept with the sun; that is, the point of the needle is made to indicate 0 on the arc of the compass-box by turning the tangent-screw connected with its arm on the opposite side of the plate. By this means the lines can be run by the needle alone in case of the temporary disappearance of the sun; but, of course, in such cases the surveyor must be sure that no local attraction is exerted.

The variation of the needle, which is noted at every station, is read off in degrees and minutes on the arc, by the edge of which the vernier of the needle-box moves.

344. Allowance for the Earth's Curvature. When long lines are run by the solar compass, either by the true meridian, or due east and west, allowance must be made for the curvature of the earth.

Thus, in running north or south, the latitude changes about one minute for every distance of 92 chains 30 links, and the side of a township requires a change on the latitude arc of 5′ 12″, the township, of course, being six miles square.

This allowance is of constant use where the surveyor fails to get an observation on the sun at noon, and is a very close approximation to the truth.

In running due east and west, as in tracing the standard parallels of latitude, the sights are set at 90° on the limb, and the line is run at right angles to the meridian.

If no allowance were made for the earth's curvature, these lines would, if sufficiently produced, reach the equator, to which they are constantly tending.

Of course, in running short lines either east or west, the variation from the parallel would be so small as to be of no practical importance; but when long sights are taken, the correction should be made by taking fore and back sights at every station, noticing the error on the back-sight, and setting off one-half of it on the fore-sight on the side towards the pole.

345. Time of Day by the Sun. The time of day is best ascertained by the solar compass when the sun is on the meridian, as at the time of making the observation for latitude.

The time thus given is that of apparent noon, and can be reduced to mean time, by merely applying the equation of time as directed in the almanac, and adding or subtracting as the sun is slow or fast.

The time, of course, can also be taken before or after noon, by bringing the sun's image between the hour lines, and noticing the position of the divided edge of the revolving arm, with reference to the graduations of the hour circle, allowing four minutes of time for each degree of the arc, and thus obtaining apparent time, which must be corrected by the equation of time as just described.

346. Caution as to the False Image. In using the compass upon the sun, if the revolving arm be turned a little one side of its proper position, a false or reflected image of the sun will appear on the silver plate in nearly the same place as that occupied by the true one. It is caused by the reflection of the true image from the surface of the arm, and is a fruitful source of error to the inexperienced surveyor. It can, however, be

readily distinguished from the real image by being much less bright, and not so clearly defined.

347. Approximate Bearings. When the bearings of lines, such as the course of a stream, or the boundaries of a forest, are not desired with the certainty given by the verniers and horizontal limb, a rough approximation of the angle they make with the true meridian is obtained by the divisions on the outside of the circular plate.

In this operation, a pencil, or thin straight edge of any sort, is held perpendicularly against the circular edge of the plate, and moved around until it is in range with the eye, the brass centre-pin, and the object observed.

The bearing of the line is then read off at the point where the pencil is placed.

348. Time for Using the Solar Compass. The solar compass, like the ordinary instrument, can be used at all seasons of the year, the most favorable time being, of course, in the summer, when the declination is north, and the days are long, and more generally fair.*

ORIGIN OF THE SYSTEM FOR THE SURVEY OF THE PUBLIC LANDS.†

349. The present system of survey of the public lands was inaugurated by a committee appointed by the Continental Congress, of which Thomas Jefferson was chairman. This committee, on May 7, 1784, reported an ordinance requiring public lands to be divided into "hundreds" of ten geographical miles square, and these again subdivided into lots of one mile square, each to be numbered from 1 to 100, commencing in the *northwestern* corner and continuing from west to east and from

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* See Article 147.

† The following pages regarding the government surveys are from "Instructions of the General Land Office to the Surveyors-General of the United States relative to the Survey of the Public Lands."

east to west consecutively. By subsequent amendment, April 26, 1785, the ordinance required the surveyors "to divide the said territory into townships of 7 miles square, by lines running due north and south, and others crossing these at right angles. The plots of the townships, respectively, shall be marked by subdivisions into sections of 1 mile square, or 640 acres in the same direction as the external lines, and numbered from 1 to 49, and these sections shall be subdivided into lots of 320 acres." This is the first record of the use of the terms "township" and "section."

This ordinance was subsequently still further amended, and as finally passed on the 20th of May, 1785, provided for townships 6 miles square, containing 36 sections of 1 mile square. The first public surveys were made under this ordinance by the direction of the Geographer of the United States.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The act of Congress, approved May 18, 1796, provided for the appointment of a surveyor-general, and directed the survey of lands northwest of the Ohio River, and above the mouth of the Kentucky River, "in which the titles of the Indian tribes have been extinguished," and among other provisions, that the "sections shall be numbered respectively,

beginning with the number one in the northeast section and proceeding west and east, alternately, through the township, with progressive numbers till the thirty-sixth be completed." This method of numbering sections, as shown by the preceding diagram, is still in use.

The act of Congress, approved Feb. 11, 1805, directs the subdivisions of the public lands into quarter-sections. The act of April 24, 1820, provides for the sale of the public lands in half-quarter-sections, and that in every case of the division of a quarter-section, the division line shall run north and south. April 5, 1832, Congress directed the subdivision of the public lands into quarter-quarters, and requiring the division line to run east and west.

350. A surveyor-general for each surveying district is appointed by the President, by and with the advice of the Senate. He is required, while in the discharge of the duties of his office, to reside in the district for which he is appointed. His term of office is four years, and he must give bonds, with sufficient security for the penal sum of $30,000, for the faithful disbursement of all public money placed in his hands, and for the faithful performance of the duties of his office. Among other duties prescribed by law and set forth in the manual, the surveyor-general is required to engage a sufficient number of skilful surveyors as his deputies, and shall cause to be surveyed, measured, and marked, without delay, all base and meridian lines through such points, and perpetuated by such monuments, and such other correction parallels and meridians, as may be prescribed by law, or by instructions from the General Land Office, in respect to the public lands within his surveying district to which the Indian title has been or may be extinguished.

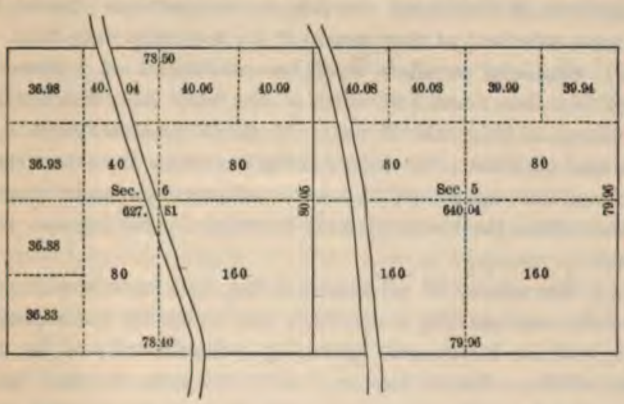
351. System of Rectangular Surveying. The public lands of the United States are ordinarily surveyed into rectangular tracts, bounded by lines conforming to the cardinal points.

The public lands shall be laid off, in the first place, into bodies of land 24 miles square, as near as may be. This shall be done by the extension of standard lines from the principal meridian every 24 miles, and by the extension from the base and standard lines, of auxiliary meridians every 24 miles. Thereafter they shall be laid off into bodies of land 6 miles square, as near as may be, called *townships*, containing, as near as may be, 23,040 acres. The townships shall be subdivided into 36 tracts, called *sections*, each containing, as near as may be, 640 acres. Any number or series of contiguous townships, situate north or south of each other, constitute a *range*.

(*a*) The law requires that the lines of the public surveys shall be governed by the true meridian, and that the township shall be *six miles square*, — two things involving in connection a mathematical impossibility. For strictly to conform to the meridian necessarily throws the township out of square, by reason of the convergency of meridians, and hence by adhering to the true meridian results the necessity of departing from the strict requirements of law, as respects the precise area of townships and the subdivisional parts thereof; the township assuming something of a trapezoidal form, which inequality develops itself more and more as such, the higher the latitude of the surveys. It is doubtless in view of these circumstances that the law provides (see Section 2 of the act of May 18, 1796) that the section of a mile square shall contain the quantity of 640 acres, *as nearly as may be;* and, morever, provides (see Section 3 of the act of May 10, 1800) in the following words: "And in all cases where the exterior lines of the townships thus to be subdivided into sections or half-sections shall exceed, or shall not extend, 6 miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west or from south to north; the sections and half-sections bounded on the northern and western lines of such townships

shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity."

Sections 5 and 6 of Township No. 6, North, Range No. 34, east, of the principal meridian, Montana, are exhibited below:



(*b*) The section lines are surveyed from *south* to north on true meridians, and from *east* to west, in order to throw the excesses or deficiencies in measurements on the north and west sides of the township, as required by law. In a case where a township has been partially surveyed, and it is necessary to complete the survey of the same, or where the character of the land is such that only the north or west portions of the township can be surveyed, this rule cannot be strictly adhered to; but in such cases must be departed from only so far as is absolutely necessary. It will also be necessary to depart from this rule where surveys close upon State or Territorial boundaries, or upon surveys extending from different meridians.

(*c*) The townships are to bear numbers in respect to the base line, either north or south of it; and the tiers of townships called "ranges" will bear numbers in respect to the meridian line, according to their relative position to it, either on the east or west.

(*d*) The 36 sections into which a township is subdivided are numbered, commencing with number *one* at the *northeast* angle of the township and proceeding west to number 6, and thence proceeding east to number 12, and so on, alternately until the number 36 is in the southeast angle. In all cases of surveys of fractional townships, the sections should bear the same numbers as they would if the township were full.

(*e*) Standard parallels shall be established at intervals of every 24 miles, north and south of the base line, and auxiliary meridians at intervals of every 24 miles, east and west of the principal meridian; the object being to confine the errors resulting from convergence of meridians and inaccuracies in measurements, within the tracts of land bounded by the lines so established.

(*f*) The survey of all principal base and meridian standard parallels, and auxiliary meridian and township lines must be made with an instrument operating independently of the magnetic needle. Burt's *improved solar compass*, or other instrument of equal utility, must be used of necessity in such cases; and it is deemed best that such instrument should be used under all circumstances. Where the needle can be relied on, however, the ordinary compass may be used in subdividing and meandering. Whenever deputies use instruments with magnetic apparatus only, they must test the accuracy of their work and the condition of their instruments by at least three observations upon a circumpolar star, upon different days, between the commencement and close of surveying operations in any given township. Deputies using instruments with solar apparatus are not required to make observations of the star Polaris, but they must test their instruments by taking the latitude daily, weather permitting, in running base, standard, meridian, and range lines, and upon three different days, during the execution of subdivisional surveys in each township. They must make complete records in their field notes, under proper dates, of the making of all observations in compliance with these instructions, showing the style and condition of the instrument in use, and

the angle formed by comparing the line run with the meridian as determined by observations.

(*g*) The construction and adjustments of all surveying instruments used in the surveying of the public lands of the United States must be tested at least once a year, and oftener if necessary, by comparison with the true meridian, established under the direction of the surveyor-general of the district; and the instruments must be so modified in construction, or in such a way corrected, as may be necessary to produce the closest possible approximation to accuracy and uniformity in the operation of all such instruments. A record will be made of such examinations, showing the number and style of the instrument, name of the maker, the quantity of instrumental error discovered by comparison, in either solar or magnetic apparatus, or both, and means taken for correction. The surveyor-general will allow no surveys to be made until the instruments to be used therefor have been approved by him.

(*h*) The township lines and the subdivision lines will usually be measured by a two-pole chain of 33.03 feet in length, consisting of 50 links, and each link being 7.92 inches long. On uniform and level ground, however, the four-pole chain may be used. The measurements will, however, always be represented according to the four-pole chain of 100 links. The four-pole chains must be adjusted to lengths of 66.06 feet. The object in adding six-hundredths of a foot to the 66 feet of a four-pole chain is to assure thereby that 66 feet will be set off upon the earth's surface without the application of a greater strain than about 20 pounds by the chainmen, thus providing for loss by vertical curvature of the chain, and at the same time avoiding the uncertain results attending the application of strains taxing its elasticity. The deputy surveyor must provide himself with a measure of the standard chain kept at the office of the surveyor-general, to be used by him as a field standard. The chain in use must be compared and adjusted with this field standard each working day; and such field standard must be returned to the surveyor-general's office for examination when his work is completed.

352. Of Tally-Pins. You will use 11 tally-pins made of steel, not exceeding 14 inches in length, weighty enough toward the point to make them drop perpendicularly, and having a ring at the top, in which is to be fixed a piece of red cloth, or something else of conspicuous color, to make them readily seen when stuck in the ground.

353. Process of Chaining. In measuring lines with a two-pole chain, every *five* chains are called a *tally;* and in measuring lines with a four-pole chain, every *ten* chains are called a *tally*, because at that distance the last of the 10 tally-pins with which the forward chainman set out will have been stuck. He then cries "tally"; which cry is repeated by the other chainman, and each registers the distance by slipping a thimble, button, or ring of leather, or something of the kind, on a belt worn for that purpose, or by some other convenient method. The hind chainman then comes up, and having counted in the presence of his fellow the tally-pins which he has taken up, so that both may be assured that none of the pins have been lost, he then takes the forward end of the chain, and proceeds to set the pins. Thus the chainmen alternately change places, each setting the pins that he has taken up, so that one is forward in all the odd, and the other in all the even, tallies. Such procedure, it is believed, tends to insure accuracy in measurement, facilitates the recollection of the distances to objects on the line, and renders a mis-tally almost impossible.

354. Levelling the Chain and Plumbing the Pins. The length of every line you run is to be ascertained by precise horizontal measurement, as nearly approximating to an air line as is possible in practice on the earth's surface. This all-important object can only be attained by a rigid adherence to the three following observances:

Ever keeping the chain *stretched* to its utmost degree of tension on even ground.

On uneven ground, keeping the chain not only stretched as

aforesaid, but horizontally *levelled.* And when ascending or descending steep ground, hills, or mountains, the chain will have to be *shortened* to one-half its length (and sometimes more), in order accurately to obtain the true horizontal measurement.

The careful plumbing of the tally-pins, so as to attain precisely the *spot* where they should be stuck. The more uneven the surface, the greater the caution needed to set the pins.

355. Marking Lines. All lines on which are to be established the legal corner boundaries are to be marked after this method, viz.: Those trees which may intercept the line must have two chops or notches on each side of them, without any other marks whatever. These are called *sight trees* or *line trees.* A sufficient number of other trees standing within 50 links of the line, on either side of it, are to be blazed on two sides diagonally, or quartering toward the line, in order to render the line conspicuous, and readily to be traced, the blazes to be opposite each other, coinciding in direction with the line where the trees stand very near it, and to approach nearer each other the farther the line passes from the blazed trees.

Where trees two inches or more in diameter are found, the required blazes must not be omitted.

Bushes on or near the line should be bent at right angles therewith, and receive a blow of the axe at about the usual height of blazes from the ground sufficient to leave them in a bent position, but not to prevent their growth.

356. On Trial or Random Lines the trees are not to be blazed, unless occasionally, from indispensable necessity, and then it must be done so guardedly as to prevent the possibility of confounding the marks of the trial line with the *true.* But bushes and limbs of trees may be lopped, and *stakes set* on the trial or random line, at every *ten* chains, to enable the surveyor on his return to follow and correct the trial line, and establish therefrom the *true line.* To prevent confusion, the temporary stakes set on the trial or random lines must be *pulled up* when the surveyor returns to establish the true line.

357. Insuperable Objects on Line; Witness Points. Under circumstances where your course is obstructed by impassable obstacles, such as ponds, swamps, marshes, lakes, rivers, creeks, etc., you will prolong the line across such obstacles by means of right-angle offsets; or, if such be inconvenient, by a traverse or trigonometrical operation, until you regain the line on the opposite side. And in case a north and south, or a true east and west, line is regained in advance of any such obstacle, you will prolong and mark the line back to the obstacle so passed, and state all the particulars in relation thereto in your field-book. And at the intersection of lines with both margins of impassable obstacles you will establish a *witness point* (for the purpose of perpetuating the intersections therewith), by setting a post, and giving in your field-book the course and distance therefrom to two trees on opposite sides of the line, each of which trees you will mark with a blaze and notch facing the post; but on the margins of navigable watercourses or navigable lakes you will mark the trees with the proper number of the fractional section, township, and range.

358. The Best Marking-Tools adapted to the purpose must be provided for marking neatly and *distinctly* all the letters and figures required to be made at corners, *Arabic* figures being used exclusively; and the deputy is always to have at hand the necessary implements for keeping his marking-tools in order.

359. Establishing Corners. To procure the faithful execution of this portion of a surveyor's duty is a matter of the utmost importance. After a true coursing and most exact measurement, the establishment of corners is the consummation of the work. If, therefore, the corners be not perpetuated in a permanent and workmanlike manner, the *great aim* of the surveying service will not have been attained.

The following are the different points for perpetuating corners, viz.:

(*a*) For township boundaries, at intervals of every 6 miles.

(*b*) For section boundaries, at intervals of every mile, or 80 chains.

(*c*) For quarter-section boundaries, at intervals of every half-mile, or 40 chains. Exceptions, however, occur, as fully set forth hereafter in that portion of the manual showing the manner of running township lines and method of subdividing.

(*d*) Meander corners are established at all those points where the lines of the public surveys intersect the banks of such rivers, bayous, lakes, or islands, as are by law directed to be meandered.

360. Miscellaneous. When a rock in place is established for a corner, its dimensions above ground must be given, and a cross (×) marked at exact corner point.

Where mounds of earth are raised "alongside" of corners on N. and S. lines, they must be placed on the W., and on the E. and W. lines on the N. side of corner. In case the character of the land is such that this cannot be done, the deputy will state in his notes instead of "alongside" "S." (on E.).

In case where pits are practicable, the deputy prefers raising a mound of stone, or stone covered with earth, as more likely to perpetuate the corner; he will use the form given for mound of stone, omitting the words "pits impracticable," and adding "covered with earth," when so established.

Where the requisite number of trees can be found within 300 links of the corner point, three (3) bearing trees should be established for every *standard* or *closing corner*, four (4) for every *corner* common to four *townships* or *sections*, and two (2) for every *quarter-section corner* or *meander corner*. In case the requisite number cannot be found within limits, the deputy must state in his field notes, after describing those established, "no other trees within limits," and "dug pits in secs. — & —," or "raised a mound of stone alongside."

Stones 18 inches and less long must be set two-thirds, and over 18 inches long, three-fourths, of their length in the ground. No stones containing less than 504 cubic inches must be used

for corners. Particular attention is called to the "summary of objects and data required to be noted," on pages — and — of these instructions, and it is expected that the deputy will thoroughly comply with the same in his work and field notes.

No mountains, swamp lands, or lands not classed as surveyable, are to be meandered, and all lines approaching such lands must be discontinued at the section or quarter-section corner.

Where, by reason of impassable objects, the south boundary of a township cannot be established, an east and west line should be run through the township, first random, and then corrected, from one range line to the other, and as far south as possible, and from such line the section lines will be extended in the usual manner, except over any fractions south of said line, which may be surveyed in the opposite direction from the section corners on the auxiliary base thus established.

When no part of the east or west boundaries can be run, both north and south boundaries will be established as true lines. Allowance for the convergency of meridians must be made whenever necessary.*

All letters and figures cut in posts or trees must be marked over with red chalk to make them still more plain and durable. Township corners common to four townships, and section corners common to four sections, are to be set diagonally in the earth, with the angles in the direction of the lines. All other corners are to be set square, with the sides facing the direction of the lines. The sizes of wooden posts, mounds, and pits, noted in foregoing descriptions of corners, are to be regarded as *minimum*, and whenever practicable to increase their dimensions, it is desirable to do so. In establishing corners, stones should be used whenever practicable; then posts; and lastly, mounds, with stake in pit.

It is expected that deputy surveyors will carefully read and familiarize themselves with these instructions, and all others

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* See Table of Convergency of Meridians at end of chapter, and explanation of same.

contained in this volume, and will instruct their assistants as to their duties before commencing work. Extra copies will be furnished the deputies for the use of their assistants.

361. Standard Quarter-Section Corners on standard lines must be established in all respects like other quarter-section corners, with the addition of the letters S.C.; and if bearing trees are established for such corners, each tree must be marked S.C. ¼ S.B.T. When a pit is dug at a meander corner, it must be 8 links from the corner on the side opposite the river or lake meandered.

The letters M.C., for "meander corner," must be marked on the side facing the river or lake meandered.

362. A Witness Corner, in addition to the marks that would be placed upon the corner for which it is a witness, must have the letters W.C., and be established in all respects like such corner.

If bearing trees are established for a witness corner, each tree must be marked W.C., in addition to the usual marks.

363. Meandering. Both banks of navigable rivers are to be meandered by taking the general courses and distances of their sinuosities.

At those points, when either the township or section lines intersect the banks of a navigable stream, corners are to be established at the time of running these lines. These are called *meander corners;* and in meandering, you are to commence at one of these corners, coursing the banks, and measuring the distance of each course from your commencing corner to the next *meander corner*. By the same method, you are to meander the opposite bank of the same river. The crossing distance *between meander corners* on same line is to be ascertained by triangulation, that the river may be accurately protracted. Rivers not classed under the statute as *navigable*, but which are well-defined natural arteries of internal communication, will only be meandered *on one bank*.

All lakes, bayous, and deep ponds which may serve as public highways of commerce must be meandered.

364. Surveying. Initial points, from which the lines of the public surveys are to be extended, must be established whenever necessary under special instructions, as may be prescribed in each case by the Commissioner of the General Land Office. The locus of such initial points must be selected with great care and due consideration for their prominence and easy identification, and must be established astronomically.

The initial point having been established, the lines of the public surveys are to be extended therefrom as follows:

365. Base Line. The base line shall be extended east and west from the initial point by the use of solar instruments or transits, as may be directed by the surveyor-general in his special written instructions. Where solar instruments are used, the deputy must test said instruments in every 12 miles of line run, by taking the latitude, or by observation on the polar star; and in all cases where he has reason to suppose that said instrument is in error, he must take an observation on the polar star; and if error be found, must make the necessary corrections before proceeding with his survey. The proper corners shall be established at each 40 and 80 chains, and at the intersection of the line with rivers, lakes, or bayous that should be meandered, in accordance with the instructions for the establishment of corners. In order to check errors in measurement, two sets of chainmen, operating independently of each other, must be employed.

Where transits are used, the line will be run by setting off at the point of departure on the principal meridians a tangent to the parallel of latitude, which will be a line falling at right angles to the said meridian. The survey will be continued on this line for twelve (12) miles, but the corners will be established at the proper points by offsets northerly from said line, at the end of each half-mile. In order to offset correctly from

the tangent to the parallel, the deputy will be guided by the table of offsets and azimuths contained in the Manual of Instructions.

As the azimuth of the tangent is shown, the angle thence to the true meridian at each mile is readily found, thus indicating the direction of the offset line. The computations are made for a distance of 12 miles, at the end of which observations on the polar star must be taken for the projection of a new tangent. The computations are also upon even degrees of latitude; offsets for intervening parallels can be readily determined by interpolation. Where offset distances quarter-section corners exceed 50 links, their direction to the parallel can be determined in like manner by interpolation for azimuth. When said distances are less than 50 links, interpolation for determining the distances will not be required.

366. Principal Meridian. The principal meridian shall be extended north and south from the initial point, by the use of solar instruments or transits, as may be directed by the surveyor-general in his special written instructions.

Where solar instruments are used, the line will be run in the same manner as prescribed for running the base line by solar instruments. Where transits are used, observations upon the polar star must be taken within each 12 miles of line run. In addition to the above general instructions, it is required that in all cases where the establishment of a new principal meridian seems to be necessary to the surveyor-general, he shall submit the matter, together with his reasons therefor, to the Commissioner of the General Land Office, and the survey of such principal meridian shall not be commenced until written authority, together with such special instructions as he may deem necessary, shall have been received from the Commissioner.

367. Standard Parallels. Standard parallels, which are also called correction lines, shall be extended east and west from the principal meridian, at intervals of every 24 miles north and south of the base line, in the same manner as prescribed for running the base line.

Auxiliary Meridians. Auxiliary meridians shall be extended north and south from the base line, at intervals of every 24 miles east and west from the principal meridian, in the same manner as prescribed for running the principal meridian.

It is contemplated that these base, principal meridian, standard, and auxiliary meridian lines shall first be extended over the territory to be surveyed, and that afterwards township and section lines shall be run, where needed, within these tracts of 24 miles square, formed by the extension of these principal lines; and each surveyor-general will therefore cause said principal lines to be extended as rapidly as practicable.

368. Exteriors, or Township Lines. The east and west boundaries of townships are always to be run from south to north on a true meridian line; and the north and south boundaries are to be run from east to west, or from west to east (according to the relation of the township to be surveyed with reference to prior surveys), on a *random* or trial line, and corrected back on a true line. The distance north or south of the township corner to be closed upon, from the point of intersection of these random lines with the east or west boundary of the township, must be carefully measured and noted. Should it happen, however, that such random line should fall short, or overrun in length, or intersect the east or west boundary more than *three chains'* distance from the township corner thereon, as compared with the corresponding boundary on the south (due allowance being made for convergency) the line, and if necessary the entire exterior boundaries of the township, must be retraced, so as to discover and correct the error. In running random lines, temporary corners are to be set at each 40 and 80 chains, and permanent corners established upon the true line as corrected back, in accordance with instructions, throwing the excess or deficiency on the west half-mile, as prescribed by law. Permanent corners are to be established, in accordance with instructions, on the east and west township boundaries at the time they are to be run. Whenever practicable,

the township lines within these tracts of 24 miles square, must be surveyed in regular order from *south* to *north; i.e.*, the exterior boundaries of the township, in any one range lying immediately north of the south boundary of such tract of 24 miles square, must first be surveyed, and the exteriors of the other three townships in said range extended therefrom, in regular order, from *south* to *north* ; and it is preferable to survey first the entire range of townships in such tract adjoining the east boundary, or adjoining the west boundary, and the other three ranges in regular sequence. In cases, however, where the character of the land is such that this rule cannot be complied with, the following will be observed. In extending the *south* or *north* boundaries of a township to the *west*, where the *southwest* or *northeast* corners cannot be established in the regular way by running a north and south line, such boundaries will be run *west on a true* line, allowing for convergency on the west half-mile; and from the township corner established at the end of such boundary, the west boundary will be run *north* or *south*, as the case may be. In extending *south* or *north* of a township to the *east*, where the *southeast* or *northeast* corner cannot be established in the regular way, the same rule will be observed, except that such boundaries will be run *east on a true* line, and the *east* boundary run *north* or *south*, as the case may be. One set of chainmen only is required in running township lines.

369. Method of Subdividing. The first mile, both on the south and east boundaries of each township you are required to subdivide, is to be carefully traced and measured before you enter upon the subdivision thereof. This will enable you to observe any change that may have taken place in the magnetic variation as it existed at the time of running the township lines, and will also enable you to compare your chaining with that upon the township lines.

Any discrepancy arising either from a change in the magnetic variation or a difference in measure is to be carefully noted in the field notes.

After adjusting your compass to a variation which you have just found will retrace the eastern boundary of the township, you will commence at the corner to Sections 35 and 36, on the south boundary, and run a line parallel to the range line, 40 chains, to the quarter-section corner, which you are to establish between Sections 35 and 36; continuing on said course 40 chains farther, you will establish the corner to Sections 25, 26, 35, and 36.

From the section corner last named, run a *random* line, without blazing, *due east*, for the corner of sections 25 and 36, on east boundary, and at 40 chains from the starting-point set a post for *temporary* quarter-section corner. If you intersect exactly at the corner, you will blaze your random line back, and establish it as the *true* line; but if your random line intersects the said east boundary either north or south of said corner, you will measure the distance of such intersection, from which you will calculate a course that will run a *true* line back to the corner from which your random started. You will establish the *permanent* quarter-section corner at a point equidistant from the two terminations of the *true* line.

From the corner of Sections 25, 26, 35, and 36, run due north between Sections 25 and 26, setting the quarter-section post, as before, at 40 chains, and at 80 chains establishing the corner of Sections 23, 24, 25, and 26. Then run a random *due east* for the corner of Sections 24 and 25 on east boundary; setting temporary quarter-section post at 40 chains; correcting back, and establishing *permanent* quarter-section corner at the equidistant point on the *true* line, in the manner directed on the line between Sections 25 and 36.

In this manner you will proceed with the survey of each successive section in the first tier until you arrive at the north boundary of the township, which you will reach in running up a random line between Sections 1 and 2. If this random line should not intersect at the corner established for Sections 1, 2, 35, and 36, upon the township line, you will note the distance that you fall east or west of the same, from which distance you

will calculate a course that will run a true line south to the corner from which your random started. If the north boundary of a township is a base or standard line, the line between Sections 1 and 2 is to be run north as a *true* line, and the closing corner established at the point of intersection with such base or standard line; and in such case, the distance from said closing corner to the nearest section or quarter-section corner on such base or standard line must be carefully measured and noted as a "connection line."

In like manner proceed with the survey of each successive tier of sections until you arrive at the fifth tier; and from each section corner which you establish upon this tier you are to run random lines to the corresponding corners established upon the range line forming the western boundary of the township; setting as you proceed each *temporary* quarter-section corner at 40 chains from the interior section corner, so as to throw the excess or deficiency of measurement on the extreme tier of quarter-sections contiguous to the township boundary; and on returning establish the *true* line, and establish thereon the *permanent* quarter-section corner.

It is not required that the deputy shall complete the survey of the first tier of sections from north to south before commencing the survey of the second or any subsequent tier, but the corner on which the random line closes must have been previously established by running the line north on which it is established, except as follows: where it is impracticable to establish such section corner in the regular manner, it may be established by running the east and west line *east* or *west*, as the case may be, *on a true line*, setting the quarter-section corner at 40 chains and the section corner at 80 chains.

Quarter-section corners, both upon north and south and upon east and west lines, are to be established at a point "equidistant" from the corresponding section corners, *except* upon the lines crossing on the north and west boundaries of the township, and in those situations the quarter-section corners will always be established at precisely 40 *chains* to the north or

west, as the case may be, of the respective section corners from which those lines respectively *start*, by which procedure the excess or deficiency in the measurements will be thrown, according to law, on the extreme tier of quarter-sections.

370. Prescribed Limits for Closing, and Length of Lines in Certain Cases. Every north-and-south section line, except those terminating in the north boundary of the township, must be 80 *chains* in length.

The east-and-west *section lines*, except those terminating in the west boundary of the township, are to be within 80 *links* of the actual distance established on the south boundary line of the township for the width of said tier of sections, and must close within 80 links north or south of the section corner.

The north boundary and south boundary of any one section, except in the extreme western tier, are to be within 80 *links* of equal length.

The meanders within each fractional section, or between two meander posts, or of an island in the interior of a section, must close within 1 chain and 50 links.

In running *random* township exteriors, if such random lines fall short or overrun in length or intersect the eastern or western boundary, as the case may be, of the township at more than 3 *chains* north or south of the true corner, the lines must be *retraced*, even if found necessary to measure the meridional boundaries of the township. One set of chainmen only is required in subdividing.

371. Subdivision of Sections. Under the provisions of the act of Congress approved Feb. 11, 1805, the course to be pursued in the subdivision of sections is to run straight lines from the established quarter-section corners — United States surveys — to the opposite corresponding corners, and the point of intersection of the lines so run will be the corner common to the several quarter-sections; or, in other words, the legal centre of the section.

In the subdivision of fractional quarter-sections where no opposite corresponding sections have been or can be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east, or west lines, as the case may be, to the watercourse, Indian boundary line, or other external boundary of such fractional section. The law presupposes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines, but in actual experience this is not always the case; hence, in order to carry out the spirit of the law, it will be necessary in running the subdivisional lines through fractional sections to adopt mean courses where the section lines are not due lines, or to run the subdivision line parallel to the section line where there is no opposite section line.

Upon the lines closing on the north and west boundaries of a township the quarter-section corners are established by the United States deputy surveyors at precisely 40 chains to the north or west of the last interior section corners, and the excess or deficiency in the measurement is thrown on the outer tier of lots, as per act of Congress approved May 10, 1800. In the subdivision of quarter-sections, the quarter-quarter corners are to be placed at points equidistant between the section and quarter-section corners, and between the quarter corners and the common centre of the section, *except* on the last half-mile of the lines closing on the north or west boundaries of a township, where they should be placed at 20 chains, proportionate measurement, to the north or west of the quarter-section corner.

The subdivisional lines of fractional quarter-sections should be run from points on the section lines intermediate between the section and quarter-section corners due north, south, east, or west, to the lake, watercourse, or reservation which renders such tracts fractional.

When there are double sets of section corners on township and range lines, the quarter corners for the sections south of the

township lines and east of the range lines are not established in the field by the United States surveyors, but in subdividing such sections said quarter corners should be so placed as to suit the *calculations of the areas of the quarter-sections adjoining the township boundaries* as expressed upon the official plot, adopting proportionate measurements where the present measurements of the north or west boundaries of the sections differ from the original measurements.

372. Re-establishment of Lost Corners. The original corners, when they can be found, must stand as the true corners they were intended to represent, even though not exactly where strict professional care might have placed them in the first instance.

As has been observed, no existing original corner can be disturbed, and it will be plain that any excess or deficiency in measurements between existing corners cannot in any degree affect the distances beyond said existing corners, but must be added or subtracted proportionately to or from the intervals embraced between the corners which are still standing.

373. Summary of Objects and Data required to be Noted. The precise length of every line run, noting all necessary offsets therefrom, with the reason and mode thereof.

The kind and diameter of all *bearing trees*, with the course and distance of the same from their respective corners, and the precise relative position of *witness corners* to the *true corners*.

The kind of materials of which corners are constructed.

Trees on line. The name, diameter, and distance on line to all trees which it intersects.

Intersections by line of *land objects*. The distance at which the line first intersects and then leaves every *settler's claim and improvements;* prairie, river, creek, or other "bottom"; or swamp, marsh, grove, and windfall, with the course of the same at both points of intersection; also the distances at which you begin to ascend, arrive at the top, begin to descend, and

reach the foot of all remarkable hills and ridges, with their courses, and *estimated* height, in feet, above the level land of the surrounding country, or above the bottom lands, ravines, or waters near which they are situated.

Intersection by line of *water objects.*

All rivers, creeks, and smaller streams of water which the line crosses; the distances on line at the points of intersection; and their *widths on line.* In cases of *navigable* streams, their width will be ascertained between the *meander corners*, as set forth under the proper head.

The land's *surface* — whether level, rolling, broken, or hilly.

The *soil* — whether first, second, third, or fourth rate.

Timber — the several kinds of timber and undergrowth, in the order in which they predominate.

Bottom lands — to be described as wet or dry; and if subject to inundation, state to what depth.

Springs of water — whether fresh, saline, or mineral, with the course of the stream flowing from them.

Lakes and ponds — describing their banks and giving their height, and also depth of water, and whether it be pure or stagnant.

Improvements — towns and villages; houses or cabins; fields, or other improvements; sugar-tree groves, sugar camps, mill seats, forges, and factories.

Coal bank or beds; *peat* or turf grounds; *minerals* and ores, with particular description of the same as to quality and extent, and all *diggings* therefor; also *salt* springs and licks. All reliable information you can obtain respecting these objects, whether they be on your immediate line or not, is to appear on the general description to be given at the end of the notes.

Roads and *trails*, with their directions whence and whither.

Rapids, cataracts, cascades, or falls of water, with the estimated height of their fall in feet.

Precipices, caves, sink holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

Natural curiosities, interesting fossils, petrifactions, organic

remains, etc.; also all ancient works of art, such as mounds, fortifications, embankments, ditches, or objects of like nature.

The *variation* of the needle must be noted at all points or places on the lines where there is found any material *change* of variation; and the positions of such points must be perfectly identified in the notes.

Besides the ordinary notes taken on line (and which must always be written down on the spot, leaving nothing to be supplied by memory), the deputy will subjoin, at the conclusion of his book, such further description or information touching any matter or thing connected with the township (or other survey) which he may be able to afford, and may deem useful or necessary to be known, with a *general description* of the township in the *aggregate*, as respects the face of the country, its soil and geological features, timber, minerals, waters, etc.

374. Specimen Field Notes of the survey of the Third Standard Parallel North, through Range No. 21 east, of the principal base and meridian in the Territory of Montana, as surveyed by James Page, U. S. Deputy Surveyor.

On the night of August 22, 1880, I took observation on the star Polaris, in accordance with instructions contained in the "Manual of Surveys," and drove pickets on the line thus established.

Survey commenced August 23, 1880, with a Burt's Improved Solar Compass.

Before commencing this survey, I test my compass on the line established last night, and find it correct. I begin at the standard corner to townships 13 north, ranges 20 and 21 east, which is a post, 4 inches square, marked:

S.C., T. 13 N., on N.; R. 21 E., S. 31, on E.; and R. 20 E., S. 36, on W. faces, with 6 notches on N., E., and W. faces, and pits N., E., and W. of post, 6 ft. dist., and mound of earth around post.

Thence I run

West, on random line, between secs. 6 and 7.
Over rolling ground.
27.15 Road to Williamsburg, course S.
40.00 Set temporary $\frac{1}{4}$ sec. cor.
78.40 Intersect west boundary of township 15 lks. S. of cor. to secs. 1, 6, 7, and 12, which is a post, 4 ft. long, 4 ins. square, marked:

T. 6 N.S. 6 on N.E.
R. 34 E.S. 7 on S.E.
R. 33 E.S. 12 on S.W.,

and S. 1 on N.W. faces, with pits, 18 × 18 × 12 ins. in each sec., $5\frac{1}{2}$ ft. dist., and mound of earth, 2 ft. high, $4\frac{1}{2}$ ft. base, around post.
Thence I run
S. 89° 54′ E. on a true line, bet. secs. 6 and 7, with same Va.
38.40 Set a sandstone, 18 × 14 × 3 ins., 12 ins. in the ground, for $\frac{1}{4}$ sec. cor., marked $\frac{1}{4}$ on N. side; dug pits 18 × 18 × 12 ins. E. and W. of stone $5\frac{1}{2}$ ft. distant, and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ base, alongside.
78.40 The cor. to secs. 5, 6, 7, and 8.
Land, rolling.
Soil, sandy; 2d rate.
No timber.

North, on a random line, bet. secs. 5 and 6.
Va. 18° 45′ E.
Over rolling ground.
40.00 Set temporary $\frac{1}{4}$ sec. cor.
Intersect N. boundary of township [illegible] lks. E. of cor. to secs. [illegible] 31, and 32, [illegible] a sandstone 30 × 12 × [illegible] marked with [illegible] on E. and one notch [illegible], and [illegible] stone, 2 ft. high, $4\frac{1}{2}$ [illegible] alongside.
Thence I run

80.00	Set a post, $4\frac{1}{2}$ ft. long, 4 ins. square, with marked stone, 12 ins. in the ground, for standard cor. to secs. 32 and 33, marked:
	S.C., T. 13 N., R. 21 E., on N.;
	S. 33, on E.; and
	S. 32, on W. faces, with 4 notches on E. and 2 notches on W. faces, and raised a mound of stone 2 ft. high, $4\frac{1}{2}$ ft. base, around post.
	Land, high and mountainous.
	Soil, sandy, gravelly, and rocky; 4th rate.
	Timber, pine, and fir, 80 chs.; mostly dead and fallen; some thick undergrowth, same.

375. Specimen Field Notes of the survey of Township No. 6 north, Range No. 34 east, of the principal base and meridian of Montana Territory.

chains.	East, on random line, bet. secs. 5 and 8.
	Va. 18° 45′ E.
	Over rolling ground.
16.40	Road to Williamsburg, course S.
40.00	Set temporary $\frac{1}{4}$ sec. cor.
79.96	Intersected N. and S. line 6 lks. N. of cor. to secs. 4, 5, 8, and 9.
	Thence I run
	N. 89° 56′ W. on true line, bet. secs. 5 and 8, with same Va.
39.98	Set a post 3 ft. long, 3 ins. square, with marked stone, 12 ins. in the ground, for $\frac{1}{4}$ sec. cor. marked $\frac{1}{4}$ S. on N. face; dug pits, 18 × 18 × 12 ins. E. and W. of post $5\frac{1}{2}$ ft. dist., and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ ft. base, around post.
79.96	The cor. to secs. 5, 6, 7, and 8.
	Land, rolling.
	Soil, sandy; 2d rate.
	No timber.

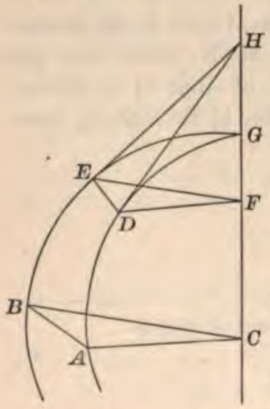
	West, on random line, between secs. 6 and 7.
	Over rolling ground.
27.15	Road to Williamsburg, course S.
40.00	Set temporary $\frac{1}{4}$ sec. cor.
78.40	Intersect west boundary of township 15 lks. S. of cor. to secs. 1, 6, 7, and 12, which is a post, 4 ft. long, 4 ins. square, marked: T. 6 N.S. 6 on N.E. R. 34 E.S. 7 on S.E. R. 33 E.S. 12 on S.W., and S. 1 on N.W. faces, with pits, 18 × 18 × 12 ins. in each sec., $5\frac{1}{2}$ ft. dist., and mound of earth, 2 ft. high, $4\frac{1}{2}$ ft. base, around post.
	Thence I run
	S. 89° 54′ E. on a true line, bet. secs. 6 and 7, with same Va.
38.40	Set a sandstone, 18 × 14 × 3 ins., 12 ins. in the ground, for $\frac{1}{4}$ sec. cor., marked $\frac{1}{4}$ on N. side; dug pits 18 × 18 × 12 ins. E. and W. of stone $5\frac{1}{2}$ ft. distant, and raised a mound of earth, $1\frac{1}{2}$ ft. high, $3\frac{1}{2}$ base, alongside.
78.40	The cor. to secs. 5, 6, 7, and 8.
	Land, rolling.
	Soil, sandy; 2d rate.
	No timber.

	North, on a random line, bet. secs. 5 and 6. Va. 18° 45′ E.
	Over rolling ground.
40.00	Set temporary $\frac{1}{4}$ sec. cor.
	Intersect N. boundary of township 20 lks. E. of cor. to secs. 5, 6, 31, and 32, which is a sandstone 30 × 12 × 6 ins., marked with 5 notches on E. and one notch on W. edges, and mound of stone, 2 ft. high, $4\frac{1}{2}$ ft. base, alongside.
	Thence I run

	S. 0° 09′ E. on a true line bet. secs. 5 and 6, with same Va.
40.05	Set a sandstone, 16 × 12 × 3 ins. 11 ins. in the ground, for ¼ sec. cor. marked ¼ on W. face; dug pits, 18 × 18 × 12 ins., N. and S. of stone, 5½ ft. dist., and raised a mound of earth, 1½ ft. high, 3½ ft. base, alongside.
80.05	The cor. to secs. 5, 6, 7, and 8.
	Land, rolling.
	Soil, sandy; 2d rate.
	No timber.

INCLINATION OF THE MERIDIAN.*

376. In projecting arcs of a great circle it is of the utmost importance that the surveyor be able to tell the inclination of the meridians for any latitude, and for any distance of eastings or westings.



In the following figure, let the two arcs AG and BG be two arcs of a quadrant of the meridian 1° of longitude apart. Let AB = the arc of 1° of longitude on the equator = 69.16 miles.

Let DE be an arc of longitude on any parallel of latitude. Also, let EH and DH be the tangents of those meridians meeting in the earth's axis produced, and corresponding to the parallel of latitude DE.

Then the line $EF = DF = \cos L = \cos AD$ or BE. Also, the angle $DFE = 1°$, and the angle DHE = the inclination of

* These articles on the inclination and convergency of meridians, and the table calculated in accordance therewith, are substantially those given in the 1886 catalogue of engineers' and surveyors' instruments, by Buff and Berger, Boston, Mass.

the meridians, which is the angle we wish to find, and which we will represent by $X°$. And because the two triangles FDE and DHE are on the same base ED, and isosceles, their vertical angles vary inversely as their sides; and we have the equation,

$$1° \times EF = X° \times EH.$$

But $\quad EF = \cos L$, and $EH = \cot L$;

hence $\quad X° \cot L = 1° \cos L$,

or $\quad X° = \cos L \div \cot L = \sin L. \qquad (a)$

That is to say,

The inclination of the meridians for any difference of longitude varies as the sine of the latitude.

Since the sine of the latitude is the inclination in decimals of a degree, for one degree of longitude, if we multiply by 3600″ we shall have the inclination in seconds of arc. Then, if we divide this by the number of miles in one degree of longitude on that latitude, we shall have the inclination due to one mile on that parallel. Thus, for

Latitude 43°	log. sin =	9.833783
Multiply by 3600″	" " =	3.556303
		3.390086
Divide by 50.66 m. = 1° long. on that L,	log. =	1.704682
48.46″ = inclination for one mile of long.		1.685404

The use of the inclination, as found by the preceding article, is to show the surveyor how much he must deflect a line of survey from the due east or west, to have it meet the parallel at a given distance from the initial point of the survey; for it will be remembered that a parallel of latitude is a *curve* having the cosine of the latitude for its radius. And the line due east or west is the tangent of the curve.

Thus, on latitude 43°, it is desired to project a six-mile line west, for the southerly line of a township.

Remembering that in an isosceles triangle the angle at the base is less than a right angle by *half the angle at the vertex*, deflect a line *towards the pole* by the inclination due to *three*

miles, — or in this case $48.46'' \times 3 = 2'.25''$; *i.e.*, *deflection* = $\frac{1}{2}$ *inclination*.

The table on next page, which was computed from the formula (a) above, gives the *inclination* for one mile, and for six miles on any parallel, from 10° to 60° of latitude; also the *convergency* for six miles, on any latitude.

377. The Convergency of the Meridian is readily found for any given distance from the corresponding inclination, by multiplying the *sine* of the inclination by the given *distance*.

Thus, for latitude 43°, the inclination for one mile is $48.46''$; the sine of which is 0.000235. This, multiplied by the number of links in a mile, which = 8,000, we have the convergency for one mile, = 1.88 links.

Multiplying this by the number of miles in a township, = 36, and we have the convergency for a township, = 67.68 links. In this manner were the convergencies of the Table computed.

378. Deflection of Range-Lines from Meridian. The second column of the table shows the surveyor how much he must deflect the range lines between the several sections of a township from the meridian, in order to make the consecutive ranges of sections in a township of uniform width, for the purpose of throwing the effects of convergency into the most westerly range of quarter-sections, agreeably to law.

Thus, say between 45° and 55° of latitude, the inclination is practically $1'$ for every mile of easting or westing. Then, bearing in mind that in the United States the surveys are regarded as projected from the east and south to the west and north, the surveyor must project the *first range-line* between the sections of a township in those latitudes $1'$ *to the left of the meridian*.

The second, $2'$; the third, $3'$; and so on to the fifth, which must be $5'$ to the left of the meridian on the east side of the township.

By this means all the convergency of the township is thrown into the *sixth*, or westerly range of sections, as the law directs.

The fourth column of the table below shows the amount of this convergency. This column is also useful in subdividing a block of territory embraced by two standard parallels and two guide meridians into townships. Thus, starting a meridian from a standard parallel on latitude 43° N., for the western boundary of a range of township, — say the first one west from the guide meridian, — and running north, say four townships, the surveyor must make a point that is *east* of the six-mile point on the northern standard parallel, 4×67.7 links $= 270.8$ links. The second meridian should fall 8×67.7 links to the *right* of the twelve-mile point.

TABLE OF INCLINATION AND CONVERGENCY OF THE MERIDIANS.

Latitude.	Inclination for one mile.	Inclination for six miles.	Convergency for one township of 36 miles.	Latitude.	Inclination for one mile.	Inclination for six miles.	Convergency for one township of 36 miles.	Latitude.	Inclination for one mile.	Inclination for six miles.	Convergency for one township of 36 miles.
°	″	′ ″	LINKS.	°	″	′ ″	LINKS.	°	′ ″	′ ″	LINKS.
10	9.18	55	13.0	27	26.52	2 39	36.9	44	50.19	5 01	70.1
11	10.13	1 01	14.2	28	27.66	2 46	38.6	45	52.00	5 12	72.6
12	11.07	1 06	15.5	29	28.85	2 53	40.2	46	53.83	5 23	75.2
13	12.02	1 12	16.8	30	30.03	3 03	41.9	47	55.67	5 34	77.8
14	12.98	1 18	18.1	31	31.26	3 07	43.6	48	57.67	5 46	80.6
15	13.96	1 24	19.4	32	32.49	3 15	45.4	49	59.83	5 59	83.5
16	14.93	1 30	20.7	33	33.83	3 23	47.2	50	1 02.00	6 12	86.5
17	15.92	1 36	22.0	34	35.17	3 31	49.1	51	1 04.17	6 25	89.7
18	16.91	1 41	23.4	35	36.50	3 39	50.9	52	1 06.67	6 40	93.0
19	17.93	1 47	24.9	36	37.83	3 46	52.7	53	1 09.17	6 55	96.4
20	18.94	1 54	26.5	37	39.17	3 55	54.7	54	1 16.67	7 10	100.0
21	19.98	2 00	27.8	38	40.67	4 04	56.8	55	1 14.33	7 26	103.7
22	21.02	2 06	29.3	39	42.17	4 13	58.8	56	1 17.17	7 43	107.6
23	22.10	2 13	30.8	40	43.67	4 22	60.9	57	1 20.00	8 00	111.8
24	23.17	2 19	32.3	41	45.17	4 31	63.1	58	1 22.00	8 19	116.2
25	24.30	2 26	33.8	42	46.85	4 41	65.4	59	1 26.66	8 40	120.9
26	25.38	2 32	35.4	43	48.52	4 51	67.7	60	1 30.00	9 00	125.7

For details of instruction in United States Government Surveying, see Hawes' System of "Rectangular Surveying," Burt's "Key to Solar Compass," and Clevenger's "Government Surveying."

CHAPTER VII.

CITY SURVEYING.

INTRODUCTION.

379. In the broadest sense, the duties of a city engineer in a large city are many and varied. His knowledge and judgment are required in the location of the city, the laying out of streets, and the fixing of suitable grades therefor, the establishment of a proper water supply, the designing of a suitable system of sewers, the improvement of the waterways, and the planning of necessary bridges and buildings. Following his judicial functions as a designer are his ministerial functions as a constructor. The field which is thus opened before him, in carrying into execution the plans for the various public works, is a very wide one.

As the borough grows and expands into the metropolis, its needs in the directions mentioned increase until a division of labor and responsibility becomes expedient and necessary. In securing the best results in engineering practice, as in other work, the tendency is towards specialties; so that in many cities, in order to secure the services of the best men, and also the best results, the numerous and important duties connected with city engineering have been separated. The province of this work, which is not a treatise on engineering, but on land surveying, makes it proper to treat in this chapter, as thoroughly as the intention and limits of the work allow, only what may be classed under the head of surveying, whether it be performed as the special work of the city or town surveyor, or as among the duties of the city engineer, — the qualifications of the

former by no means fitting a man to perform the varied duties of the latter.

Although this work is intended for the instruction of the student, not of the experienced surveyor, and hence in many things may go into details which to the latter may seem unimportant, it is impossible in the limits of a chapter to impart a thorough knowledge of the duties of a city or town surveyor, — indeed, even to mention all his duties and the many operations and methods which only a long and varied practice can impart. General methods will be given and discussed, but any surveyor of a practical turn of mind will have his own methods of performing much of the routine work pertaining to his situation.

It is not in harmony with the plan of this work to go into the statement in this chapter of any elaborate theories regarding surveying and the instruments used therein, but to endeavor to give some methods which are found to be applicable in practice and to give good practical results. A thorough knowledge of any one good method of performing a certain work is of much more value to the student than a misty idea of numerous methods.

Under the two leading heads of this chapter, field instruments and work and office instruments and work, theoretical discussions will not be entered into; not because they do not possess much value, but because we conceive that they are not adapted to the student's present needs and most rapid advancement. Under the former head, in the light of the work which is likely to engage the greater part of the surveyor's time, field instruments and methods of using them will be described. Under the latter, the nature of office plans and records will be described, the instruments and methods used in the work of producing the plans having been described in other chapters.

In dividing land and locating the boundaries between parties it is evident that the greater the value or the prospective value of said lands, the more delicate should be the instruments, and the more exact the methods used in the work. The methods and instruments which would for all practical purposes be sufficiently exact for the location of a line fence in the country,

where land might be purchased for $100 per acre, would not at all meet the requirements in locating in a city a line between two parties on land worth $100 per front foot. This fact becomes the more evident when we consider that the structures placed upon party lines in a city are so much more substantial and permanent in their nature than those thus located in the country. To meet these considerations we shall find that while some of the methods of land surveying previously described in this work, and the instruments used therein, are applicable to the purposes of city surveying, many of the methods will be more exact, and the instruments more numerous and delicate.

Following the plan heretofore pursued in this work, we will, before discussing the work of the city surveyor, describe the instruments (not described in previous chapters) of most general use in his work, and explain their adjustments and the general methods of using them. These instruments are the transit and rods, steel tapes, measuring-rods, pocket-thermometer, hand-level, spring-balance, plummet, Y-level, levelling-rods, and rod-levels.

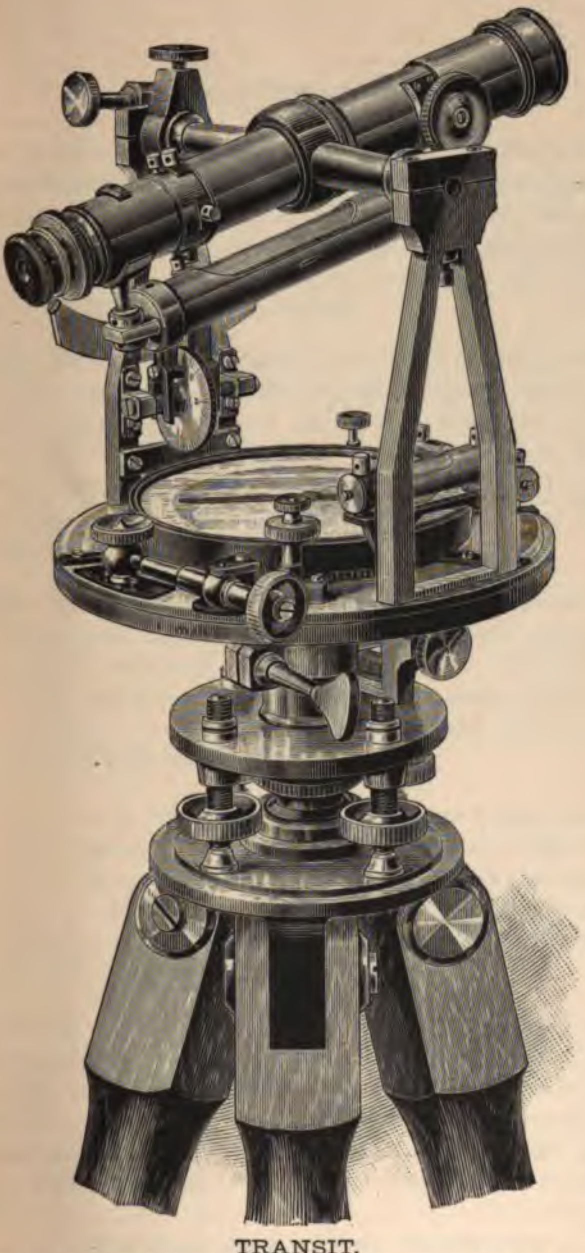
SECTION I.

Instruments, their Adjustments and General Uses.

A. FIELD INSTRUMENTS.

380. The Transit. Full description of the transit, its adjustment and uses, may be found in Chapter II.

381. As precision is the distinguishing feature of city and town surveying, the magnetic needle, which is usually found upon the transits, is in this work of but little use. Angles in carefully made surveys are now taken on the horizontal graduated circle of the transit. The instructions already given in this work regarding the magnetic needle are sufficient reason for the



TRANSIT,

WITH GRADIENTER, LEVEL TO TELESCOPE, AND VERTICAL ARC, AS MADE BY YOUNG & SONS, PHILADELPHIA, PA.

above. It is, however, desirable that in each city and town the true meridian should be determined and permanently marked. Besides being useful in many other ways which will suggest themselves, it will be of great use as an aid in determining the situation of lines described by their bearings in old deeds, the date of the old survey being known.

382. The stadia-hairs * and vertical circle for stadia-measurements are useful attachments, and the telescope should by all means have a long level-tube attached, as this is of much use in city and town work in running grade lines and in levelling for short distances. After the level and the manner of using it have been described, the operation of running a grade line will be explained.

383. **Rods**. Besides the usual iron-pointed wooden rods, very convenient rods, or pickets, for use with the transit, may be made of gas-pipe about three-quarters of an inch in diameter drawn out on one end to a point, and painted in alternate sections of red and white, — red preferred to black because against red the cross-hairs can be seen.

384. It is by no means as easy a matter to run a straight line with a transit as at first thought it may seem to the student. After the selection of suitable weather, reversing at every extension, care in handling the instrument, and with a corresponding degree of care on the part of assistants, the results are not always what the most careful would desire.

385. In marking a line with stakes, it is convenient to have stake-wood which, in cross-section, has one dimension greater than the other. If, in setting the stake, it always be placed with its broader side towards the instrument, its position will afterwards tell one at a glance in which direction the line was run. This is important when several stakes are set on different

* See Articles 148 to 152, Stadia Measurements.

lines near their intersection, as it will often be the means of avoiding confusion and the resulting errors.

386. Steel Tapes, etc. Before making any important measurements for a city or town, it is necessary, in order to avoid subsequent confusion, that a standard of measurement should be adopted. In many parts of an old city or town the introduction of a new standard would bring inextricable confusion. If there be a standard, even though it has not been carefully preserved, it should, if possible, be ascertained and regarded. When, however, it is at the option of the surveyor to select his standard, the United States standard should, as tending to uniformity, be adopted in this country. Standard rods may be procured of the government. With these rods tape lines and other instruments used for a line purpose should be compared, and the variation noted. It is desirable, also, for purposes of comparison, that a standard, 50 feet or 100 feet, at a known temperature, should be carefully laid down with these rods in the corridor of some building, or in some other convenient place.

Very accurate measuring may be done with graduated wooden rods properly shod with metal ends. These rods are necessarily of but moderate length; hence, work with them is correspondingly slow. For city work, steel tapes are now in very general use; and, when properly handled, give very satisfactory results. They are of different lengths and of different widths. For measuring full hundreds over tolerably level ground the narrow tape, $\frac{3}{32}$ inch wide and 200 feet long, is very convenient. For general city use the 100-feet tape, $\frac{3}{8}$ inch in width, is most convenient.

387. As a rule measurements will be made with the tape in a horizontal position. If not so held, the measurements will afterwards be reduced to the horizontal. In order to determine the horizontal, a hand-level is used to ascertain the difference in elevation of the ground at the two ends of the tape. A cut and description of this convenient little instrument is given below.

Locke's Hand-Level consists of a brass tube about 6 inches long, having, as shown in the figure, a small level on top and near the object end, there being also an opening in the tube beneath, through which the bubble can be seen, as reflected by a glass prism, immediately under the level. Both ends of the tube are closed by plain glass settings to exclude the dust, and there is at the inner end of the sliding or eye tube a semicircular convex lens, which serves to magnify the level bubble, and cross-wire underneath, while it allows the object to be clearly seen through the open half of the tube.

The cross-wire is fastened to a little frame moving under the level-tube, and adjusted to its place by the small screw shown on the end of the level-case. The level of any object in line with the eye of the observer is determined by sighting upon it through the tube, and bringing the air-bubble of the level into a position where it is bisected by the cross-wire.

A short telescope is sometimes applied in place of the plain glass ends, enabling levels to be taken at greater distances and with increased accuracy.

If one or both ends of the tape be held up, the point on the ground vertically under the end of the tape will be determined by means of the plummet, which here needs no description further than to say that its sides should make such an angle with each other as not to prevent the observer when using it from seeing its point; neither should it be so long as to be unsteady.

In all extended and important measurements regard must be had in using the steel tape to standard, temperature, sag, and wind.

Before using a tape its relation to the standard should be

determined by comparison with the standard, marked as previously described, and the variation noted.

388. All important measurements, no matter at what temperature made, should be reduced to a standard temperature; for if, at a certain temperature, we determined with a steel tape the distance apart of two points, at a higher temperature that distance on the same tape would be less because the tape is longer; or, at a lower temperature, greater, because the tape is shorter. The temperature of the air at the time of measurement is ascertained by means of a small thermometer which can be exposed with the tape, and which is so protected that, when not in use, it can be safely carried in the pocket. The standard temperature to which all measurements should be reduced may be taken at pleasure. The correction for expansion and contraction of the steel tape by heat and cold is 0.000006 per unit per degree F.

389. When the tape is held suspended, it will always sag in a vertical direction. Hence the horizontal distance between the extreme graduations will be less than if there were no sag. For this reason, when used to measure the distance between two points, it will, without correction, give a result too great; when used without correction to lay down a given distance, it will give it too small. While a formula may be derived by which to make a correction for sag, it will be found quite as satisfactory to determine it by actual trial. The amount of sag will of course depend upon the tension, or pull. This may be regulated by using at one end of the tape a small spring-balance. It is, however, very desirable that on important work the same men at the same ends of the tape should make all measurements. The experience gained in working together will be a most important factor in securing uniform results.

The effect of wind is in the same direction as that of sag. While much of the work of the surveyor, particularly that involving short measurements, must be done regardless of wind,

no good results in long and important measurements can be secured in windy weather. The best correction for wind is to wait for a calm. In windy weather a narrow tape, as it exposes less surface to the wind, is useful.

390. To illustrate what has been said in regard to the corrections to be applied to measurements made with the steel tape, let us suppose two examples.

First. With a steel tape 100 feet long ($\frac{3}{8}$ inch wide) suspended each length at one or both ends, the temperature of the air being 79° F., the distance on the tape between two points is found to be 550 feet $6\frac{7}{8}$ inches. If the tape is $\frac{1}{8}$ inch longer than the standard, and parts of its length proportionately longer, the standard temperature, 60° F., and the sag $\frac{1}{4}$ inch in 100 feet, what are the corrections, and what is the actual distance between the points?

On account of differing from the standard, as the tape is too long, the distance obtained is too short; the correction for standard is therefore additive. On account of difference in temperature, the temperature being higher than the standard, as the tape is too long, the distance obtained is too short; the correction for temperature is therefore additive. On account of the sag, as the tape is thereby made too short, the distance obtained is too long; the correction for sag is therefore subtractive.

Correction for standard:

$\frac{1}{8}$ in. $\times 5\frac{1}{2} = \frac{11}{16}$ in. additive.

Correction for temperature $(79° - 60° = 19°)$:

0.000006 ft. $\times$ 550 $\times$ 19 = 0.0627 ft.

0.0627 ft. $\times$ 12 = 0.7524 in. $= \frac{12}{16}$ in. additive.

Correction for sag:

$\frac{1}{4}$ in. $\times 5\frac{1}{2} = \frac{22}{16}$ in. subtractive.

Total correction:

$+\frac{11}{16}$ in. $+\frac{12}{16}$ in. $-\frac{22}{16}$ in. $= +\frac{1}{16}$ in. additive.

Actual distance between points:

$$550 \text{ ft. } 6\tfrac{7}{8} \text{ in.} + \tfrac{1}{16} \text{ in.} = 550 \text{ ft. } 6\tfrac{15}{16} \text{ in.}$$

Second. Suppose it be required, — other things being as before, — to locate with the steel tape, when the temperature of the air is 52° F., two points which shall at the standard temperature be 225 feet $4\frac{1}{2}$ inches apart.

What length on the tape must be taken?

Correction for standard:

$$\tfrac{1}{8} \text{ in.} \times 2\tfrac{1}{4} = \tfrac{9}{32} \text{ in. subtractive.}$$

Correction for temperature $(60° - 52° = 8°)$:

$$0.000006 \text{ ft.} \times 225 \times 8 = 0.0108 \text{ ft.}$$
$$0.0108 \text{ ft.} \times 12 = 0.1296 \text{ in.} = \tfrac{4}{32} \text{ in. additive.}$$

Correction for sag:

$$\tfrac{1}{4} \text{ in.} \times 2\tfrac{1}{4} = \tfrac{18}{32} \text{ in. additive.}$$

Total correction:

$$-\tfrac{9}{32} \text{ in.} + \tfrac{4}{32} \text{ in.} + \tfrac{18}{32} \text{ in.} = +\tfrac{13}{32} \text{ in. additive.}$$

Length to be taken on tape:

$$225 \text{ ft. } 4\tfrac{1}{2} \text{ in.} + \tfrac{13}{32} \text{ in.} = 225 \text{ ft. } 4\tfrac{29}{32} \text{ in.}$$

When the tape is not suspended, correction for sag will not be made.

In short and less important measurements the same attention to corrections is not necessary.

In practice, the above method has been found to give satisfactory results.

391. In placing stakes to hold measurements, it is best, and in harmony with the method suggested for placing them on instrument lines, to set them with the greater dimension of cross-section in the direction in which the measurement is being made.

Measuring is a very important part of the work of the surveyor. Even when done with the greatest care, it is difficult to obtain results *entirely* satisfactory.

Measurements which are to be directly compared, or are to be used in connection, as in locating parallel lines, should be made under circumstances as nearly as possible identical. Experience and a correct idea of the importance of the work will enable the surveyor to determine the degree of accuracy therein necessary.

Levelling-Instruments.

392. The Y-Level. Of the different varieties of the levelling-instrument, that termed the Y-level has been almost universally preferred by American engineers, on account of the facility of its adjustment and superior accuracy.

The engraving represents a twenty-inch Y-level as made by W. and L. E. Gurley, Troy, N.Y.

393. The Telescope has at each end a ring of bell-metal, turned very truly, and both of exactly the same diameter; by these it revolves in the wyes, or can be at pleasure clamped in any position when the clips of the wyes are brought down upon the rings, by pushing in the tapering-pins.

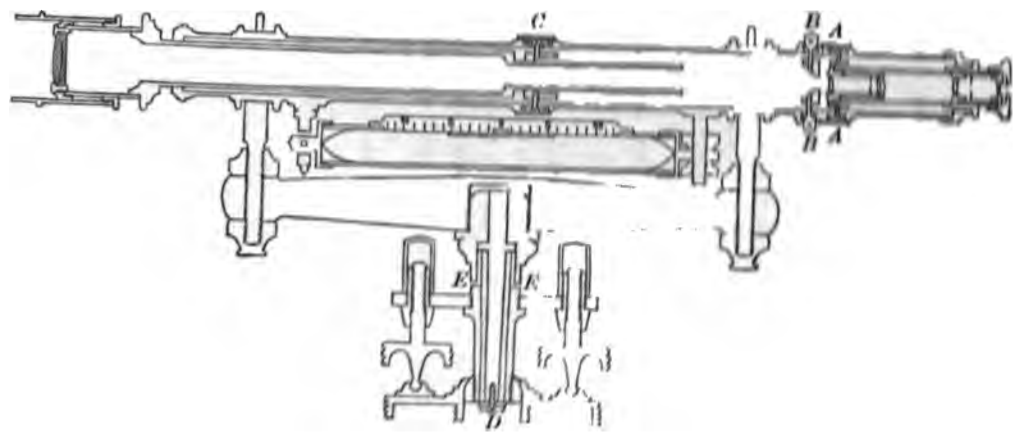
394. The Level or ground bubble tube is attached to the under side of the telescope, and furnished at the different ends with the usual movements, in both horizontal and vertical directions.

The aperture of the tube, through which the glass vial appears, is about 5¼ inches long, being crossed at the centre by a small rib or bridge, which greatly strengthens the tube.

The level-scale which extends over the whole length is graduated into tenths of an inch, and figured at every fifth division, counting from zero at the centre of the bridge; the scale is set close to the glass.

The bubble vial is made of thick glass tube, selected so as to have an even bore from end to end, and finely ground on its upper interior surface, that the run of the air-bubble may be uniform throughout its whole range.

20-INCH Y-LEVEL.



SECTIONAL VIEW OF 20-INCH Y-LEVEL.

395. The Wyes are made large and strong, of the best bell-metal, and each has two nuts, both being adjustable with the ordinary steel pin.

The clips are brought down on the rings of the telescope-tube by the Y-pins, which are made tapering, so as to clamp the rings very firmly.

The clip of one of the wyes has a little pin projecting from it, which, entering a recess filed in the edge of the ring, insures the vertical position of the level and cross-wire.

396. The Level-Bar is made round, of the best bell-metal, and shaped so as to possess the greatest strength in the parts most subject to sudden strains.

Connected with the level-bar is the head of the tripod-socket.

397. The Tripod-Socket is compound; the interior spindle D, sectional view, upon which the whole instrument is supported, is made of steel, and nicely ground, so as to turn evenly and firmly in a hollow cylinder of bell-metal; this, again, has its exterior surface fitted and ground to the main socket EE of the tripod-head.

The bronze cylinder is held upon the spindle by a washer and screw, the head of the last having a hole in its centre, through which the string of the plumb-bob is passed.

The Adjustments.

398. The three adjustments of the level which the surveyor usually has to attend to are the following:

1. *To adjust the line of collimation*, or, in other words, to bring both wires into the optical axis, so that their point of intersection will remain on any given point during an entire revolution of the telescope.

2. *To bring the level-bubble parallel* with the bearings of the Y-rings, and with the longitudinal axis of the telescope.

3. *To adjust the wyes*, or to bring the bubble into a position at right angles to the vertical axis of the instrument.

399. To Adjust the Line of Collimation, set the tripod firmly, remove the Y-pins from the clips, so as to allow the telescope to turn freely, clamp the instrument to the tripod-head, and, by the levelling and tangent screws, bring either of the wires upon a clearly marked edge of some object, distant from 100 to 500 feet.

Then, with the hand, carefully turn the telescope half-way around, so that the same wire is compared with the object assumed.

Should it be found above or below, bring it half-way back by moving the capstan-head screws at right angles to it, remembering always the inverting property of the eye-piece; now bring the wire again upon the object, and repeat the first operation until it will reverse correctly.

Proceed in the same manner with the other wire until the adjustment is completed.

Should both wires be much out, it will be well to bring them nearly correct before either is entirely adjusted.

When this is effected, unscrew the covering of the eye-piece centring-screws, shown in the sectional view at *AA*, and move each pair in succession with a small screw-driver, until the wires are brought into the centre of the field of view.

The inverting property of the eye-piece does not affect this operation, and the screws are moved direct.

To test the correctness of the centring, revolve the telescope, and observe whether it appears to shift the position of an object.

Should any movement be perceived, the centring is not perfectly effected.

It may here be repeated, that in all telescopes the position and adjustment of the line of collimation depends upon that of the object-glass; and, therefore, that the movement of the eye-piece does not affect the adjustment of the wires in any respect.

When the centring has been once effected, it remains permanent, the cover being screwed on again to conceal and protect it from derangement at the hands of the curious or inexperienced operator.

400. To Adjust the Level-Bubble. Clamp the instrument over either pair of levelling-screws, and bring the bubble into the centre of the tube.

Now turn the telescope in the wyes, so as to bring the level-tube on either side of the centre of the bar. Should the bubble run to the end, it would show that the vertical plane passing through the centre of the bubble was not parallel to that drawn through the axis of the telescope-rings.

To correct the error, bring the bubble entirely back, with the capstan-head screws, which are set in either side of the level-holder, placed usually at the object end of the tube.

Again bring the level-tube over the centre of the bar, and the bubble to the centre; turn the level to either side, and, if necessary, repeat the correction until the bubble will keep its position, when the tube is turned half an inch or more to either side of the centre of the bar.

The necessity for this operation arises from the fact that when the telescope is reversed end for end in the wyes in the other and principal adjustment of the bubble, we are not certain of placing the level-tube in the same vertical plane; and therefore it would be almost impossible to effect the adjustment without a lateral correction.

Having now, in great measure, removed the preparatory difficulties, we proceed to make the level-tube parallel with the bearings of the Y-rings.

To do this, bring the bubble into the centre with the levelling-screws, and then, without jarring the instrument, take the telescope out of the wyes and reverse it end for end. Should the bubble run to either end, lower that end, or, what is equivalent, raise the other by turning the small adjusting-nuts, on one end of the level, until by estimation half the correction is made;

again bring the bubble into the centre, and repeat the whole operation, until the reversion can be made without causing any change in the bubble.

It would be well to test the lateral adjustment, and make such correction as may be necessary in that, before the horizontal adjustment is entirely completed.

401. To Adjust the Wyes. Having effected the previous adjustments, it remains now to describe that of the wyes, or, more precisely, that which brings the level into position at right angles to the vertical axis, so that the bubble will remain in the centre during an entire revolution of the instrument.

To do this, bring the level-tube directly over the centre of the bar, and clamp the telescope firmly in the wyes, placing it, as before, over two of the levelling-screws, unclamp the socket, level the bubble, and turn the instrument half-way around, so that the level-bar may occupy the same position with respect to the levelling-screws beneath.

Should the bubble run to either end, bring it half-way back by the Y-nuts on either end of the bar; now move the telescope over the other set of levelling-screws, bring the bubble again into the centre, and proceed precisely as above described, changing to each pair of screws, successively, until the adjustment is very nearly perfected, when it may be completed over a single pair.

The object of this approximate adjustment is to bring the upper parallel plate of the tripod-head into a position as nearly horizontal as possible, in order that no essential error may arise, in case the level, when reversed, is not brought precisely to its former situation. When the level has been thus completely adjusted, if the instrument is properly made, and the sockets well fitted to each other and the tripod-head, the bubble will reverse over each pair of screws in any position.

Should the surveyor be unable to make it perform correctly, he should examine the outside socket carefully to see that it sets securely in the main socket, and also notice that the clamp does not bear upon the ring which it encircles.

When these are correct, and the error is still manifested, it will probably be in the imperfection of the interior spindle.

After the adjustments of the level have been effected, and the bubble remains in the centre, in any position of the socket, the surveyor should turn the telescope in the wyes until the pin on the clip of the wye will enter the little recess in the ring to which it is fitted, and by which is insured the vertical position of the spirit-level and cross-wire.

When the pin is in its place, the vertical wire may be applied to the edge of a building; and in case it should not be parallel with it, two of the cross-wire screws that are at right angles to each other may be loosened, and by the screws outside, the cross-wire ring turned until the wire is vertical; the line of collimation must then be corrected again and the adjustments of the level will be complete.

402. To Use the Level. Set the legs firmly into the ground. The bubble should then be brought over each pair of levelling-screws successively and levelled in each position, any correction that may appear necessary being made in the adjustments.

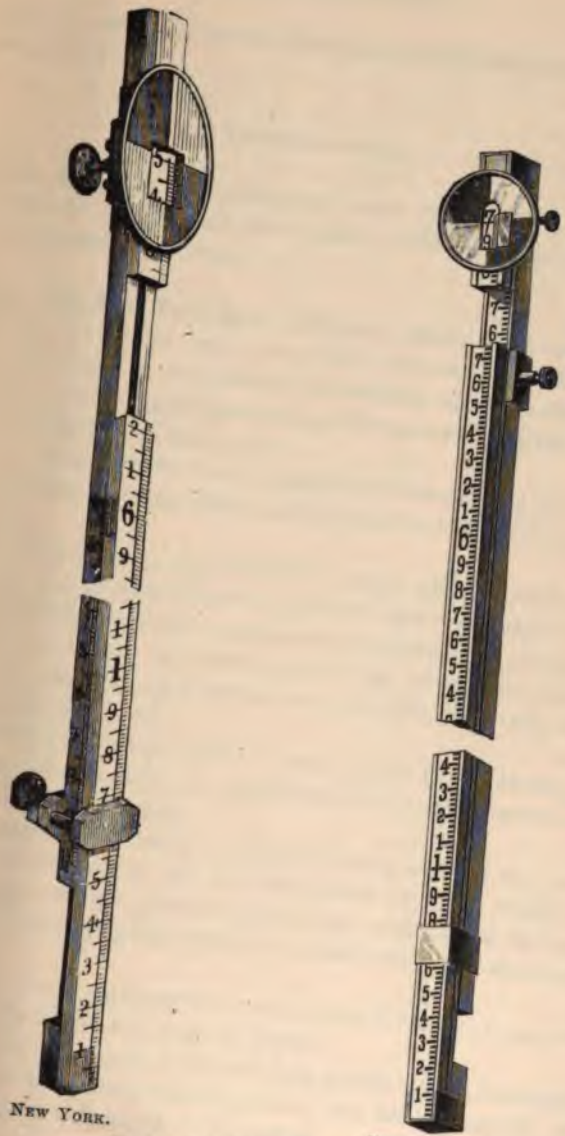
Bring the wires precisely in focus and the object distinctly in view, so that all errors of parallax may be avoided.

This error is seen when the eye of an observer is moved to either side of the centre of the eye-piece of a telescope, in which the foci of the object and eye-glasses are not brought precisely upon the cross-wires and object; in such a case the wires will appear to move over the surface, and the observation will be liable to inaccuracy.

In all instances the wires and object should be brought into view so perfectly that the cross-wires will appear to be fastened to the surface, and will remain in that position however the eye is moved.

Care should be exercised during an observation, lest the hand touching the instrument inadvertently, or a foot placed near the leg of the tripod, impair the adjustment.

The weight of a level having a 20-inch telescope, with level-



NEW YORK. PHILADELPHIA.

LEVELLING-RODS.

ling-head, exclusive of the tripod, is between thirteen and fourteen pounds.

Levelling-Rods.

403. The various levelling-rods used by American engineers are made in two or more parts, which slide from each other as they are extended in use.

404. The New York Rod. This rod, which is shown in the engraving as cut in two, so that the ends may be exhibited, is made of maple, in two pieces, but sliding one from the other, the same end being always held on the ground, and the graduations starting from that point.

The graduations are made to tenths and hundredths of a foot, the tenth figures being black, and the feet marked with a large red figure.

The front surface, on which the target moves, reads to $6\frac{1}{2}$ feet; when a greater height is required, the horizontal line of the target is fixed at that point, and the upper half of the rod, carrying the target, is moved out of the lower, the reading being now obtained by a vernier on the graduated side, up to an elevation of 12 feet.

The target is round, made of thick sheet brass, having, to strengthen it still more, a raised rim, which also protects the paint from being defaced.

The target moves easily on the rod, being kept in any position by the friction of the two flat plates of brass which are pressed against two alternate sides, by small spiral springs, working in little thimbles attached to the band which surrounds the rod.

There is also a clamp-screw on the back, by which it may be securely fastened to any part of the rod.

The face of the target is divided into quadrants by horizontal and vertical diameters, which are also the boundaries of the alternate colors with which it is painted.

The colors usually preferred are white and red; sometimes white and black.

The opening in the face of the target is a little more than a tenth of a foot long, so that in any position a tenth or a foot figure can be seen on the surface of the rod.

The right edge of the opening is chamfered, and divided into ten equal spaces, corresponding with nine-hundredths on the rod; the divisions start from the horizontal line which separates the colors of the face.

The vernier, like that on the side of the rod, reads to thousandths of a foot.

The clamp, which is screwed fast to the lower end of the upper sliding-piece, has a movable part which can be brought by the clamp-screw firmly against the front surface of the lower half of the rod, and thus the two parts immovably fastened to each other without marring the divided face of the rod.

405. The Philadelphia Rod. This rod is made of two strips of cherry, each about $\frac{3}{4}$ inch thick by $1\frac{1}{2}$ inches wide and 7 feet long, connected by two metal sleeves, the lower one of which has a clamping-screw for fastening the two parts together when the rod is raised for a higher reading than 7 feet.

Both sides of the back strip and one side of the front one are planed out $\frac{1}{16}$ inch below the edges; these depressed surfaces are painted white, divided into feet, tenths and hundredths of a foot, and the feet and tenths figured.

The front piece reads from the bottom upward to 7 feet, the foot figures being red and an inch long, the tenth figures black and eight-tenths of an inch long. When the rod is extended to full length, the front surface of the rear half reads from 7 to 13 feet, and the whole front of the rod is figured continuously and becomes a self-reading rod 13 feet long.

The back surface of the rear half is figured from 7 to 13 feet, reading from the top down; it has a vernier also by which the rod is read to two-hundredths of a foot as it is extended. The target is round and made of sheet-brass, raised

on the perimeter to increase its strength, and is painted in white and red quadrants; it has also a scale on its chamfered edge, reading to two-hundredths of a foot.

When a level of less than 7 feet is desired, the target is moved up or down the front surface, the rod being closed together and clamped; but when a greater height is required, the target is fixed at 7 feet and the rear half slid out, the scale on the back giving the readings like those of the target to two-hundredths of a foot.

This rod is so graduated that the leveller is enabled to take the reading direct from it, the rodman's duties being simply to hold the rod vertical over the points. It is hence called a *self-reading* or *speaking rod*.

406. The Rod-Level. The figures below represent a level recently devised, for the more accurate plumbing of levelling-rods.

ROD-LEVEL.

ROD-LEVEL AS APPLIED TO A ROD.

The left-hand figure shows it when folded for convenience in carrying. Its convenience and value commend it to general favor.

407. Levelling is measuring in a vertical direction. In his treatise on levelling, Frederick W. Simms says: "Levelling is the art of tracing a line at the surface of the earth which shall

cut the directions of gravity everywhere at right angles. . . . The direction of gravity invariably tends towards the centre of the earth, and may be considered as represented by a plumb-line when hanging freely, and suspended beyond the sphere of attraction of the surrounding objects. . . . The operation of levelling may be defined as the art of finding how much higher or lower any one point is than another, or, more properly, the difference of their distances from the centre of the earth."

A surface like that of still water may be called a level surface. The curve formed by the intersection with such a surface of a vertical plane is a *line of true level;* a line tangent to the latter is a *line of apparent level.*

Levelling is the art of determining the differences of elevation of two or more points, or of determining how much one point is above or below a line of true level passing through the other point.

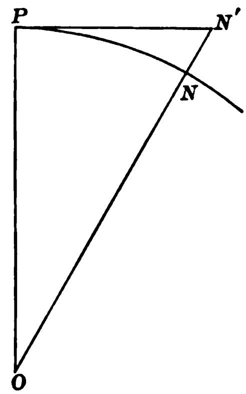
408. From the foregoing it is evident that, on account of the curvature of the earth, a horizontal line is not really throughout its length a level line; that of two points in the same level line each will have its own horizon. Hence, in levelling, the effect of the curvature of the earth upon the comparative elevations of different points must be taken into consideration. The effect of the curvature is to make objects appear lower than they really are.

The air nearer the surface of the earth is denser than that farther removed from the surface. This difference in density, causing refraction of light, will affect the elevation of a point as observed through the telescope of a level, so that it also must be taken into consideration. Its effect is to make objects appear higher than they really are. The error caused by refraction is one-seventh as great as that caused by curvature.

Let us first find an expression for the correction due to the curvature of the earth. That is —

409. *To find the deviation from its tangent of a line of true level.*

Let O represent the centre of the earth, PN a line of true level, and PN' its tangent, or a line of apparent level. The distance NN' corresponding to the length of sight PN is required.



From Geometry,

$$\overline{PN'}^2 = NN'(2\,ON + NN');$$

or, $$NN' = \frac{\overline{PN'}^2}{2\,ON + NN'}.$$

For ordinary distances, the length of the arc may be regarded as that of the tangent, and NN' as inconsiderable in comparison with $2\,ON$, the diameter of the earth. Therefore, calling the length of sight d, the correction c, and the radius of the earth r, we have

$$c = \frac{d^2}{2r},$$

and the correction for refraction

$$= \frac{1}{7}c = \frac{1}{7} \times \frac{d^2}{2r} = \frac{d^2}{14r};$$

then the correction due to curvature and refraction, which we will call C, is

$$c - \frac{1}{7}c = \frac{d^2}{2r} - \frac{d^2}{14r};$$

or, $$C = \frac{3d^2}{7r}.$$

This correction must be added to the height of the object as found by the level.

In practice, the necessity for using the above formula is avoided whenever it is possible to set the level at equal distances from the points whose difference of height is required.

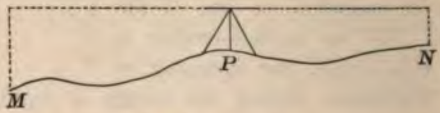
EXERCISES.

1. Assuming the diameter of the earth 7,926 miles, show that for a mile sight $c =$ about 8 inches. Find the value of C for the same distance.

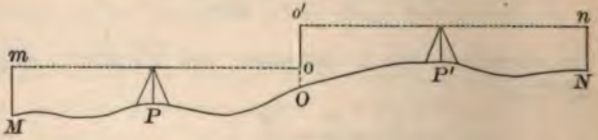
2. What is the correction due to curvature for half a mile?

3. What is the length of sight when C equals one-tenth of a foot?

4. Show that, practically, the correction for curvature in feet is equal to two-thirds the square of the distance in miles.



410. If two points M, N, whose difference of elevation is required, can be observed upon from some point P about equidistant * from them, not necessarily in their line, set up the level at P, and note the reading of a rod held vertically over each point. The difference of the two readings will indicate the difference of level required.



411. If the above method is impracticable, set up the instrument at some point P — either in or out of the line, no matter which — from which a rod may be observed on the first station M, and also on another point O in the direction of N, about equidistant with M from the instrument. Remove the level to a

* Placing the instrument in this position lessens the effects of inaccurate adjustment and renders unnecessary the corrections indicated in Article 409.

new position P', whence observe again the rod on O, also the rod reading at N.

The difference between the readings of the rod at M and O shows how much higher the latter is than the former, and in like manner the difference of the readings at O and N gives the difference in elevation of these points, and so on, no matter what the number of stations. The difference in height of M and N

$$= Mm - Oo + Oo' - Nn;$$

or,

$$Mm + Oo' - Oo - Nn$$
$$= Mm + Oo' - (Oo + Nn).$$

Calling Mm and Oo' back-sights, and the other two, fore-sights, we perceive that the difference of level of two points is shown by subtracting the sum of the fore-sights from the sum of the back-sights.

412. Again, in levelling, we measure, by means of the rod, how much lower than the line of sight (height of instrument) certain points are. Thus we may determine the relative elevations of the points. Suppose, for example, it be required to determine the difference in elevation of any two points. For reasons already given, set the level equally distant from the points. If this cannot be done, and both observations have to be taken from one of the stations, especially if the distance between them is considerable, correction as previously described must be made. But in this case suppose it is possible; and suppose that when held on one point, the rod reads 7.255; that is, this point may be considered 7.255 below the line of sight, and 4.755 when held on the other; then the first may be considered $7.255 - 4.755$, or 2.500 farther than the second below the line of sight, or lower than the second.

413. Suppose it be required to determine the difference in elevation between two points, of which one is so much higher than the other that the rod is too short to give a reading on both points for one position of the instrument. In such a case

one or more auxiliary points, called turning-points (T.P.), must be used, and their relative elevations determined. Suppose the reading on the first point is 0.824, and on a turning-point is 10.432; the latter is then 9.608 below the former. Now the instrument must be moved and set up so as to obtain a reading on the turning-point; and (we will suppose) on the other of the given points. Suppose that on the former it is 1.302, and on the latter 8.634; the latter is then 7.332 below the turning-point, or $9.608 + 7.332$, or 16.940, below the first of the two given points.

The first sight taken after setting up the level is called a back-sight, or plus sight; those taken after this, and before the instrument is moved, are called fore-sights or minus sights. As the difference of the readings of the rod on two points gives their difference of elevation, the difference of the sum of the plus sights, and the sum of the minus sights on T.P.'s and the last point will give the difference in elevation of the extreme points. In the above example

$$\begin{array}{r} 0.824 \\ 1.302 \\ \hline 2.126 \end{array} \qquad\qquad \begin{array}{r} 10.432 \\ 8.634 \\ \hline 19.066 \end{array}$$

$$19.066 - 2.126 = 16.940, \text{ as before.}$$

This is used as a check on level-notes.

In extended levelling, permanent elevations fixed during the progress of the work for future reference are called bench marks or benches (B.M.).

414. In levelling, it is customary to refer all elevations to an assumed level plane, called the plane of reference, the datum plane, or simply the datum. Points are then said to be so much above or below the datum. As this plane may be assumed at pleasure, it is generally so taken as to be lower than any point whose elevation is to be determined. In city levelling this plane may be assumed at the height of mean low water.

which elevation may be called zero. Then a point which has the elevation 125.37 will be 125.37 above low water.

If two points have the elevations 125.375 and 105.213 respectively, the former is 125.375 − 105.213, or 20.162 higher than the latter.

The datum having once been determined, its elevation, or that of a point a known distance above it, should be permanently fixed for future reference and comparison.

415. The levels for profile given under Street Grades, on page 365, show how the field notes in levelling may be kept. The elevation of the bench-mark from which they start is 51.415 above the datum. The first plus sight is 7.030, which, added to 51.415, gives 58.445, the height of the instrument (H.I.) above the datum. The first minus sight, which is on a turning-point (T.P.), is 0.870, which, subtracted from 58.445, gives 57.575, the height of the T.P. above the datum. The instrument is then moved, set up again in a convenient place, and the work proceeds.

At one setting of the instrument, the elevations of any points, besides the turning-point, which are not too high or too low to be reached, may be ascertained. It is evident that if any error be made at a T.P., all the following elevations will thereby be affected; but if made at one of these other points, only the elevation of that point will be affected. Hence the importance of careful observations at T.P's.

In the above-mentioned form for the keeping of the field notes, all the observations (Obs.) are set in one column. If desired, plus sights and minus sights may be set in different columns; and of minus sights, those on turning-points may be set in a column by themselves. It will then be easy to apply the check before described. However, the form given is in practice very convenient.

EXERCISE.

Tabulate in both of the above forms, also in the form headed

Sta.	+ S.	H. I.	− S.	Elevation.	Remarks.

the following level notes:

Height of B.M.	100.000.
Obs. on B.M.	5.132.
" " Sta. 0	6.28.
" " " 1	7.12.
" " " 2	8.84.
" " T.P. 3	9.780.
From new position of inst. obs. on Sta. 3,	2.160.
Obs. on Sta. 4	5.89.
" " " 5	7.92.
" " " 6	10.18.
" " T.P. 7	12.020.
Again on " 7	1.260.
Obs. on Sta. 8	4.23.
" " " 9	5.87.
" " " 10	6.94.

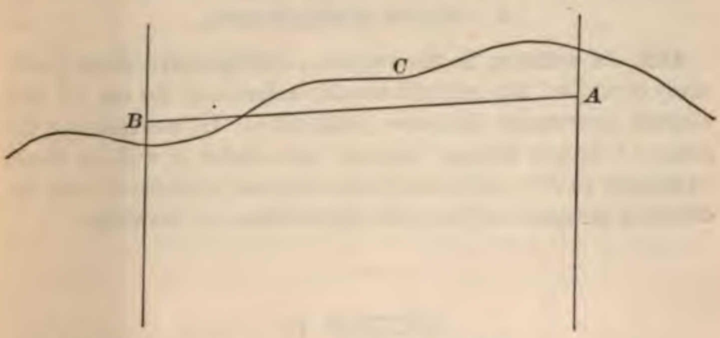
416. Wind and sunshine affect the accuracy of levelling, as of work with the transit. For very good work it is desirable to have a calm day on which the sun is obscured by clouds. In addition to a proper manipulation of the instrument, the sights should not be longer than from 200 to 300 feet, the rod should be held vertical, and the rodman should select for turning-points good and firm points on stones, pegs, etc., on which the rod may be freely turned or spun around.

417. Numerous bench-marks should be located in convenient places. In a city such places are at the intersections of streets, on door-sills of buildings which have become thoroughly settled, on roots of trees, etc. There are many other suitable places which will suggest themselves.

418. In city work, in making a circuit of levels for the establishment of grade elevations and bench-marks, the work should check out with no greater error than 0.01 foot in three miles.

In levelling, as in all other work, regard must be had to the difference between actual mistakes, the results of carelessness, and the degree of accuracy actually obtainable by the observer.

We will now describe a general method of running a grade-line with the transit. In the figure the irregular line represents the profile of the ground, and the straight line the grade-line.



Let it be required to run a grade-line from A, elevation 30.29, to B, elevation 28.79; elevation of plug or ground at A 33.49, at B 27.26; therefore cut at A 3.20 and fill at B 1.53.

Set the transit over A; and, using the long level-tube, take the elevation from a convenient bench. Suppose the H.I. is found to be 38.21; then the length of the rod for marking the grade-line (called working height) is $38.21 - 30.29 = 7.92$. The rod will then be taken to B and held on the plug. But as the plug is 1.53 below the grade-line at B, the target, when the rod is held for grade on that plug, will be set at $7.92 + 1.53 = 9.45$. When thus held, the observer will set the horizontal cross-hair on the middle of the target and clamp the telescope. The line of sight will then be a line parallel with the grade-line and 7.92 above it. Care must be taken to use the rod 7.92, and

not 9.45, as the working height. Measurements may now be made from the line of sight to determine the cut to the grade-line at any intermediate point.

Suppose at C the rod read 5.97; then the cut at that point is $7.92 - 5.97 = 1.95$.

How would you proceed if the instrument were set at B?

The cuts or fills to grade at any points may be determined by taking the elevations of the ground at those points and calculating the grade elevations at the same points. The difference of elevation will be the cut or fill required.

B. OFFICE INSTRUMENTS.

419. In addition to the various drawing-instruments previously described the student should understand the use of that elegant instrument the polar planimeter. In ascertaining the areas of figures having irregular boundaries it will be found extremely useful. He should also become acquainted with the different methods for the rapid reproduction of drawings.

SECTION II.

WORK.

420. The work of the city surveyor may be divided into two classes: first, public work, or that which he is called upon to perform for the city government; second, private work, or that which he performs for private citizens. The former is generally connected with the streets; the latter, with the property between them.

Again, all of his work may be classed as field work or office work, the former of which we will now consider.

A. FIELD WORK.

421. Public Work. There are many and varied natural features and artificial influences affecting the original location

of a town or city. To the thoughtful student many of these will readily suggest themselves. While in the choice of a site the surveyor may have a voice, it is more than probable that his work will commence upon a site already selected. We will now describe some of his more important duties as performed for the town or city government.

422. Street Lines. The city consists of streets for public use, and of the blocks bounded by them, the land in which is divided and sold to individuals for their private use. Hence we have first to consider the general plan or arrangement of the streets, their widths (the distances between house lines), and their distances apart. There are many general plans which may be adopted, or may be used as the foundation for new ones. When general convenience and the economical division of property are considered, I believe there is none which better meets the requirements than that which is characterized by two systems of parallel streets crossing at right angles. With this general arrangement, and some well-located diagonal avenues, we have the lay-out of a beautiful and convenient city.

The general directions of the streets should be such that the greatest number may during the day be visited by the sunshine. This will be accomplished if one set of parallel streets runs in a northeasterly and southwesterly direction.

Every important street should be at least 60 feet wide, while some of the main streets should be at least 100 feet wide, with avenues even wider. The streets will then admit freely air and sunshine, which latter is too often in narrow streets cut off by tall buildings; while the avenues will be in harmony with their design as elegant thoroughfares.

Another important consideration which affects the width of streets is the expense of paving and of keeping them in order.

The distances of the streets from each other will vary very much, according to the purposes for which the included property is to be used, and how it is to be divided. They may vary

from 300 to 600 feet. The sidewalks will be from one-fifth to one-fourth of the width of the streets.

In small towns an elaborate design will not be attempted; but it is always best to have in view the possibilities of future growth.

423. With the transit, the surveyor will run and extend street lines, and will turn off required horizontal angles on the horizontal graduated circle of that instrument. It is convenient to work upon the centre lines of the streets. Two base lines having been carefully located at right angles with each other, the centre lines of the two sets of streets will, with the most reliable measuring-instruments at the disposal of the surveyor, be carefully located parallel with them respectively. If the land is quite level, a 200-foot steel tape is useful. If it be inclined and irregular, a 100-foot tape is better suited to the purpose. In any case, the hand-level, plummet, thermometer, etc., should be used. The work, like all work of the surveyor, should be carefully checked by a test of the different angles and distances. All this work should be done with the greatest care. It is desirable, in order to guard against future difficulties in regard to measurements by other parties, to make streets and block distances a little full; that is, greater than they are actually required to be — say about one-fourth of an inch in 100 feet. As the work progresses, it will be properly marked with stakes, as before described. After the satisfactory location of the centre lines of the street, the house lines may easily be located therefrom.

424. The work of the surveyor may be not in laying out and regulating a new town, but in connection with one already laid out. The extensions of the old town may be carried on in harmony with the plan already existing, or they may be on a plan altogether different, and after the manner already described for a new town. He will find that the already built-up portions of the town have been previously regulated, or that they have

not been. If they have been, it is advisable in carrying on the work therein to adhere as closely as possible to established lines, elevations, standard of measurement, etc., lest any alterations should lead to expensive and unnecessary legal complications. If the town has never been regulated, the first steps will be to regulate its streets. In doing this a complete survey will be required. Instrument lines will be carefully located with the transit on all streets, and the angles at their intersections determined. These lines will be the basis for the location, by offsets, of all buildings, fences, etc. As the survey goes on, the results will be carefully plotted to a conveniently large scale; and from the completed plot, an advantageous location of the streets may be determined upon. They will then be located upon the ground to correspond. All important measurements will be made, as before described, with the steel tape, with all the corrections carefully attended to. Offsets to fences, etc., need not be made with so much care, and the corrections will, as a rule, be superfluous. During the progress of the work in an old town, as in a new one, all important lines will be carefully marked with stakes, and upon permanent objects, as houses, etc.

425. The streets in any city or town having been satisfactorily located according to the general plan, it is necessary, in order to preserve work already done, and to prevent conflict in future work, that the location of the street lines should be preserved. On account of the perishable nature of wooden stakes, and the fact that they may soon be disturbed, it is necessary to use something more permanent. This is generally found in stones. Mere stones, or monuments used for permanently holding the lines of streets, are differently located and are of different sizes, depending upon their location. Sometimes they are placed in the sidewalks 5 feet from the house lines. Then they need not be more than 4 or 5 inches square and 2 feet in length. The line is determined by a small hole drilled in the top of the stone. Sometimes the top of the stone is placed below the surface of the pavement; sometimes it is placed flush

therewith. Larger stones set in the intersections of the streets, where their centre lines cross, are very conveniently situated for use, and afford a very satisfactory means of marking street lines. On account of their more exposed position, they must be larger than those previously described, and should be set with the greatest care, the materials around them being well packed and rammed. They should be paved about and well protected from danger from traffic. The stones should be square in cross-section about 3 feet long, about 8 inches square on the top, and about 1 foot square on the bottom, the top and bottom being at right angles with the axis of the stone. The line is determined as before by a hole drilled in the top of the stone. From their situation we call these stones centre stones. It is well also to mark substantial buildings standing at the corners of streets with their distances from the house lines of the streets, these distances having been carefully determined by measurements. In general, a line having once been determined upon as satisfactory, every available means should be employed to preserve its location, as any change would obviously be attended with inconvenience and danger.

426. Street Grades. In the selection of a site for a town, and in the location of the streets of a town or city, a topographical map will be of much service. This map will show at a glance the shape of the ground under consideration. If the surface of the earth were cut by horizontal planes 5, 10, 20, or more feet apart, and the curves in which these planes intersect the surface were projected upon a horizonal plane, the resulting lines would be called contour lines or contours. These curves would represent points of the same elevation. Their distances apart would represent relative inclination in the ground, the curves being nearer as the ground is steeper. The determination of these contours is an important feature in topographical surveying. In addition to its other uses, such a map would be of service in locating sewers, also in fixing proper elevations and grades for streets. The field work necessary in the prep-

aration of topographical maps, which we will briefly notice, may be done as follows: Two sets of parallel lines having been located at right angles with each other by means of the transit and tape, the level will be set up, and a number of points at any one elevation above the datum found with the level and the rod, and their locations with reference to the two sets of lines determined. Another set of points as far above or below the former as the planes are apart will in like manner be determined and located, and so on until the entire ground has been gone over. The above method of topographical surveying in determining contours is not a very rapid one. The stadia method is more rapid, and is well adapted to large areas. In addition to the usual horizontal cross-hair in the transit, two others are introduced, one above and one below the former. The instrument has also a vertical circle. The stadia-hairs are so arranged that when the level rod is held at a certain distance from the transit, a certain number of feet on the rod is included between them. The *distance* of any point from the instrument can be determined, as it varies with the number of feet intercepted on the rod. The line of sight must be at right angles to the rod; if it is not, a calculation must be made to determine the distance. By this distance and a horizontal angle the point is located horizontally.* The *elevation* of the point above the station at which the instrument is placed is obtained by observing on the rod a point as much above the ground as the telescope is, and taking the vertical angle. The product of the horizontal distance and the tangent of the angle will give the required difference in elevation. The plane table also has been much used in making topographical surveys.

Street grades themselves will be determined upon in the office, after the necessary data has been obtained in the field.

427. A very convenient method of obtaining the data necessary for the determination of elevations and grades for the streets is to obtain a continuous profile of the ground on the

* See Chapter II., Stadia Measurements, Articles 148–152.

centre line of each street. The work is done in the following manner: The level having been set up, and the height of instrument determined from a convenient bench-mark, an elevation will be taken on a level plug set at the intersection of the centre lines of two streets. Elevations will then be taken at stations, say 50 feet apart, about on the centre line, measurements with the tape being commenced at the intersection before mentioned, and made carefully enough to avoid any error that might affect the work. In addition to the elevations at the stations, elevations should be taken at any intermediate points where the shape of the ground abruptly changes; and the points should be located by measurement. These intermediate points are called pluses. When the next intersection is reached, measurements will be commenced anew, and the levelling continued in the same manner. Elevations on level plugs at intersections, on turning-points, and on benches, which, if not previously established should be established as the work progresses, should be carefully taken with the target. The elevations for the profile should be read without the target to the nearest hundredth. Such circuits should be made in levelling for profiles, and the levelling on the cross-streets should be so carried on as to check the work in every way. The level notes, taken as described for the profile of the centre line of a street, are shown below. They are from actual practice. The datum is mean low water in the ——— River, the elevation of which is taken as zero. The manner of plotting these notes, and of determining grade lines is given under the head Office Work.

428. In order to avoid errors in giving grade lines, the grade elevations at the intersections of streets should be permanently marked. This may be done by placing the centre stones before described so that their tops shall be at the grade elevation. In order to preserve these elevations in case of the removal or disturbance of the stones, bench-marks should be established on convenient door-sills, and in other safe and con-

LEVELS ON FIFTH AVENUE, SOUTHERLY FROM MARYLAND AVENUE.

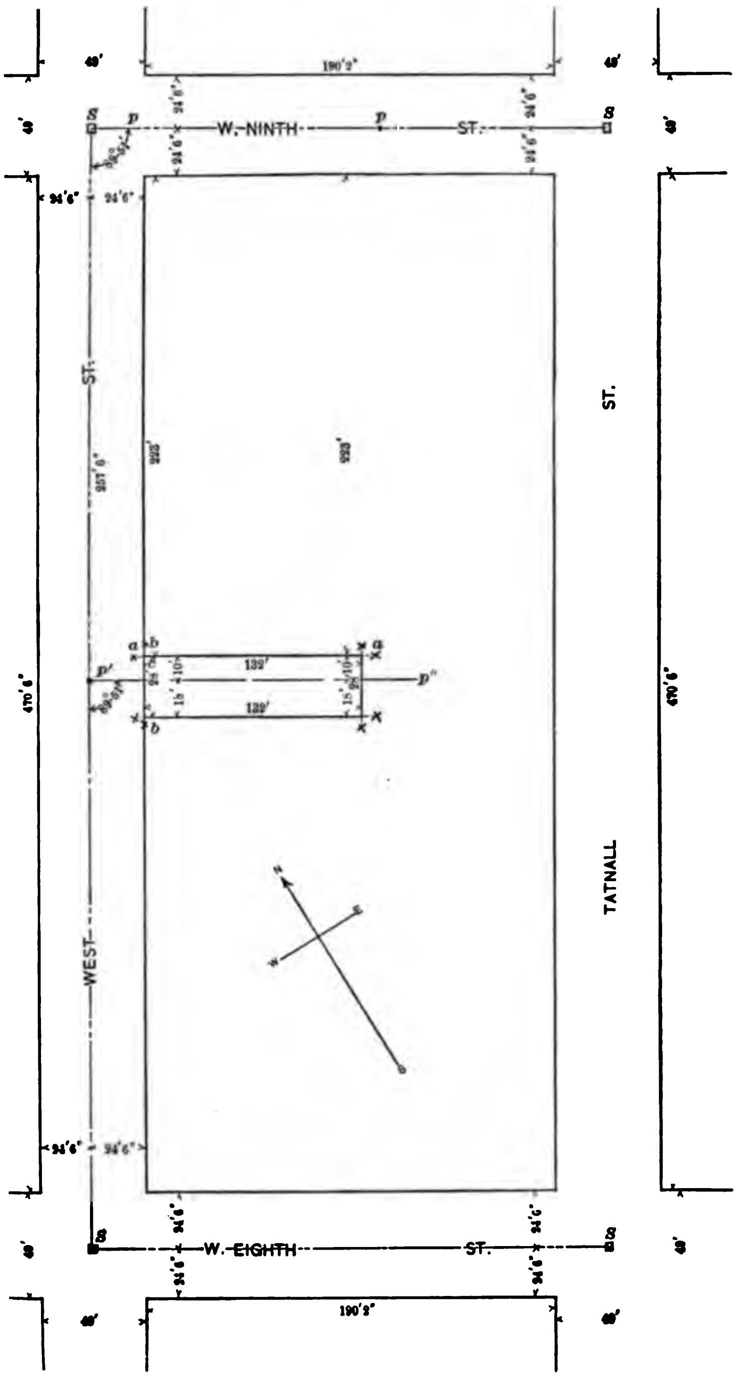
FOR PROFILE. Nov. 21, 1886, A.M.

STA.	OBS.	H.I.	EL.		REMARKS.
B.M.			51.415	....	On west end of door-sill, etc.
+	7.030	58.445	....		
P.	0.870	....	57.575		
+	10.005	67.580	....		
B.M. & P.	1.300	....	66.280	....	On highest point of red rock, etc.
+	0.900	67.180	....		
Sta. 0.	0.000	....	67.180	....	Plug middle of 5th and Md. Aves.
0 + 25.	1.55	....	65.63		
0 + 35.	0.28	....	66.90		
1.	1.50	....	65.68	....	50-ft. Sta. meas. south from mid. of Md. Ave.
2.	3.91	....	63.27		
3.	6.20	....	60.98		
4.	8.83	....	58.35		
5.	11.80	....	55.38		
6.	13.20	....	53.98		
7.		....	....		
Plug & P.	11.352	....	55.828	....	Plug centre 5th Ave. and Anchorage St.
+	4.365	60.193	....		
B.M.	5.480	....	54.713	....	Temporary — on plug near fence, etc.
Sta. 1.	5.13	....	55.06	....	50-ft. sta. meas. south from middle of Anchorage St.
2.	4.65	....	55.54		
3.	4.93	....	55.26		
4.	5.69	....	54.50		
5.	7.26	....	52.93		
6.	11.00	....	49.19		
Plug 6 + 34.	12.224	....	47.969		Plug centre 5th Ave. and Brown St.

venient places. Besides serving as benches for the stones, these bench-marks will be used in doing very close final levelling, the tops of the stones being too uneven for that purpose.

429. Marking of Lines and Grades. The lines and grades of the streets having been finally determined, and the means of preserving them having been established, the marking of these lines and grades for any public work, as street extension and grading, curb setting, sewer and water-pipe laying, etc., can be readily done. Street lines will be run with the transit; and, in the manner previously described, grade lines will be run with the same instrument. The marking of street lines and grades for the purposes mentioned, the giving of lines and elevations for other public work, and measurements of various kinds, as of earthwork, constitute the principal part of the field work to be done for the town or city government by the city or town surveyor; or, as the officer who does this work may have more extended duties, the principal part of the *surveying* to be done by the city engineer.

430. Private Work. Continuing the description of the field work of the town or city surveyor, we will notice the second general class in which his work is comprised; that is, work for individuals, or private work. In general, — for other duties in this connection will fall to his lot, such as surveying large tracts according to methods already described, etc., — this work will consist in marking property lines and in giving grades and elevations. As a rule, in a town or city more property lines are marked for buildings than for any other purpose. When the surveyor is called upon to locate the lines of a lot, his first inquiry will be as to the data by which to locate them. It is of course understood that in this connection the only power of the surveyor is to locate lines according to given data, not, as many persons seem to think, to establish of his own volition new lines. So we will inquire what is proper data for locating such lines. In general, the


W. NINTH ST.
W. EIGHTH ST.
WEST ST.
TATNALL ST.
190'2"
470'6"
257'6"
223'
132'
24'6"
40'

party desiring to have the lines of a lot marked will produce his deed for the property. The young surveyor will be inclined to think that the distances given in deeds are, as to the location of lines, final. This is not always the case. When walls, alleys, stones, and other permanent landmarks are called for, and can be found, they will take precedence of distances in locating lines. When walls, fences, and other holdings prove undisputed possession for a period of years, though they may not be described in the deed, they govern. In such cases it would be superfluous to mark lines. In towns and cities lots are now as a rule located from the streets. Let us take, in marking the lines for a lot, an example from actual practice. The description taken from the deed is definite, and is as follows:

Beginning at the easterly side of West Street, between Eighth and Ninth Streets, at the distance of 223 feet from the southerly side of Ninth Street; thence easterly, parallel with Ninth Street, 132 feet to a corner; thence southerly, parallel with West Street, 28 feet to a corner; thence westerly, parallel with the first-described line and Ninth Street, 132 feet to the aforesaid easterly side of West Street; and thence thereby, northerly, 28 feet to the place of beginning. The lot is located as shown in the sketch. The owner desired to have marked upon the ground, for use in building, the two lines parallel with Ninth Street and the line of the easterly side of West Street. In order that they may not be removed in making excavations for cellars, walls, etc., the nail plugs to mark the lines are set 3 or 4 feet outside of the lot. In the sketch, S, S, S, S represent the stone monuments set at the intersections of the centre lines of the streets to mark lines and grade elevations. Each street is 49 feet wide. In marking the lot, points p, p, will be taken in the centre line of Ninth Street. From these points (if there are no obstructions that prevent) measurements will be made parallel with West Street. Twenty-four feet six inches, half the width of Ninth Street, and 223 feet, the distance from the southerly side of Ninth

Street to the northerly side of the lot, will be laid down, and nails placed in nail plugs at a, a, to mark the northerly line of the lot. From these the southerly line will be located. In a similar manner the front and back lines will be located. Lines strained from a to a and from b to b will cross at c, giving a corner of the lot, the nail plugs being undisturbed as the work of building progresses.

If, on account of impassable obstacles, as buildings, walls, etc., a measurement cannot be made from Ninth Street to the place for the nail plug a back of the lot, the marking of the side lines will be done as follows: The southeast angle at the intersection of Ninth and West Streets, 89° 51′, if not known, will be taken. In addition to the points taken in the centre line of West Street for use in locating the front and back lines of the lot, an additional point p' will be taken, and at this point the angle 89° 51′ will be thrown in, and the random line $p'p''$ located parallel with Ninth Street. On this random line points for the location of the side lines will be taken. Now, suppose the point p' is found by measurement to be 257 feet and 6 inches from the centre of Ninth Street (all corrections having been made), or 233 feet from the southerly side thereof. Then the northerly side line will be located by measuring northerly from the line $p'p''$ 10 feet, and the southerly side line by measuring southerly from the line $p'p''$ 18 feet. If the surveyor is in possession of an instrument thoroughly reliable for use in angular measurements, the latter method of marking side lines is to be preferred. When one measurement is made along a sidewalk where there are no obstructions, and the other through fences and over various obstructions, it is hardly possible to obtain the degree of accuracy that may be obtained by the angular method. Sometimes it may be necessary to turn off an angle from the random line in order to locate the back line of a lot. The location of lines is often marked by nails in fences, measurements to houses, walls, etc., instead of by nails in plugs.

After the street lines have been located and marked, the

work in each block should be done independently of the other blocks.

In the intervals between routine work it is desirable, in connection with gathering other data, to take and record in a suitable book, for use as described above, the angles at the intersections of the streets, thus saving time in marking the lines of lots.

The location from the deed of the lines of a lot is not always so easy as in the example given. It is frequently the case that the distances given are indefinite; sometimes none are given. In such cases, in the absence of established holdings, or other means of determining the location of property lines, the matter must be settled by an arrangement between adjoining owners.

In some cases a lot is described in whole or part without distances, but as bounded by the property of other owners. In such a case the location of the lines may, if the descriptions in the deeds of these other proprietors are sufficiently definite, be determined by marking the lines of the other lots.

431. The city or town surveyor will frequently be called upon for surveys to locate new lines with reference to the street lines, or for surveys of tracts of land in or adjoining the city or town. In such cases his manner of working will be based upon the methods of land-surveying already described.

Private parties will frequently require, for use in building operations, the marking of grade lines. This will be done in the manner previously described. In marking the grade and height of the building line in front of a lot, it will very often be found convenient to mark the tops of the front line plugs as so much above or below grade elevation.

B. OFFICE WORK.

432. Like the field work, the office work of the surveyor may be classified as *Public Work* and *Private Work.*

433. Public Work. All field notes should be sufficiently elaborate to be understood by those who may have occasion to

refer to them. They should be carefully arranged and indexed like all other office records for convenient reference. Plots of work should be made whenever they will aid in the preservation and proper understanding of work done in the field. When plans are sent from the office, copies should always be retained.

434. It is desirable that, besides the necessary general plans of the town or city, the surveyor should have in his office two sets of plans, of a size convenient for handling, representing the city in sections. For these plans a horizontal scale of 100 feet to the inch is suitable.

The first set should represent street lines. On them should be placed all the street lines, and, in figures, the widths of streets and block distances, also the location of street monuments, measurements made from time to time between centres, angles at the intersections of the centre lines of streets, and any other data of a like nature giving information in regard to horizontal measurements, whether of lines or angles.

The second set should represent street grades. On them should be placed, as on those of the other set, the street lines and, in figures, the widths of streets, block distances, and location of street monuments. In addition, there should be placed upon them the profiles of the centre lines of the streets. These plans will be used in determining grade lines for the streets, which, after they have been determined, will be placed upon the plans, with the grade elevations (G.E.) and surface elevations at the intersections of the centre lines of streets, grade elevations at curb corners, and any other data giving information in regard to vertical measurements. The street lines having been laid down, we will explain, in connection with the accompanying sketch copied from a plan in actual use, how the data given on page 365 would be used in placing upon the plan the profile of the centre line of Fifth Avenue, and then how the plan would be used in determining suitable grades for the streets.

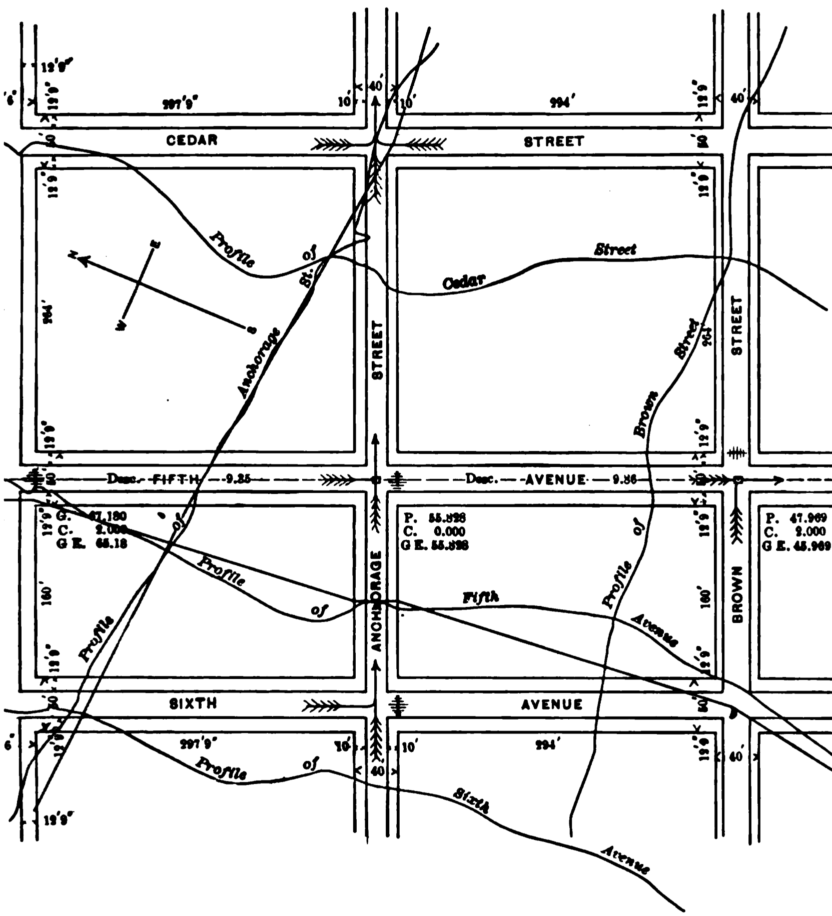
435. If the points whose elevations have been determined by the level be connected by a line in a vertical plane, such a line is called a profile. The block distance from Maryland Avenue to Anchorage Street is 297 feet and 9 inches, from Anchorage Street to Brown Street is 294 feet, from Cedar Street to Fifth Avenue is 264 feet, and from Fifth Avenue to Sixth Avenue is 160 feet. Maryland Avenue is 64 feet and 6 inches wide, Anchorage and Brown Streets each 40 feet wide, and Cedar Street, Fifth Avenue, and Sixth Avenue each 50 feet wide. The sidewalks on Cedar Street and on Fifth, Sixth, and Maryland Avenues are 12 feet and 9 inches wide, and on Anchorage and Brown Streets are 10 feet wide. By the use of the profile of Fifth Avenue we will illustrate how the profiles of the centre lines of the streets are placed upon the plan. The irregular lines represent profiles. The profile is commenced by considering the centre line of Fifth Avenue, as drawn on the plan, to have the elevation 67.180, which is the elevation in the notes for the surface of the ground at the intersection of the centre lines of Fifth and Maryland Avenues. The stations and pluses as given in the notes are then laid down by scale on the centre line of Fifth Avenue, in the order in which they were taken in the field, beginning at the centre of Maryland Avenue. The elevation at each of the points thus located is then plotted, in a perpendicular to the centre line at that point, with reference to the centre line elevation 67.180. In this case the points obtained will all fall below the centre line. These points are points in the profile, and, being joined, will give the profile as shown. The profile of Fifth Avenue having been started at the elevation of the ground at the intersection of Fifth and Maryland Avenues, is said to be swung on Maryland Avenue. In the sketch, the profiles of Cedar Street and Sixth Avenue also are swung on Maryland Avenue. Those of Anchorage and Brown Streets are swung on Cedar Street.

436. A little thought will make it evident to the student that, as the differences of elevation are small as compared with the

horizontal distances, if both were plotted to the same scale, or, as we say, if the vertical and horizontal scales were made equal, the differences in elevation will scarcely be apparent. This is remedied by conveniently exaggerating the vertical scale. For example, if the horizontal scale be made 100 feet to the inch, the vertical scale might be made 10 feet to the inch. In the sketch the two scales have this ratio.

EXERCISE. Let the student select scales, and, in the manner described above, prepare a profile from the field notes given on page 356.

437. Having thus plotted the streets and profiles in a large area, we may, by use of the plan thus made, determine suitable grades for the streets. This will involve careful study of the shape of the ground, location of watercourses, probable location of sewers, and effect upon property. The effect of a proposed grade for one street upon those which it crosses must be particularly noticed. To properly perform this work involves that knowledge and judgment which can only be acquired by long experience. The straight lines drawn in connection with the profiles represent the surface grades of the finished streets. In fixing the grade for Fifth Avenue, those of the other streets having been taken into consideration, it was found best to have a cut of 2 feet at Maryland Avenue, no cut or fill at Anchorage Street, and a cut of 2 feet at Brown Street. The elevations of the surface at the intersections of Fifth Avenue with Maryland Avenue, Anchorage Street and Brown Street, are respectively 67.180 on the ground, 55.828 and 47.969 on plugs flush with the ground. The grade line having been fixed, the grade elevations (G.E.) at the centres are respectively 65.180, 55.828, and 45.969, and the descents 9.35 feet and 9.86 feet, as shown in the sketch. The nature of grades will depend much upon local considerations. Grades should always be steep enough to secure proper drainage. The inclination should not be less than 1 in 100. Considering the accumulations of dirt on many of our city streets, from 1 to 1.5 in 100 is to be preferred.


CEDAR
STREET
FIFTH
AVENUE
SIXTH
AVENUE
ANCHORAGE
STREET
BROWN
STREET
Profile of Cedar Street
Profile of Anchorage St.
Profile of Brown Street
Profile of Fifth Avenue
Profile of Sixth Avenue
Desc. 9.35
Desc. 9.36
P. 55.898
C. 0.000
G E. 55.898
P. 47.969
C. 2.000
G E. 45.969
N
S
E
W

438. In streets in which surface water is carried on the streets, some streets will carry the water in gutters across others. In the sketch such streets are indicated by having arrows drawn in their directions across intersections. In this manner Fifth Avenue carries the water across Brown Street, and Anchorage Street carries it across Fifth Avenue. The water flowing on Fifth Avenue, from Maryland Avenue towards Anchorage Street, will turn into Anchorage Street. The opposite side of Anchorage Street, at the house line, will be a knuckle as high as the centre of the street; and the water will flow from that point towards Brown Street. In fixing grades great care must be taken to so arrange them that one street shall not be overtaxed with water from the others. An outlet for the surface water is formed in the natural watercourses.

If the grade of Anchorage Street were very heavy, so that if continued across it would make one side of Fifth Avenue much higher than the other, it would be desirable to break the grade of Anchorage Street at the curb lines of Fifth Avenue, giving only sufficient fall to carry the water across the Avenue.

439. If the section is sewered, and if the sewers are made large enough to carry the surface water, the gutters across the streets will be dispensed with, and inlets to the sewers placed at the curb corners of the blocks.

440. It is often convenient and useful to have plotted on separate streets the profile and grades of each street.

441. Besides making street and grade plans, it will be a part of the office work of the surveyor to plot, in the usual manner of plotting such work, the surveys made in and about the city or town, for both the city and individuals.

442. In some cities a registry of property is kept. The plotting of lots in suitable record books, and the keeping up of the records, will be a part of the city surveyor's work.

443. Private Work. This includes the preparation of any plans ordered for their own use by parties other than those connected with the city government.

CONCLUSION.

444. The student must bear in mind that he can never, from books, learn to be an accomplished surveyor. The practice is ever in advance of the books. Though he should store his mind with book knowledge upon the subject, he will yet be wanting in the knowledge and readiness regarding actual work which can only be acquired by a long experience. Many operations which can with difficulty be understood from pages of explanation, will, when their actual performance is seen, be comprehended in a short time. Again, there is that which can *never* be learned from books; that is, the judgment which must be constantly exercised in practising the delicate duties of a city surveyor. Among other things, this judgment will teach him to be *very* cautious about giving voluntary advice, and careful in giving even that which is requested; to perform his duties conscientiously, and to keep clear of all entangling alliances. Let him learn everything connected with a complete performance of his work, from the work of the axeman up; that, when he directs, he may do it with the same grace with which he should ever follow the directions of his superiors.

The practice of city surveying is a most excellent drill. If conscientiously performed, it will develop careful and thoughtful habits. However, in practice the student will also have to learn to avoid "fussing" over work, and to proportion to the importance of the work in hand the time and care spent upon a particular work.

BOOKS.

445. Valuable information regarding the matters treated of in this chapter will be found in the following publications:

The manuals and catalogues of instrument-makers.

"A Treatise on the Principles and Practice of Levelling," by Frederick W. Simms; published by D. Van Nostrand, New York.

"A Descriptive Treatise on Mathematical Drawing-Instruments," by William F. Stanley; published by E. & F. N. Spon, New York and London.

"A Manual of Drafting Instruments and Operations," by S. Edward Warren; published by John Wiley & Son, New York.

"The Draughtsman's Handbook of Plan and Map Drawing," by George S. André; published by E. & F. N. Spon, New York and London.

The student of surveying who wishes to extend his studies into the field of city engineering will find information upon that subject in the numerous works upon its special branches, and in the current technical periodicals of that class. Much information regarding present American practice in city engineering will be found in the series of papers on "Municipal Engineering" now being published in "Engineering News." When completed, these in book form will make a very useful volume.

CHAPTER VIII.

MINE SURVEYING.

446. The survey of underground excavations (mines) to determine their position and extent may be principally for the purpose of projecting the points upon a horizontal plane as in land surveying.

But in strata of high inclination and in cavernous spaces various vertical projections will be needed to complete the graphical representation of the workings; and in fissure veins the elevation may be more important than the plan.

447. Surveys to depict areas underground may be made with surveyors' compass and chain, but generally now the transit or theodolite is used to take the angles, and the steel tape to measure the distances, and in some mines the tape may be with advantage hundreds of feet in length; but generally 50 feet for the chain or 100 feet for the tape are most convenient lengths.

448. The surveyor and each assistant, of course, requires a lamp, and "the sights" are ranged with lamp and plummet, the sight from the instrument being taken upon the flame of the miner's lamp (or candle, it may be) suitably held at the plummet line, which is held to depend from a point fixed or to be fixed in the "roof" or over a point in the "bottom." The plummet string itself may be seen within 300 feet. A chain-pin (arrow) can be used to plumb the light over or under a point. It is advised to display the light at a station *for sight only*, and therefore in moving it, for any reason, other than vertically, in giving the point, it should be hidden from the observer.

The point may be marked by a nail in the timber cap or sill, or be a nail in a peg; the place of the point in smooth roof is to be made conspicuous by a ring of white paint around it, and also as it may be by reference marks at the sides (pillars) of the passage-way.

It is a refinement to use a lamp which is also a plummet, and further to place an extra lamp on the bottom under it; two lights seen in the vertical line making its place more certain, and helping to decide that the sight is ready to be taken.*

It may happen that the line of reflection from standing water can be taken for the line of incidence of a light held under a point, when the roof droops between, the passage being "in swamp" there.

The surveyor's lamp is made entirely of brass or copper, so as not to affect the magnetic needle of the instrument.

For use in low openings the tripod of the instrument must be one of short legs (an extra set of shifting legs will answer the purpose), or have extension legs.

It has been suggested to use two extra tripods, one to set up in advance, for keeping the place of fore-sight and for receiving the instrument alone, carried forward to be mounted there at the same exact spot with facility, while the tripod, left standing at the last place of the instrument, marks the point for back-sight with equal certainty: thus each of three tripods taking its turn in being at a place for fore-sight, remaining there for mounting the instrument upon it, and still remaining for back-sight after the instrument is taken for mounting at next station. There are obvious objections to this in the weight of the luggage, and that only some instruments are made for such ready separate handling.

Some rays of light must be thrown into the telescope at its object end to make visible the cross-hairs therein. This is generally done by the surveyor, while taking a sight, holding his

* Eckley B. Coxe devised the plummet lamp, and also a form of it with wire-gauze covering, like the Davy Safety Lamp, for use where fire-damp may be expected.

lamp in his left hand at the front, but a little to one side of the object-glass. A reflector mounted at the object end is a help. One is a silvered flat ring, standing bias, about 2 inches forward from a collar which is slipped over the object end of the telescope. It reflects light into the instrument as an annular beam. Another one is a diminutive hemisphere which scatters light caught from the lamp into the tube.

The change to, and the equable temperature of, the mine require the trying and favor the making of the ordinary adjustments of the instrument there.

449. Stations are generally made only at the angle points of survey lines, and are therefore not regularly distanced. They may be numbered, lettered, or designated by the total distance from the zero of the measurements of their line. Intermediate points are made on the line where, opposite to lateral openings, other lines of survey or important short connections by measurement merely may start. The corners of chambers along the passage may be noted by distance without making points; the size and position of parts of chambers being afterwards taken and noted by sketch with dimensions relatively marked thereon, there being mostly a parallelism in the rock measures which simplifies the position and shape that chambers take, so that no special survey of directions is regularly required for them.

450. Angles between vertical planes of sight (in azimuth) are noted for obtaining the courses as reduced courses from the initial course of survey, by the successive additions and subtractions to it and from it of the angles as taken, and modified according to the series of 90° in each quadrant of the circle.

The initial course had better be referred to true meridian, and comparison with bearings made with allowance for the variation (declination) of the needle. But it has always been recognized that the course, in degrees and minutes, of a quadrant — and therefore liable to mistakes as to the particular one of four quadrants — would be absolute if the full circle be graduated

around to 90°, 180°, 270°, and 360°, in the successive quadrants. While it is not agreed whether north or south shall be the zero, the direction of graduation with the movement of the hands on the dial of a watch or clock is conventionally fixed. The bearings will be a key to which zero was used in the notes.

451. It is but seldom that in drifts of mines the alignment as well as the grade requires adjustment to the regularity of straight lines and curves similar to surface railroads; for the tram-cars will run around very sharp turns, and for them there is therefore no necessity of expensive improvements in line. But when a locomotive is to be used, or wire-rope haulage is to be introduced, there is apt to be a call for regulation of the line, with regard, especially, to minimum radius of curvature.

Unlike the longer, flat curves of a railroad, — designated according to the American system by the even angular deflections from each other of chords of 100 feet, — these sharper curves will go by assumed even radii (in length not less than ten times the gauge of track), and the deflection angles for running them in by the instrument upon short chords will have to be calculated.

One-half the chord divided by the radius will equal the sine of the angle of deflection from tangent, which is half the angle that two such equal chords will make with each other, and also half the angle at the centre of the circle subtended by the chord. From any point on the circular curve as a position of the instrument, successive deflections of the angle will fix the ends of consecutive chords as measured in. Shorter chords (like those less than 100 feet in a railroad curve) have deflection angles approximately proportional to their lengths.

For ranging the line of direction of a passage that is being opened into the solid, two points for placing lights are given at the start, necessarily near together, until the prolongation of open space allows testing the line by the instrument and giving new points of line. From the three points of a curve line that mark the chords of half the arc, obviously, by simple measure-

ments, a like fourth point may be derived as the face (breast) of the working is advanced. In driving a passage-way describing a semicircle — to save weakening pillar at foot of shaft — a long, curved gas-pipe was used in ranging around. A large-scale working plot showing offsets secures the proper location of curving and branching passages.

Outside, besides the fixing of projected curves by deflection angles as above, the laying off of points of arc intermediate on the chord is by foot-rule measurement of ordinates at right angles.

But without strict regard to data, an expedient way of uniting two intersecting straight lines of track by a circular curve (as an arc starting from the one straight line at any distance short of the apex of the lines and ending on the other line an equal distance from the apex) is to find points by linear measurement merely. Assuming any tangential distance back from apex to P.C. (point of curve), the beginning, and the same to P.T. (point of tangent), the end of curve, we find a third point of the arc, its middle, as a point midway between the middle of the chord of the whole arc and the apex. One-fourth of this *versed sine* will be the versed sine (middle ordinate) to be erected on each chord of half the arc for points of the arc. And any other middle ordinates will be as the squares of their arcs or chords.

This principle applies in rounding off intersecting grades into vertical curves, either convex or concave; by vertical allowances and according to horizontal distances, starting with that at the apex and proceeding similarly to the foregoing as to subdivisions.

The laying off of curves by chords and versed sine so derived does not require knowledge of length of radius or of amplitude of angle. But when the extent of circular arc between two tangents is to be determined by the length of radius, the tangential distance from apex will equal radius multiplied by natural tangent of half the angle of intersection; and between P.C. and P.T. there will be the same measures of chord as there are of chord angles in angle of intersection.

452. In the note-book the left-hand page is used for stations, distances, angles, courses (reduced), and bearings (magnetic), and the opposite right-hand page for offset distances — marked relative to a perpendicular line dividing the page, together with sketches and remarks. The notes should begin at the bottom of the pages and proceed upwards, to appear as on the plan to which their results are to be transferred, in their proper relation of position and observation forward.

The plan of underground work is begun with the plotted network of the lines of survey, then the outline of parts excavated is drawn in detail, and these are shaded, as the places become closed in and abandoned, to distinguish what is open work at any period.

The scale of maps showing the workings, etc., of coal mines is now fixed by law in many of the States at 1 : 1200 as the least; that is, at not less than 1 inch for 100 feet; the purpose of the maps being to aid the official inspection and regulation of the mines for securing the health and safety of the miners. The plan will generally require to show the relation of the workings to surface openings, watercourses, and bounding lines, and to improvements, such as buildings, roads, and railroads.

The line of outcrops (exposure at the surface of the ground of the mineral beds) within its range will appear on the map, but general topographical detail is reserved for the extended small-scale maps of the surface, which will represent what may be learned of mineral indications also; from which data in advance of the workings may be derived and confirmed by special explorations, as of proof-holes and deep boring. But upon the mine plan such elevations (heights of surface above datum) as seem most essential, such as principal ones along the outcrops, highest points of hills, and lowest of streams should be mapped.

The use of the *pantograph*, for reducing the irregular figures of mine plans with all details from one scale to another, has found much approval; and the *planimeter* is liked for labor-saving and accuracy in determining such areas.

453. In *veins*, the work being deep and narrow, and pursued from *levels* or galleries (horizons of working) generally about 60 feet apart in height, plans of these levels, drawn in different colors to distinguish them, are superimposed on the map of general plan. They show the openings, — the gangways, the cross-cuts, etc., — with the defining lines of the walls of the vein, and may embrace other separations of the mineral. Longitudinal elevation and vertical cross-sections will show the shafts and other connections between the levels, together with the *chambers*, whether open, filled in, or caved.

Ore bodies occurring detached and of the most varying dimensions, though often resembling each other as lenticular in shape, make the workings appear in plan, elevation, and cross-section, as the results of exploration in patches. Shafts in the vein will be parallel to pitch of one wall, and therefore varying from the vertical.

A stratified bed that is to be operated upon, — opened, and won by mining, — may be conceived as a seam of uniform small thickness extending within limits as a plane surface and in relative position defined by the "strike" (the course of all its level lines, which will all be parallel) and its "dip" (the greatest pitch at right angles to the course of the level-line). But upon the large scale the seam occurs of variable thickness, and with lines of level changing in direction and not parallel at different elevations, to the degree that instead of a plane it is a warped surface.

The arrangement of permanent works upon the surface of the ground with reference to the lay of the bed as well as the topography and improvements existing or suited to it, the favorable connection of the lines of haulage and drainage inside, with all to govern outside, present to the mind of the mathematical surveyor applications of the theorems of Descriptive Geometry, as included in adaptation to the ends of practical economy.

454. **Location** upon the surface of the ground of the plan of inside work, is a repetition of courses and distances outside in the

same vertical planes. Any particular portion of the workings in progress can thus be compared in natural scale upon actual plan of surface of the ground over them.

Overlaid plans with elevations and cross-sections of workings, such as were described for workings in veins, are required to show the development in high pitching beds. The "lifts" or levels in such of coal are 100 yards apart, measured on line of pitch.

Overlaid plans of different parallel seams worked through same shaft are also made, but without systematic elevation and cross-section; the connections (shafts, slopes, or tunnels) between the beds being through barren ground, and limited to the exigencies of hoisting, draining, and ventilating.

455. Following the determination in azimuth by courses and distances of the passages in the mine is the determination of their changes in level by the spirit levelling-instrument and the level-rod (as a separate operation, even if the transit be a combined instrument having a parallel spirit level attached to its telescope), the work being quite similar to such above ground. But the rod must be limited in height to the low spaces where it is to be used, and is preferably marked with red figures for the feet, and white figures for the tenths, upon a black ground. The top of a simple white target is safer to take, however, than the reading from the instrument of the figures themselves. For accuracy, sights, as above ground, should be limited to 300 feet in distance from the instrument.

From the elevations of points taken by levelling, contour lines can be shown on plan as the mineral bed is exploited.

Blue is the conventional color for these contour lines and the figures marking their elevation above the datum, on a mine plan, and *brown* suits for the contrasted surface elevations.

456. Levelling along passage-ways for the purpose of fixing better gradients of hauling-roads, or for fall of water by rectification of undulating bottom to improve drainage, requires sta-

tions especially chained in at regular distances of 50 feet or less; the marks being temporary ones on the sides to serve for taking the levels and to be referred to as to heights in grading, when the *variation* of level of bottom from the *grade* of a station governs the cutting or filling of bottom there, or change of the whole cross-section in height, as it may be. For the adoption of suitable gradients along an extended line, a longitudinal vertical section is drawn, called a profile, which exhibits the relation of ground-line levels, and allows the fixing of grade with assurance. The profile may include the line of top as well as of bottom, with section of rock measures to be affected by "ripping" of the roof and "cutting" of bottom.

457. **A Drift** or passage along with the measures of a bed will make undulating grade, if course be followed; and if the drainage-rise be allowed to govern, the alignment will be sacrificed.

Tunnelling, however, being arbitrary, across the measures, is mostly upon directed line and grade. *Slopes* are mostly upon directed course; but if within the measures of an inclined bed will mostly be variable in grade. So with an *adit*, driven to give drainage outfall to the surface. For it, shortening of the distance will probably be the governing condition principally.

458. For the workings at high pitch, the determination of horizontal and vertical components of the distances on the sloping lines of top and bottom in a bed, and "hanging wall" and "foot wall" in a vein, will bring the vertical arc of the instrument into requisition, for obtaining the vertical angle, which is always taken as the full angle above the horizontal. Vertical sections, besides such longitudinal ones following broken line of passage within a stratum and showing only adjacent rock measures, may be made of particular places where there is *folding*, or *fault*, of the measures, and for geological or more general purposes they may exhibit the lay and thickness of the various rocks up to the surface, which will as a correct

margin show the outcroppings in profile. Vertical sections may be projections upon planes that traverse the measures according to various conditions, and may be constructed of related points from the map that were not determined for their relevancy to this purpose.

It seems that vertical arcs have had *versed sines* corresponding to radius 1 marked around them for the purpose of telling the allowance upon slope measurement to obtain corresponding horizontal distances, the versed sine being the difference between the hypothenuse as the radius and the horizontal base as the cosine of the vertical right-angled triangle formed; and the slope length for a given horizontal distance would be greater, according to the versed sine of the angle.

Vertical arcs have had *tangents* as rises corresponding to the unit of horizontal distance for the different angles marked upon them.

A method of dividing the arc according to the *sines*, without the intervention of the equal graduation into degrees necessarily, is the subject of a contribution to "Van Nostrand's Engineering Magazine" for July, 1876, and is appended at the end of this chapter.

459. The measurement down deep borings or shafts is best made by special flat steel wire, with suitable plummet heavy enough to insure its making the wire line taut.

The transfer of points down a shaft, as of two to determine a base line for connecting surveys below with those on the surface of the ground, is made by very heavy plummets attached to ordinary wire run off of reels. A portable box to contain the reels, their cranks, and the plummets, is convenient; the best arrangement being that of reels fixed in a frame that stays in the box. The suspended plummets are to be received below each in a bucket of water, or, if hanging from considerable height, in some thicker liquid to settle the wire lines to a steady position for ranged observation by the instrument below. And the observation will be easier upon wire that is whitened there by chalk or paint after being placed.

The plummets in the shaft of the Washington Monument, for showing changes in the verticality of the structure, are steadied in vessels containing a mixture of glycerine and molasses.

460. For taking courses on pitches at high angles an extra telescope on the axis extended to the outside of one of the standards of transit has been used. Another *mining transit* has for the same purpose the sweep of the telescope to the vertical position, made possible by having its standards made inclined to overhang. But the *object-prism* placed before the object-glass, allowing sighting at true right angles in any plane, seems most simply to fulfil the requirements for sighting up or down, as well as sidewise, and is a ready means applicable to the telescope of any ordinary instrument. A transit adapted in any of these ways for taking vertical sights enables the points of base line, as transferred by plummets to the bottom of the shaft, to be tested and compared with the extended line across the pit top, provided the atmosphere be clear in the shaft and obstructions do not intervene. The vertical adjustment of the instrument itself would be tested by this check, the usual test being on high objects, with reversal of standards to opposite sides by turning the horizontal plates.

A heavy, substantial, simple transit, not weighted with "attachments," is the most reliable.

461. The use of the *hanging compass* and of the *hanging clinometer* of the olden time is retained in small and crooked passages of some metalliferous mines. And their subsidiary use in excavations inconvenient of access or footing of the ordinary (the standing instruments) has lately been recommended as of wider application, and they have been introduced into this country. Each of the instruments is to hang by its two hooks, turned opposite ways, to the cord that marks the line. The compass-box levels itself by its gimbals (double trunnions), like a ship's compass, in the frame of which the flat hooks with long bearings in line are a part. The clinometer

hangs as a vertical arc with plummet to give the inclination of the cord from the horizon, while the compass gives the needle course. The cord is stretched from one low stout tripod to another, or in a curving space may be fastened to a gimlet screwed into side timber beyond intersecting point or angle of two cords. The tripod serves as a stool also for the assistant holding cord to the point on it firmly. The distances are accurately measured along the cord by applying a graduated rod to it. The horizontal and vertical components of the measurements have to be calculated for plotting on plan and section. In the old mining regions of Europe the surface surveys were also carried on with the same appliances. With care and patience surprisingly good results in locating connections were attained. The old instruments were graduated in hours and minutes, and the English designations of *dial* and *dialling* for the mine compass and operations with it seem to refer to the same original division of its circle. It seems strange to learn that the plotting was protracted by the same compass (swung there on horizontal plate used for straight edge), reference being had to a meridian line fixed in the office, and the drawing-table being a smooth and level stone slab resting on foundation independent of the office floor.

462. Formerly, when topography was used more for the picturing of the plan of landscape in mapping the features for the information of the tourist or the military commander, than for the projection of the contour accurately to fit the location of artificial ways of the different kinds to the ground, *hachures* were used to indicate character of sloping elevations, and they survive in use upon small-scale maps, to indicate mountain chains. They are intended to be lines of pitch, drawn close together so as to graduate changes naturally, and they should be broken at the intersection of the successive level planes with the surface to make terraces however narrow, and suggest level stages in measure of elevation. Now we have on topographical plans *contour lines* to represent the lines of suc-

cessive levels, say 10 feet apart in rise. They are plotted by connecting all points of elevation that may be determined over the area with regard to the requirements of accuracy in noting the changes; and they may be considered the margins made by a body of water that had successively risen or receded 10 feet in height at a time over the area. They are to be marked by their elevation above the lowest datum plane, preferably over that of mean tide of the ocean. They turn upon themselves where they enclose a peak or a basin — according as the next ones indicate them as higher or lower in the series; they are farther apart in horizontal distance as slopes are flatter, and where two or more coincide for any distance there is a precipice.

These points of even elevations of the ground are determined from the levels run along the survey lines, and the cross-section profiles taken at the stations of the lines—slopes being taken at right angles to the line with *straight edge pole* and *clinometer* or *plummet slope level* applied to it. Each of these angle instruments having a vertical graduated arc, the former with arm hinged at centre of arc and carrying a spirit-level to ascertain the vertical angle included between the levelled arm and the slope of the straight edge under it; the latter, by the departure from the perpendicular of the plummet, showing the equal departure from the horizontal of the straight edge.

From the profile of each slope sketched in the field-book and marked with distances and degrees of rise and fall across the survey line, the successive even 10-foot points can be laid off on plan, regard being had in starting with elevation of station to the partial changes required for the first even 10-foot point each way. A scale of horizontal distances for each degree of the arc, to gain 10 feet rise, is made by the topographer of Bristol-board to lay off the points derived by sloping at the stations, and saves the plotting of the profile of cross-section.

The topographer prefers to draw the contours in the field as taken, using demi-sheets of paper that can be joined at their margins, and upon each of which a portion of the line corresponding to its number is plotted, the line having dots along it, spacing the successive stations intermediate of the angle points

of line, and having the elevations corresponding in pencil alongside. The sheets are held in a box that is carried by a shoulder-strap, and the side of which is used in the field as a drawing-board, the particular sheet in use at the time being tacked on it.

463. The topographer will sketch in the streams, buildings, etc., with reference to measurements however, and will have special lines with small compass, etc., run for him to make contour connections. The operations will rise to the scope of plane-table work, if the drawing-board have a socket with clamps, and be mounted and levelled (by applying a loose hand-level) upon a tripod; the ruler used on it having small compass sights screwed to its ends for sighting to objects and fixing their position on plot by the graphic triangulation of intersected sight-lines from different stations on the survey-line; the station on plot when over its place on ground having a needle stuck upright in it, that has a sealing-wax head for convenient handling, for the purpose of resting the ruler against when sighting. Interpolation, or resection, is the reverse sighting from without the line over the plot to two or three poles on stations of the line or other previously located objects, to attain position, it being understood that the plane table stands with plot in proper relative position always. Secondary triangulation will extend the area of topographic sketching, but this should be checked by connections beyond with surveyed lines and levels.

The Locke level may be used for taking rises by finding all the points in sight that are at a level of the eye, and, in connection with the levelling-rod, the fall of ground may also be determined by this instrument. For gently undulating ground the use of it is better than sloping.

464. Contour lines are drawn 10 feet apart in elevation on most plans of extended land and other surveys that are measured in detail, but it is obvious that cases occur where for large-scale work they are taken closer in elevation or farther for small-scale mapping. In the former case of large-scale work they may be required exactly as elevations directly located by

spirit levelling-instrument, in the latter case as the approximation from altitudes taken in a few places by the barometer.

The scope of their usefulness on plans for projecting improvements it would be difficult to describe exhaustively. They may be for use in locating the drives and walks and terraces, etc., of a park; the shaping of grounds, under-draining, etc., about a residence; the laying out of streets, etc., in a hilly town; the leading of streams of water, large or small, for all purposes in partial or wholly artificial channels, for navigation, water power and supply, irrigation, etc.; the location of roads and railroads with regard to ease of construction and of favorable gradients, as well as the uses in mining directly, and location of all surface erections collateral thereto or elsewhere, collectively known as "the Works."

ANGULAR CROSS-SECTIONING.

By F. Z. Schellenberg, C.E.

Written for "Van Nostrand's Engineering Magazine," July, 1876.

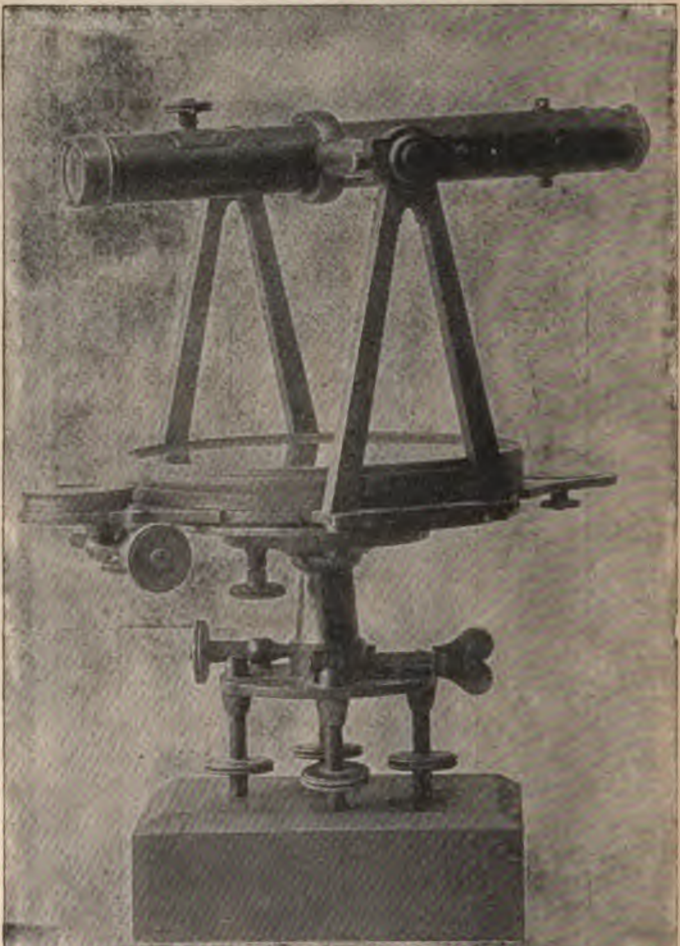
A most direct and expeditious method to get differences in level between points in sight is by the use of a vertical arc graduated to the successive sines 1, 2, 3, . . . 100, in quadrant, for the radius of arc 100.

Multiplying the distance measured in hundreds on the slope by the rate per hundred indicated on the arc gives the difference in level in units. In the higher parts of the arc the corresponding cosines may be marked for deriving horizontal distances.

The applicability of this graduation to such purposes, as described under this caption by R. Bell, C.E., in May number, is obvious, as may also be its use for more extended profiles, for geological cross-sections, for road-grading, or wherever between points obtained by the levelling-instrument its accuracy is not indispensable.

A clinometer thus graduated enables contour lines for topographical work to be most readily determined. The table following gives the 100 points in the quadrant in terms of the common graduation of 90° to the quadrant.

Vertical Distance for 100 measured on Slope.	Horizontal Distance for 100 measured on Slope.	Angle with Horizon.	Vertical Distance for 100 measured on Slope.	Horizontal Distance for 100 measured on Slope.	Angle with Horizon.
0	....	0° 00′	51	....	30° 40′
1	....	0° 34′	52	....	31° 20′
2	....	1° 09′	53	....	32° 00′
3	....	1° 43′	54	....	32° 41′
4	....	2° 18′	55	83.5	33° 22′
5	99.9	2° 52′	56	....	34° 03′
6	....	3° 26′	57	....	34° 45′
7	....	4° 01′	58	....	35° 27′
8	....	4° 35′	59	....	36° 09′
9	....	5° 10′	60	80.0	36° 52′
10	99.5	5° 44′	61	....	37° 35′
11	....	6° 19′	62	....	38° 19′
12	....	6° 54′	63	....	39° 03′
13	....	7° 28′	64	....	39° 48′
14	....	8° 03′	65	76.0	40° 32′
15	98.9	8° 38′	66	....	41° 18′
16	....	9° 12′	67	....	42° 04′
17	....	9° 47′	68	....	42° 51′
18	....	10° 22′	69	....	43° 38′
19	....	10° 57′	70	71.4	44° 26′
20	98.0	11° 32′	71	....	45° 14′
21	....	12° 07′	72	....	46° 03′
22	....	12° 43′	73	....	46° 53′
23	....	13° 18′	74	....	47° 44′
24	....	13° 53′	75	66.2	48° 35′
25	96.8	14° 29′	76	....	49° 28′
26	....	15° 04′	77	....	50° 21′
27	....	15° 40′	78	....	51° 16′
28	....	16° 16′	79	....	52° 11′
29	....	16° 51′	80	60.0	53° 08′
30	95.4	17° 27′	81	....	54° 06′
31	....	18° 04′	82	....	55° 05′
32	....	18° 40′	83	....	56° 06′
33	....	19° 16′	84	....	57° 08′
34	....	19° 53′	85	52.7	58° 13′
35	93.7	20° 29′	86	....	59° 19′
36	....	21° 06′	87	....	60° 28′
37	....	21° 43′	88	....	61° 39′
38	....	22° 20′	89	....	62° 52′
39	....	22° 57′	90	43.6	64° 09′
40	91.6	23° 35′	91	....	65° 30′
41	....	24° 12′	92	....	66° 56′
42	....	24° 50′	93	....	68° 26′
43	....	25° 28′	94	....	70° 03′
44	....	26° 06′	95	31.2	71° 48′
45	89.3	26° 45′	96	....	73° 44′
46	....	27° 23′	97	....	75° 56′
47	....	28° 02′	98	....	78° 31′
48	....	28° 41′	99	....	81° 54′
49	....	29° 20′	100	00.0	90° 00′
50	86.6	30° 00′			



TRANSIT,

AS FIRST MADE IN 1831 BY THE INVENTOR, WILLIAM J. YOUNG, PHILADELPHIA, PA.

APPENDIX.

THE JUDICIAL FUNCTIONS OF SURVEYORS.*

When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his actions by facts which lead him away from the results which theoretically he ought to reach.

Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan, all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is, that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed, it is impossible to determine by the record, by the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats were

* By Chief Justice Cooley of the Supreme Court of Michigan.

more accurately surveyed, as indeed they should have been; for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear.

The fact probably is, that town surveys are quite as inaccurate as those made under authority of the general government. It is now upwards of fifty years since a major part of the public surveys, in what is now the State of Michigan, were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold and the work of improvement began. A generation has passed away since they were converted into cultivated farms, and few, if any, of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must have been relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush-fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing was substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary. If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determine the boundary line between the proprietors.

However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though

the effect be to make one half-quarter section 90 acres, and the one adjoining 70; for parties buy, or are supposed to buy, in reference to these monuments, and are entitled to what is within their lines, and no more, be it more or less. While the witness trees remain, there can generally be no difficulty in determining the locality of the stakes. When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men, whose theoretical education is thought to make them experts, who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and would have been if placed correctly. This is a serious mistake. The problem is now the same that it was before: To ascertain by the best lights of which the case admits where the original lines were. The mistake above referred to is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed. An act passed in 1869 (Compiled Laws, 593), amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

"*Second.* Extinct interior section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

"*Third.* Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners; in all other cases, at its proportionate distance between the nearest original corners on the same line."

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near. To

estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct.

If the monument was set for an interior section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word "extinct" may be used in this connection: one, the sense of physical disappearance; the other, the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man loses his deed, he shall lose his land altogether. But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and Federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether State or Federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged. The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost, until after he has thoroughly sifted the evidence, and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary, when no other is attainable; and the surveyor should inquire when it originated, how and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or

may not have been trustworthy; but to bring these lines into discredit, when the people concerned do not question them, not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance, and perhaps discredit, since in a legal controversy the law, as well as common sense, must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. Stewart *v.* Carleton, 31 Mich. Reports, 270; Diehl *v.* Zanger, 39 Mich. Reports, 601. And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession must often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees and no monuments to govern, except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these without looking farther.

Now it may perhaps be known in a particular case that a certain monument still remaining was the starting-point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and used for many years, and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot-owners quarrel, and one of them calls in a surveyor that he may be sure that his neighbor

shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was or not, and the first result is, he notifies the lot-owners that there is error in the street line, and that all fences should be moved, say, one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot-owners will generally allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result? It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. McNamara *v.* Seaton, 82 Ill. Reports, 498; Bunce *v.* Bidwell, 43 Mich. Reports, 542. Public policy requires that such lines be not lightly disturbed or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go as an officer of the law is to express his opinion where the monument should be at the same time that he imparts the information

to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer, and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument for any considerable period as strong, if not conclusive, evidence of such settlement. The peace of the community absolutely requires this rule. Foyce *v.* Williams, 26 Mich. Reports, 332. It is not long since that in one of the leading cities of the State an attempt was made to move houses two or three rods into a street, on the ground that a survey, under which the street had been located for many years, had been found on a more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor, where boundaries are in dispute, must be varied by the circumstances. (1) He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties express or implied have rendered them unimportant. By monuments in the case of government surveys we mean, of course, the corner and quarter stakes; blazed lines or marked trees on the lines are not monuments; they are merely guides or finger-posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. (2) If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any State statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts, giving due prominence to the acts of parties concerned, and always keeping in mind, first, that neither his opinion nor his survey can be conclusive upon parties concerned; and, second, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and same rules that will govern theirs. On town plats if a surplus or

deficiency appears in a block when the actual boundaries are compared with the original figures, and there is no evidence to fix the exact location of the stakes which marked the division into lots, the rule of common sense and the law is that the surplus or deficiency is to be apportioned between the lots on an assumption that the error extended alike to all parts of the block. O'Brien *v.* McGrane, 29 Wis. Reports, 446; Quinnin *v.* Reixers, 46 Mich. Reports, 605.

It is always possible when corners are extinct that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course it is desirable that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines. The subject to which allusion will now be made is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless, it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that meander lines are boundary lines, and all in front of them remains unsold. Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and while some courts have held

that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the State while conceding to the owners of the bank all riparian rights. The practical difference is not very important. In this State the rule that the centre line is the boundary line is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation in some cases with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the centre line of the river. Of course the original survey supposes that each purchaser of land on the stream has a water-front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and in some cases, if it were continued in the same direction to the centre line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation or other valuable use of the water, while the other might have a water-front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless, it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule. Extend the line of division between the two parcels from

the meander line to the centre line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water-front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water-lots which appear as such on town plats. Bay City Gas Light Co. *v.* The Industrial Works, 28 Mich. Reports, 182. It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out. When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties, and may constitute a sufficient guide when peculiar cases arise. Each riparian lot-owner ought to have a line on the legal boundary, namely, the centre line of the stream, proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream, when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the

original admeasurement. Wherever that was the case the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of government sale afterwards, for the reason that the rights of the bank owners are fixed by their purchase: when making that they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it cannot afterwards be taken from them. It is believed, however, that the Federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the Great Lakes.

On all the little lakes of the State, which are mere expansions near their mouths of the rivers passing through them, — such as the Muskegon, Pere Marquette, and Manistee, — the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. Rice *v.* Ruddiman, 10 Mich. 125. If such a lake were circular, the lines would converge to the centre; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for some considerable distance, in order to determine where one may lawfully moor vessels or rafts for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. Lorman *v.* Benson, 8 Mich. 18; People's Ice Co. *v.* Steamer Excelsior, recently decided.

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is, that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them. As regards the lakes and ponds of the State, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not: (1) To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them? (2) What public rights exist in them? (3) If there are islands in them which were not surveyed out and sold by the United States, can this be done now? Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the Great Lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the Great Lakes would belong to the United States until sold, and might be surveyed and measured at any time. The right to take fish in the lakes or to cut ice is public, like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore-owners; but, so far as these public rights can be the subject of ownership, they belong to the State, not the United States; and so, it is believed, does the bed of a lake also. Pollard *v.* Hagan, 3 Howard's U. S. Reports. But such rights are not generally considered proper subjects of sale, but, like the right to make use of the public highways, they are held by the State in trust for all the people. What is said of the large lakes may, perhaps, be said also of many of the interior lakes of the State; such, for example, as Houghton,

Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size, — say, even a mile across, — there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon, before. But the general rules for the extension of shore lines which have already been laid down should govern such cases, or at least should serve as guides in their settlement.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the States may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation. If the State should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as a trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the State could not drain, even for this purpose, against the will of the shore-owners, unless their rights were appropriated and paid for. Upon many questions that might arise between the State as owner of the

bed of a little lake and the shore-owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not, and cannot be, judicial officers, but in a great many cases they act in a *quasi* judicial capacity, with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

TABLE I.

THE

COMMON OR BRIGGS LOGARITHMS

OF THE

NATURAL NUMBERS

From 1 to 10000.

1—100

N	log	N	log	N	log	N	log	N	log
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	30	1.47 712	50	1.69 897	70	1.84 510	90	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	40	1.60 206	60	1.77 815	80	1.90 309	100	2.[illegible]
N	log	N	log	N	log	N	log	N	[illegible]

TABLE I.

THE

COMMON OR BRIGGS LOGARITHMS

OF THE

NATURAL NUMBERS

From 1 to 10000.

1—100

N	log	N	log	N	log	N	log	N	log
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	30	1.47 712	50	1.69 897	70	1.84 510	90	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	40	1.60 206	60	1.77 815	80	1.90 309	100	2.00 000
N	log	N	log	N	log	N	log	N	log

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
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241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
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252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
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258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
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260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
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265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
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273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
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275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
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285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
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290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 67[illegible]
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299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	[illegible]	[illegible]
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301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
309	48 996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
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316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
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333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
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335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
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338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
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342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
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348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
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353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
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356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
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359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
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367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
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373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
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387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
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390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
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394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
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398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
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400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	[illegible]
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402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
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413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
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428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
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431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
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640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
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642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
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651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
N	0	1	2	3	4	5	6	7	8	9

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700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 19[illegible]
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 24[illegible]
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 30[illegible]
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
N	0	1	2	3	4	5	6	7	[illegible]	[illegible]

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800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	80 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 07$\underline{5}$	93 080	93 08$\underline{5}$	93 090
853	93 09$\underline{5}$	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 42$\underline{5}$	93 430	93 43$\underline{5}$	93 440	93 44$\underline{5}$
860	93 4$\underline{5}$0	93 45$\underline{5}$	93 460	93 46$\underline{5}$	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 30$\underline{5}$	94 310	94 31$\underline{5}$	94 320	94 32$\underline{5}$	94 330	94 33$\underline{5}$	94 340	94 34$\underline{5}$
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 66$\underline{5}$	94 670	94 67$\underline{5}$	94 680	94 68$\underline{5}$	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 90$\underline{5}$	94 910	94 91$\underline{5}$	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 09$\underline{5}$	95 100	95 10$\underline{5}$	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 26$\underline{5}$	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 40$\underline{5}$	95 410	95 41$\underline{5}$	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	*97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	[illegible]
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	[illegible]
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	[illegible]
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	[illegible]
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	[illegible]	[illegible]
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	[illegible]	[illegible]
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	[illegible]	[illegible]
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	[illegible]	[illegible]
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	[illegible]	[illegible]
N	0	1	2	3	4	5	6	7	[illegible]	[illegible]

TABLE II.

APPROXIMATE EQUATION OF TIME.

DATE.	MINUTES.	DATE.	MINUTES.	DATE.	MINUTES.	DATE.	MINUTES.
Jan. 1	4 Clock faster than Sun	Apr. 1	4 Clock Faster.	Aug. 9	5 Faster	Oct. 27	16 Clock Slower than Sun
" 3	5	" 4	3	" 15	4	Nov. 15	15
" 5	6	" 7	2	" 20	3	" 20	14
" 7	7	" 11	1	" 24	2	" 24	13
" 9	8	" 15	0	" 28	1	" 27	12
" 12	9			" 31	0	" 30	11
" 15	10	" 19	1 Clock Slower			Dec. 2	10
" 18	11	" 24	2	Sept. 3	1 Clock Slower	" 5	9
" 21	12	" 30	3	" 6	2	" 7	8
" 25	13	May 13	4	" 9	3	" 9	7
" 31	14	" 29	3	" 12	4	" 11	6
Feb. 10	15	June 5	2	" 15	5	" 13	5
" 21	14	" 10	1	" 18	6	" 16	4
" 27	13	" 15	0	" 21	7	" 18	3
Mar. 4	12			" 24	8	" 20	2
" 8	11	" 20	1 Clock Faster	" 27	9	" 22	1
" 12	10	" 25	2	" 30	10	" 24	0
" 15	9	" 29	3	Oct. 3	11		
" 19	8	July 5	4	" 6	12	" 26	1 Faster
" 22	7	" 11	5	" 10	13	" 28	2
" 25	6	" 28	6	" 14	14	" 30	3
" 28	5			" 19	15		

TABLE III.

THE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS:

From 0° to 0° 3′, or 89° 57′ to 90°, for every second;
From 0° to 2°, or 88° to 90°, for every ten seconds;
From 1° to 89°, for every minute.

NOTE. To all the logarithms −10 is to be appended.

log sin **0°** log tan = log sin
log cos = 10. 00 000

″	0′	1′	2′	″	″	0′	1′	2′	″
0	—	6. 46 373	6. 76 476	60	30	6. 16 270	6. 63 982	6. 86 167	30
1	4. 68 557	6. 47 090	6. 76 836	59	31	6. 17 694	6. 64 462	6. 86 455	29
2	4. 98 660	6. 47 797	6. 77 193	58	32	6. 19 072	6. 64 936	6. 86 742	28
3	5. 16 270	6. 48 492	6. 77 548	57	33	6. 20 409	6. 65 406	6. 87 027	27
4	5. 28 763	6. 49 175	6. 77 900	56	34	6. 21 705	6. 65 870	6. 87 310	26
5	5. 38 454	6. 49 849	6. 78 248	55	35	6. 22 964	6. 66 330	6. 87 591	25
6	5. 46 373	6. 50 512	6. 78 595	54	36	6. 24 188	6. 66 785	6. 87 870	24
7	5. 53 067	6. 51 165	6. 78 938	53	37	6. 25 378	6. 67 235	6. 88 147	23
8	5. 58 866	6. 51 808	6. 79 278	52	38	6. 26 536	6. 67 680	6. 88 423	22
9	5. 63 982	6. 52 442	6. 79 616	51	39	6. 27 664	6. 68 121	6. 88 697	21
10	5. 68 557	6. 53 067	6. 79 952	50	40	6. 28 763	6. 68 557	6. 88 969	20
11	5. 72 697	6. 53 683	6. 80 285	49	41	6. 29 836	6. 68 990	6. 89 240	19
12	5. 76 476	6. 54 291	6. 80 615	48	42	6. 30 882	6. 69 418	6. 89 509	18
13	5. 79 952	6. 54 890	6. 80 943	47	43	6. 31 904	6. 69 841	6. 89 776	17
14	5. 83 170	6. 55 481	6. 81 268	46	44	6. 32 903	6. 70 261	6. 90 042	16
15	5. 86 167	6. 56 064	6. 81 591	45	45	6. 33 879	6. 70 676	6. 90 306	15
16	5. 88 969	6. 56 639	6. 81 911	44	46	6. 34 833	6. 71 088	6. 90 568	14
17	5. 91 602	6. 57 207	6. 82 230	43	47	6. 35 767	6. 71 496	6. 90 829	13
18	5. 94 085	6. 57 767	6. 82 545	42	48	6. 36 682	6. 71 900	6. 91 088	12
19	5. 96 433	6. 58 320	6. 82 859	41	49	6. 37 577	6. 72 300	6. 91 346	11
20	5. 98 660	6. 58 866	6. 83 170	40	50	6. 38 454	6. 72 697	6. 91 602	10
21	6. 00 779	6. 59 406	6. 83 479	39	51	6. 39 315	6. 73 090	6. 91 857	9
22	6. 02 800	6. 59 939	6. 83 786	38	52	6. 40 158	6. 73 479	6. 92 110	8
23	6. 04 730	6. 60 465	6. 84 091	37	53	6. 40 985	6. 73 865	6. 92 362	7
24	6. 06 579	6. 60 985	6. 84 394	36	54	6. 41 797	6. 74 248	6. 92 612	[illegible]
25	6. 08 351	6. 61 499	6. 84 694	35	55	6. 42 594	6. 74 627	6. 92 861	[illegible]
26	6. 10 055	6. 62 007	6. 84 993	34	56	6. 43 376	6. 75 003	6. 93 109	[illegible]
27	6. 11 694	6. 62 509	6. 85 289	33	57	6. 44 145	6. 75 376	6. 93 355	[illegible]
28	6. 13 273	6. 63 006	6. 85 584	32	58	6. 44 900	6. 75 746	6. 93 599	[illegible]
29	6. 14 797	6. 63 496	6. 85 876	31	59	6. 45 643	6. 76 112	6. 93 843	[illegible]
30	6. 16 270	6. 63 982	6. 86 167	30	60	6. 46 373	6. 76 476	6. 94 [illegible]	[illegible]
″	59′	58′	57′	″	″	59′	58′	[illegible]	[illegible]

log cot = log cos
log sin = 10. 00 000 **89°** log cos

′ ″	log sin	log tan	log cos	″ ′
0 0	—	—	10.00000	0 60
10	5.68 557	5.68 557	10.00000	50
20	5.98 660	5.98 660	10.00000	40
30	6.16 270	6.16 270	10.00000	30
40	6.28 763	6.28 763	10.00000	20
50	6.38 454	6.38 454	10.00000	10
1 0	6.46 373	6.46 373	10.00000	0 59
10	6.53 067	6.53 067	10.00000	50
20	6.58 866	6.58 866	10.00000	40
30	6.63 982	6.63 982	10.00000	30
40	6.68 557	6.68 557	10.00000	20
50	6.72 697	6.72 697	10.00000	10
2 0	6.76 476	6.76 476	10.00000	0 58
10	6.79 952	6.79 952	10.00000	50
20	6.83 170	6.83 170	10.00000	40
30	6.86 167	6.86 167	10.00000	30
40	6.88 969	6.88 969	10.00000	20
50	6.91 602	6.91 602	10.00000	10
3 0	6.94 085	6.94 085	10.00000	0 57
10	6.96 433	6.96 433	10.00000	50
20	6.98 660	6.98 661	10.00000	40
30	7.00 779	7.00 779	10.00000	30
40	7.02 800	7.02 800	10.00000	20
50	7.04 730	7.04 730	10.00000	10
4 0	7.06 579	7.06 579	10.00000	0 56
10	7.08 351	7.08 352	10.00000	50
20	7.10 055	7.10 055	10.00000	40
30	7.11 694	7.11 694	10.00000	30
40	7.13 273	7.13 273	10.00000	20
50	7.14 797	7.14 797	10.00000	10
5 0	7.16 270	7.16 270	10.00000	0 55
10	7.17 694	7.17 694	10.00000	50
20	7.19 072	7.19 073	10.00000	40
30	7.20 409	7.20 409	10.00000	30
40	7.21 705	7.21 705	10.00000	20
50	7.22 964	7.22 964	10.00000	10
6 0	7.24 188	7.24 188	10.00000	0 54
10	7.25 378	7.25 378	10.00000	50
20	7.26 536	7.26 536	10.00000	40
30	7.27 664	7.27 664	10.00000	30
40	7.28 763	7.28 764	10.00000	20
50	7.29 836	7.29 836	10.00000	10
7 0	7.30 882	7.30 882	10.00000	0 53
10	7.31 904	7.31 904	10.00000	50
20	7.32 903	7.32 903	10.00000	40
30	7.33 879	7.33 879	10.00000	30
40	7.34 833	7.34 833	10.00000	20
50	7.35 767	7.35 767	10.00000	10
8 0	7.36 682	7.36 682	10.00000	0 52
10	7.37 577	7.37 577	10.00000	50
20	7.38 454	7.38 455	10.00000	40
30	7.39 314	7.39 315	10.00000	30
40	7.40 158	7.40 158	10.00000	20
50	7.40 985	7.40 985	10.00000	10
9 0	7.41 797	7.41 797	10.00000	0 51
10	7.42 594	7.42 594	10.00000	50
20	7.43 376	7.43 376	10.00000	40
30	7.44 145	7.44 145	10.00000	30
40	7.44 900	7.44 900	10.00000	20
50	7.45 643	7.45 643	10.00000	10
10 0	7.46 373	7.46 373	10.00000	0 50
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
10 0	7.46 373	7.46 373	10.00000	0 50
10	7.47 090	7.47 091	10.00000	50
20	7.47 797	7.47 797	10.00000	40
30	7.48 491	7.48 492	10.00000	30
40	7.49 175	7.49 176	10.00000	20
50	7.49 849	7.49 849	10.00000	10
11 0	7.50 512	7.50 512	10.00000	0 49
10	7.51 165	7.51 165	10.00000	50
20	7.51 808	7.51 809	10.00000	40
30	7.52 442	7.52 443	10.00000	30
40	7.53 067	7.53 067	10.00000	20
50	7.53 683	7.53 683	10.00000	10
12 0	7.54 291	7.54 291	10.00000	0 48
10	7.54 890	7.54 890	10.00000	50
20	7.55 481	7.55 481	10.00000	40
30	7.56 064	7.56 064	10.00000	30
40	7.56 639	7.56 639	10.00000	20
50	7.57 206	7.57 207	10.00000	10
13 0	7.57 767	7.57 767	10.00000	0 47
10	7.58 320	7.58 320	10.00000	50
20	7.58 866	7.58 867	10.00000	40
30	7.59 406	7.59 406	10.00000	30
40	7.59 939	7.59 939	10.00000	20
50	7.60 465	7.60 466	10.00000	10
14 0	7.60 985	7.60 986	10.00000	0 46
10	7.61 499	7.61 500	10.00000	50
20	7.62 007	7.62 008	10.00000	40
30	7.62 509	7.62 510	10.00000	30
40	7.63 006	7.63 006	10.00000	20
50	7.63 496	7.63 497	10.00000	10
15 0	7.63 982	7.63 982	10.00000	0 45
10	7.64 461	7.64 462	10.00000	50
20	7.64 936	7.64 937	10.00000	40
30	7.65 406	7.65 406	10.00000	30
40	7.65 870	7.65 871	10.00000	20
50	7.66 330	7.66 330	10.00000	10
16 0	7.66 784	7.66 785	10.00000	0 44
10	7.67 235	7.67 235	10.00000	50
20	7.67 680	7.67 680	10.00000	40
30	7.68 121	7.68 121	10.00000	30
40	7.68 557	7.68 558	9.99999	20
50	7.68 989	7.68 990	9.99999	10
17 0	7.69 417	7.69 418	9.99 999	0 43
10	7.69 841	7.69 842	9.99 999	50
20	7.70 261	7.70 261	9.99 999	40
30	7.70 676	7.70 677	9.99 999	30
40	7.71 088	7.71 088	9.99 999	20
50	7.71 496	7.71 496	9.99 999	10
18 0	7.71 900	7.71 900	9.99 999	0 42
10	7.72 300	7.72 301	9.99 999	50
20	7.72 697	7.72 697	9.99 999	40
30	7.73 090	7.73 090	9.99 999	30
40	7.73 479	7.73 480	9.99 999	20
50	7.73 865	7.73 866	9.99 999	10
19 0	7.74 248	7.74 248	9.99 999	0 41
10	7.74 627	7.74 628	9.99 999	50
20	7.75 003	7.75 004	9.99 999	40
30	7.75 376	7.75 377	9.99 999	30
40	7.75 745	7.75 746	9.99 999	20
50	7.76 112	7.76 113	9.99 999	10
20 0	7.76 475	7.76 476	9.99 999	0 40
′ ″	log cos	log cot	log sin	″ ′

0°

′ ″	log sin	log tan	log cos	″ ′
20 0	7.76 475	7.76 476	9.99 999	0 **40**
10	7.76 836	7.76 837	9.99 999	50
20	7.77 193	7.77 194	9.99 999	40
30	7.77 548	7.77 549	9.99 999	30
40	7.77 899	7.77 900	9.99 999	20
50	7.78 248	7.78 249	9.99 999	10
21 0	7.78 594	7.78 595	9.99 999	0 **39**
10	7.78 938	7.78 938	9.99 999	50
20	7.79 278	7.79 279	9.99 999	40
30	7.79 616	7.79 617	9.99 999	30
40	7.79 952	7.79 952	9.99 999	20
50	7.80 284	7.80 285	9.99 999	10
22 0	7.80 615	7.80 615	9.99 999	0 **38**
10	7.80 942	7.80 943	9.99 999	50
20	7.81 268	7.81 269	9.99 999	40
30	7.81 591	7.81 591	9.99 999	30
40	7.81 911	7.81 912	9.99 999	20
50	7.82 229	7.82 230	9.99 999	10
23 0	7.82 545	7.82 546	9.99 999	0 **37**
10	7.82 859	7.82 860	9.99 999	50
20	7.83 170	7.83 171	9.99 999	40
30	7.83 479	7.83 480	9.99 999	30
40	7.83 786	7.83 787	9.99 999	20
50	7.84 091	7.84 092	9.99 999	10
24 0	7.84 393	7.84 394	9.99 999	0 **36**
10	7.84 694	7.84 695	9.99 999	50
20	7.84 992	7.84 994	9.99 999	40
30	7.85 289	7.85 290	9.99 999	30
40	7.85 583	7.85 584	9.99 999	20
50	7.85 876	7.85 877	9.99 999	10
25 0	7.86 166	7.86 167	9.99 999	0 **35**
10	7.86 455	7.86 456	9.99 999	50
20	7.86 741	7.86 743	9.99 999	40
30	7.87 026	7.87 027	9.99 999	30
40	7.87 309	7.87 310	9.99 999	20
50	7.87 590	7.87 591	9.99 999	10
26 0	7.87 870	7.87 871	9.99 999	0 **34**
10	7.88 147	7.88 148	9.99 999	50
20	7.88 423	7.88 424	9.99 999	40
30	7.88 697	7.88 698	9.99 999	30
40	7.88 969	7.88 970	9.99 999	20
50	7.89 240	7.89 241	9.99 999	10
27 0	7.89 509	7.89 510	9.99 999	0 **33**
10	7.89 776	7.89 777	9.99 999	50
20	7.90 041	7.90 043	9.99 999	40
30	7.90 305	7.90 307	9.99 999	30
40	7.90 568	7.90 569	9.99 999	20
50	7.90 829	7.90 830	9.99 999	10
28 0	7.91 088	7.91 089	9.99 999	0 **32**
10	7.91 346	7.91 347	9.99 999	50
20	7.91 602	7.91 603	9.99 999	40
30	7.91 857	7.91 858	9.99 999	30
40	7.92 110	7.92 111	9.99 998	20
50	7.92 362	7.92 363	9.99 998	10
29 0	7.92 612	7.92 613	9.99 998	0 **31**
10	7.92 861	7.92 862	9.99 998	50
20	7.93 108	7.93 110	9.99 998	40
30	7.93 354	7.93 356	9.99 998	30
40	7.93 599	7.93 601	9.99 998	20
50	7.93 842	7.93 844	9.99 998	10
30 0	7.94 084	7.94 086	9.99 998	0 **30**
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
30 0	7.94 084	7.94 086	9.99 998	0 **30**
10	7.94 325	7.94 326	9.99 998	50
20	7.94 564	7.94 566	9.99 998	40
30	7.94 802	7.94 804	9.99 998	30
40	7.95 039	7.95 040	9.99 998	20
50	7.95 274	7.95 276	9.99 998	10
31 0	7.95 508	7.95 510	9.99 998	0 **29**
10	7.95 741	7.95 743	9.99 998	50
20	7.95 973	7.95 974	9.99 998	40
30	7.96 203	7.96 205	9.99 998	30
40	7.96 432	7.96 434	9.99 998	20
50	7.96 660	7.96 662	9.99 998	10
32 0	7.96 887	7.96 889	9.99 998	0 **28**
10	7.97 113	7.97 114	9.99 998	50
20	7.97 337	7.97 339	9.99 998	40
30	7.97 560	7.97 562	9.99 998	30
40	7.97 782	7.97 784	9.99 998	20
50	7.98 003	7.98 005	9.99 998	10
33 0	7.98 223	7.98 225	9.99 998	0 **27**
10	7.98 442	7.98 444	9.99 998	50
20	7.98 660	7.98 662	9.99 998	40
30	7.98 876	7.98 878	9.99 998	30
40	7.99 092	7.99 094	9.99 998	20
50	7.99 306	7.99 308	9.99 998	10
34 0	7.99 520	7.99 522	9.99 998	0 **26**
10	7.99 732	7.99 734	9.99 998	50
20	7.99 943	7.99 946	9.99 998	40
30	8.00 154	8.00 156	9.99 998	30
40	8.00 363	8.00 365	9.99 998	20
50	8.00 571	8.00 574	9.99 998	10
35 0	8.00 779	8.00 781	9.99 998	0 **25**
10	8.00 985	8.00 987	9.99 998	50
20	8.01 190	8.01 193	9.99 998	40
30	8.01 395	8.01 397	9.99 998	30
40	8.01 598	8.01 600	9.99 998	20
50	8.01 801	8.01 803	9.99 998	10
36 0	8.02 002	8.02 004	9.99 998	0 **24**
10	8.02 203	8.02 205	9.99 998	50
20	8.02 402	8.02 405	9.99 998	40
30	8.02 601	8.02 604	9.99 998	30
40	8.02 799	8.02 801	9.99 998	20
50	8.02 996	8.02 998	9.99 998	10
37 0	8.03 192	8.03 194	9.99 997	0 **23**
10	8.03 387	8.03 390	9.99 997	50
20	8.03 581	8.03 584	9.99 997	40
30	8.03 775	8.03 777	9.99 997	30
40	8.03 967	8.03 970	9.99 997	20
50	8.04 159	8.04 162	9.99 997	10
38 0	8.04 350	8.04 353	9.99 997	0 **22**
10	8.04 540	8.04 543	9.99 997	50
20	8.04 729	8.04 732	9.99 997	40
30	8.04 918	8.04 921	9.99 997	30
40	8.05 105	8.05 108	9.99 997	20
50	8.05 292	8.05 295	9.99 997	10
39 0	8.05 478	8.05 481	9.99 997	0 **21**
10	8.05 663	8.05 666	9.99 997	50
20	8.05 848	8.05 851	9.99 997	40
30	8.06 031	8.06 034	9.99 997	30
40	8.06 214	8.06 217	9.99 997	20
50	8.06 396	8.06 399	9.99 997	10
40 0	8.06 578	8.06 581	9[illegible]	[illegible]
′ ″	log cos	log cot	[illegible]	[illegible]

89°

′ ″	log sin	log tan	log cos	″ ′
40 0	8.06 578	8.06 581	9.99 997	0 **20**
10	8.06 758	8.06 761	9.99 997	50
20	8.06 938	8.06 941	9.99 997	40
30	8.07 117	8.07 120	9.99 997	30
40	8.07 295	8.07 299	9.99 997	20
50	8.07 473	8.07 476	9.99 997	10
41 0	8.07 650	8.07 653	9.99 997	0 **19**
10	8.07 826	8.07 829	9.99 997	50
20	8.08 002	8.08 005	9.99 997	40
30	8.08 176	8.08 180	9.99 997	30
40	8.08 350	8.08 354	9.99 997	20
50	8.08 524	8.08 527	9.99 997	10
42 0	8.08 696	8.08 700	9.99 997	0 **18**
10	8.08 868	8.08 872	9.99 997	50
20	8.09 040	8.09 043	9.99 997	40
30	8.09 210	8.09 214	9.99 997	30
40	8.09 380	8.09 384	9.99 997	20
50	8.09 550	8.09 553	9.99 997	10
43 0	8.09 718	8.09 722	9.99 997	0 **17**
10	8.09 886	8.09 890	9.99 997	50
20	8.10 054	8.10 057	9.99 997	40
30	8.10 220	8.10 224	9.99 997	30
40	8.10 386	8.10 390	9.99 997	20
50	8.10 552	8.10 555	9.99 996	10
44 0	8.10 717	8.10 720	9.99 996	0 **16**
10	8.10 881	8.10 884	9.99 996	50
20	8.11 044	8.11 048	9.99 996	40
30	8.11 207	8.11 211	9.99 996	30
40	8.11 370	8.11 373	9.99 996	20
50	8.11 531	8.11 535	9.99 996	10
45 0	8.11 693	8.11 696	9.99 996	0 **15**
10	8.11 853	8.11 857	9.99 996	50
20	8.12 013	8.12 017	9.99 996	40
30	8.12 172	8.12 176	9.99 996	30
40	8.12 331	8.12 335	9.99 996	20
50	8.12 489	8.12 493	9.99 996	10
46 0	8.12 647	8.12 651	9.99 996	0 **14**
10	8.12 804	8.12 808	9.99 996	50
20	8.12 961	8.12 965	9.99 996	40
30	8.13 117	8.13 121	9.99 996	30
40	8.13 272	8.13 276	9.99 996	20
50	8.13 427	8.13 431	9.99 996	10
47 0	8.13 581	8.13 585	9.99 996	0 **13**
10	8.13 735	8.13 739	9.99 996	50
20	8.13 888	8.13 892	9.99 996	40
30	8.14 041	8.14 045	9.99 996	30
40	8.14 193	8.14 197	9.99 996	20
50	8.14 344	8.14 348	9.99 996	10
48 0	8.14 495	8.14 500	9.99 996	0 **12**
10	8.14 646	8.14 650	9.99 996	50
20	8.14 796	8.14 800	9.99 996	40
30	8.14 945	8.14 950	9.99 996	30
40	8.15 094	8.15 099	9.99 996	20
50	8.15 243	8.15 247	9.99 996	10
49 0	8.15 391	8.15 395	9.99 996	0 **11**
10	8.15 538	8.15 543	9.99 996	50
20	8.15 685	8.15 690	9.99 996	40
30	8.15 832	8.15 836	9.99 996	30
40	8.15 978	8.15 982	9.99 995	20
50	8.16 123	8.16 128	9.99 995	10
50 0	8.16 268	8.16 273	9.99 995	0 **10**
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
50 0	8.16 268	8.16 273	9.99 995	0 **10**
10	8.16 413	8.16 417	9.99 995	50
20	8.16 557	8.16 561	9.99 995	40
30	8.16 700	8.16 705	9.99 995	30
40	8.16 843	8.16 848	9.99 995	20
50	8.16 986	8.16 991	9.99 995	10
51 0	8.17 128	8.17 133	9.99 995	0 **9**
10	8.17 270	8.17 275	9.99 995	50
20	8.17 411	8.17 416	9.99 995	40
30	8.17 552	8.17 557	9.99 995	30
40	8.17 692	8.17 697	9.99 995	20
50	8.17 832	8.17 837	9.99 995	10
52 0	8.17 971	8.17 976	9.99 995	0 **8**
10	8.18 110	8.18 115	9.99 995	50
20	8.18 249	8.18 254	9.99 995	40
30	8.18 387	8.18 392	9.99 995	30
40	8.18 524	8.18 530	9.99 995	20
50	8.18 662	8.18 667	9.99 995	10
53 0	8.18 798	8.18 804	9.99 995	0 **7**
10	8.18 935	8.18 940	9.99 995	50
20	8.19 071	8.19 076	9.99 995	40
30	8.19 206	8.19 212	9.99 995	30
40	8.19 341	8.19 347	9.99 995	20
50	8.19 476	8.19 481	9.99 995	10
54 0	8.19 610	8.19 616	9.99 995	0 **6**
10	8.19 744	8.19 749	9.99 995	50
20	8.19 877	8.19 883	9.99 995	40
30	8.20 010	8.20 016	9.99 995	30
40	8.20 143	8.20 149	9.99 995	20
50	8.20 275	8.20 281	9.99 994	10
55 0	8.20 407	8.20 413	9.99 994	0 **5**
10	8.20 538	8.20 544	9.99 994	50
20	8.20 669	8.20 675	9.99 994	40
30	8.20 800	8.20 806	9.99 994	30
40	8.20 930	8.20 936	9.99 994	20
50	8.21 060	8.21 066	9.99 994	10
56 0	8.21 189	8.21 195	9.99 994	0 **4**
10	8.21 319	8.21 324	9.99 994	50
20	8.21 447	8.21 453	9.99 994	40
30	8.21 576	8.21 581	9.99 994	30
40	8.21 703	8.21 709	9.99 944	20
50	8.21 831	8.21 837	9.99 994	10
57 0	8.21 958	8.21 964	9.99 994	0 **3**
10	8.22 085	8.22 091	9.99 994	50
20	8.22 211	8.22 217	9.99 994	40
30	8.22 337	8.22 343	9.99 994	30
40	8.22 463	8.22 469	9.99 994	20
50	8.22 588	8.22 595	9.99 994	10
58 0	8.22 713	8.22 720	9.99 994	0 **2**
10	8.22 838	8.22 844	9.99 994	50
20	8.22 962	8.22 968	9.99 994	40
30	8.23 086	8.23 092	9.99 994	30
40	8.23 210	8.23 216	9.99 994	20
50	8.23 333	8.23 339	9.99 994	10
59 0	8.23 456	8.23 462	9.99 994	0 **1**
10	8.23 578	8.23 585	9.99 994	50
20	8.23 700	8.23 707	9.99 994	40
30	8.23 822	8.23 829	9.99 993	30
40	8.23 944	8.23 950	9.99 993	20
50	8.24 065	8.24 071	9.99 993	10
60 0	8.24 186	8.24 192	9.99 993	0 **0**
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
0 0	8.24 186	8.24 192	9.99 993	0 60
10	8.24 306	8.24 313	9.99 993	50
20	8.24 426	8.24 433	9.99 993	40
30	8.24 546	8.24 553	9.99 993	30
40	8.24 665	8.24 672	9.99 993	20
50	8.24 785	8.24 791	9.99 993	10
1 0	8.24 903	8.24 910	9.99 993	0 59
10	8.25 022	8.25 029	9.99 993	50
20	8.25 140	8.25 147	9.99 993	40
30	8.25 258	8.25 265	9.99 993	30
40	8.25 375	8.25 382	9.99 993	20
50	8.25 493	8.25 500	9.99 993	10
2 0	8.25 609	8.25 616	9.99 993	0 58
10	8.25 726	8.25 733	9.99 993	50
20	8.25 842	8.25 849	9.99 993	40
30	8.25 958	8.25 965	9.99 993	30
40	8.26 074	8.26 081	9.99 993	20
50	8.26 189	8.26 196	9.99 993	10
3 0	8.26 304	8.26 312	9.99 993	0 57
10	8.26 419	8.26 426	9.99 993	50
20	8.26 533	8.26 541	9.99 993	40
30	8.26 648	8.26 655	9.99 993	30
40	8.26 761	8.26 769	9.99 993	20
50	8.26 875	8.26 882	9.99 993	10
4 0	8.26 988	8.26 996	9.99 992	0 56
10	8.27 101	8.27 109	9.99 992	50
20	8.27 214	8.27 221	9.99 992	40
30	8.27 326	8.27 334	9.99 992	30
40	8.27 438	8.27 446	9.99 992	20
50	8.27 550	8.27 558	9.99 992	10
5 0	8.27 661	8.27 669	9.99 992	0 55
10	8.27 773	8.27 780	9.99 992	50
20	8.27 883	8.27 891	9.99 992	40
30	8.27 994	8.28 002	9.99 992	30
40	8.28 104	8.28 112	9.99 992	20
50	8.28 215	8.28 223	9.99 992	10
6 0	8.28 324	8.28 332	9.99 992	0 54
10	8.28 434	8.28 442	9.99 992	50
20	8.28 543	8.28 551	9.99 992	40
30	8.28 652	8.28 660	9.99 992	30
40	8.28 761	8.28 769	9.99 992	20
50	8.28 869	8.28 877	9.99 992	10
7 0	8.28 977	8.28 986	9.99 992	0 53
10	8.29 085	8.29 094	9.99 992	50
20	8.29 193	8.29 201	9.99 992	40
30	8.29 300	8.29 309	9.99 992	30
40	8.29 407	8.29 416	9.99 992	20
50	8.29 514	8.29 523	9.99 992	10
8 0	8.29 621	8.29 629	9.99 992	0 52
10	8.29 727	8.29 736	9.99 991	50
20	8.29 833	8.29 842	9.99 991	40
30	8.29 939	8.29 947	9.99 991	30
40	8.30 044	8.30 053	9.99 991	20
50	8.30 150	8.30 158	9.99 991	10
9 0	8.30 255	8.30 263	9.99 991	0 51
10	8.30 359	8.30 368	9.99 991	50
20	8.30 464	8.30 473	9.99 991	40
30	8.30 568	8.30 577	9.99 991	30
40	8.30 672	8.30 681	9.99 991	20
50	8.30 776	8.30 785	9.99 991	10
10 0	8.30 879	8.30 888	9.99 991	0 50
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
10 0	8.30 879	8.30 888	9.99 991	0 50
10	8.30 983	8.30 992	9.99 991	50
20	8.31 086	8.31 095	9.99 991	40
30	8.31 188	8.31 198	9.99 991	30
40	8.31 291	8.31 300	9.99 991	20
50	8.31 393	8.31 403	9.99 991	10
11 0	8.31 495	8.31 505	9.99 991	0 49
10	8.31 597	8.31 606	9.99 991	50
20	8.31 699	8.31 708	9.99 991	40
30	8.31 800	8.31 809	9.99 991	30
40	8.31 901	8.31 911	9.99 991	20
50	8.32 002	8.32 012	9.99 991	10
12 0	8.32 103	8.32 112	9.99 990	0 48
10	8.32 203	8.32 213	9.99 990	50
20	8.32 303	8.32 313	9.99 990	40
30	8.32 403	8.32 413	9.99 990	30
40	8.32 503	8.32 513	9.99 990	20
50	8.32 602	8.32 612	9.99 990	10
13 0	8.32 702	8.32 711	9.99 990	0 47
10	8.32 801	8.32 811	9.99 990	50
20	8.32 899	8.32 909	9.99 990	40
30	8.32 998	8.33 008	9.99 990	30
40	8.33 096	8.33 106	9.99 990	20
50	8.33 195	8.33 205	9.99 990	10
14 0	8.33 292	8.33 302	9.99 990	0 46
10	8.33 390	8.33 400	9.99 990	50
20	8.33 488	8.33 498	9.99 990	40
30	8.33 585	8.33 595	9.99 990	30
40	8.33 682	8.33 692	9.99 990	20
50	8.33 779	8.33 789	9.99 990	10
15 0	8.33 875	8.33 886	9.99 990	0 45
10	8.33 972	8.33 982	9.99 990	50
20	8.34 068	8.34 078	9.99 990	40
30	8.34 164	8.34 174	9.99 990	30
40	8.34 260	8.34 270	9.99 989	20
50	8.34 355	8.34 366	9.99 989	10
16 0	8.34 450	8.34 461	9.99 989	0 44
10	8.34 546	8.34 556	9.99 989	50
20	8.34 640	8.34 651	9.99 989	40
30	8.34 735	8.34 746	9.99 989	30
40	8.34 830	8.34 840	9.99 989	20
50	8.34 924	8.34 935	9.99 989	10
17 0	8.35 018	8.35 029	9.99 989	0 43
10	8.35 112	8.35 123	9.99 989	50
20	8.35 206	8.35 217	[illegible]	40
30	8.35 299	8.35 310	[illegible]	30
40	8.35 392	8.35 403	[illegible]	20
50	8.35 485	8.35 497	[illegible]	10
18 0	8.35 578	8.35 590	[illegible]	0 42
10	8.35 671	8.35 682	[illegible]	50
20	8.35 764	8.35 775	[illegible]	40
30	8.35 856	8.35 867	[illegible]	30
40	8.35 948	[illegible]	[illegible]	20
50	8.36 040	[illegible]	[illegible]	10
19 0	8.36 131	[illegible]	[illegible]	0 41
10	8.36 223	[illegible]	[illegible]	50
20	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]
40	8.36 496	[illegible]	[illegible]	[illegible]
50	8.36 587	[illegible]	[illegible]	[illegible]
20 0	8.36 678	[illegible]	[illegible]	[illegible]
′ ″	log cos	log cot	log sin	[illegible]

′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
20 0	8.36 678	8.36 689	9.99 988	0 **40**	**30** 0	8.41 792	8.41 807	9.99 985	0 **30**
10	8.36 768	8.36 780	9.99 988	50	10	8.41 872	8.41 887	9.99 985	50
20	8.36 858	8.36 870	9.99 988	40	20	8.41 952	8.41 967	9.99 985	40
30	8.36 948	8.36 960	9.99 988	30	30	8.42 032	8.42 048	9.99 985	30
40	8.37 038	8.37 050	9.99 988	20	40	8.42 112	8.42 127	9.99 985	20
50	8.37 128	8.37 140	9.99 988	10	50	8.42 192	8.42 207	9.99 985	10
21 0	8.37 217	8.37 229	9.99 988	0 **39**	**31** 0	8.42 272	8.42 287	9.99 985	0 **29**
10	8.37 306	8.37 318	9.99 988	50	10	8.42 351	8.42 366	9.99 985	50
20	8.37 395	8.37 408	9.99 988	40	20	8.42 430	8.42 446	9.99 985	40
30	8.37 484	8.37 497	9.99 988	30	30	8.42 510	8.42 525	9.99 985	30
40	8.37 573	8.37 585	9.99 988	20	40	8.42 589	8.42 604	9.99 985	20
50	8.37 662	8.37 674	9.99 988	10	50	8.42 667	8.42 683	9.99 985	10
22 0	8.37 750	8.37 762	9.99 988	0 **38**	**32** 0	8.42 746	8.42 762	9.99 984	0 **28**
10	8.37 838	8.37 850	9.99 988	50	10	8.42 825	8.42 840	9.99 984	50
20	8.37 926	8.37 938	9.99 988	40	20	8.42 903	8.42 919	9.99 984	40
30	8.38 014	8.38 026	9.99 987	30	30	8.42 982	8.42 997	9.99 984	30
40	8.38 101	8.38 114	9.99 987	20	40	8.43 060	8.43 075	9.99 984	20
50	8.38 189	8.38 202	9.99 987	10	50	8.43 138	8.43 154	9.99 984	10
23 0	8.38 276	8.38 289	9.99 987	0 **37**	**33** 0	8.43 216	8.43 232	9.99 984	0 **27**
10	8.38 363	8.38 376	9.99 987	50	10	8.43 293	8.43 309	9.99 984	50
20	8.38 450	8.38 463	9.99 987	40	20	8.43 371	8.43 387	9.99 984	40
30	8.38 537	8.38 550	9.99 987	30	30	8.43 448	8.43 464	9.99 984	30
40	8.38 624	8.38 636	9.99 987	20	40	8.43 526	8.43 542	9.99 984	20
50	8.38 710	8.38 723	9.99 987	10	50	8.43 603	8.43 619	9.99 984	10
24 0	8.38 796	8.38 809	9.99 987	0 **36**	**34** 0	8.43 680	8.43 696	9.99 984	0 **26**
10	8.38 882	8.38 895	9.99 987	50	10	8.43 757	8.43 773	9.99 984	50
20	8.38 968	8.38 981	9.99 987	40	20	8.43 834	8.43 850	9.99 984	40
30	8.39 054	8.39 067	9.99 987	30	30	8.43 910	8.43 927	9.99 984	30
40	8.39 139	8.39 153	9.99 987	20	40	8.43 987	8.44 003	9.99 984	20
50	8.39 225	8.39 238	9.99 987	10	50	8.44 063	8.44 080	9.99 983	10
25 0	8.39 310	8.39 323	9.99 987	0 **35**	**35** 0	8.44 139	8.44 156	9.99 983	0 **25**
10	8.39 395	8.39 408	9.99 987	50	10	8.44 216	8.44 232	9.99 983	50
20	8.39 480	8.39 493	9.99 987	40	20	8.44 292	8.44 308	9.99 983	40
30	8.39 565	8.39 578	9.99 987	30	30	8.44 367	8.44 384	9.99 983	30
40	8.39 649	8.39 663	9.99 987	20	40	8.44 443	8.44 460	9.99 983	20
50	8.39 734	8.39 747	9.99 986	10	50	8.44 519	8.44 536	9.99 983	10
26 0	8.39 818	8.39 832	9.99 986	0 **34**	**36** 0	8.44 594	8.44 611	9.99 983	0 **24**
10	8.39 902	8.39 916	9.99 986	50	10	8.44 669	8.44 686	9.99 983	50
20	8.39 986	8.40 000	9.99 986	40	20	8.44 745	8.44 762	9.99 983	40
30	8.40 070	8.40 083	9.99 986	30	30	8.44 820	8.44 837	9.99 983	30
40	8.40 153	8.40 167	9.99 986	20	40	8.44 895	8.44 912	9.99 983	20
50	8.40 237	8.40 251	9.99 986	10	50	8.44 969	8.44 987	9.99 983	10
27 0	8.40 320	8.40 334	9.99 986	0 **33**	**37** 0	8.45 044	8.45 061	9.99 983	0 **23**
10	8.40 403	8.40 417	9.99 986	50	10	8.45 119	8.45 136	9.99 983	50
20	8.40 486	8.40 500	9.99 986	40	20	8.45 193	8.45 210	9.99 983	40
30	8.40 569	8.40 583	9.99 986	30	30	8.45 267	8.45 285	9.99 983	30
40	8.40 651	8.40 665	9.99 986	20	40	8.45 341	8.45 359	9.99 982	20
50	8.40 734	8.40 748	9.99 986	10	50	8.45 415	8.45 433	9.99 982	10
28 0	8.40 816	8.40 830	9.99 986	0 **32**	**38** 0	8.45 489	8.45 507	9.99 982	0 **22**
10	8.40 898	8.40 913	9.99 986	50	10	8.45 563	8.45 581	9.99 982	50
20	8.40 980	8.40 995	9.99 986	40	20	8.45 637	8.45 655	9.99 982	40
30	8.41 062	8.41 077	9.99 986	30	30	8.45 710	8.45 728	9.99 982	30
40	8.41 144	8.41 158	9.99 986	20	40	8.45 784	8.45 802	9.99 982	20
50	8.41 225	8.41 240	9.99 986	10	50	8.45 857	8.45 875	9.99 982	10
29 0	8.41 307	8.41 321	9.99 985	0 **31**	**39** 0	8.45 930	8.45 948	9.99 982	0 **21**
10	8.41 388	8.41 403	9.99 985	50	10	8.46 003	8.46 021	9.99 982	50
20	8.41 469	8.41 484	9.99 985	40	20	8.46 076	8.46 094	9.99 982	40
30	8.41 550	8.41 565	9.99 985	30	30	8.46 149	8.46 167	9.99 982	30
40	8.41 631	8.41 646	9.99 985	20	40	8.46 222	8.46 240	9.99 982	20
50	8.41 711	8.41 726	9.99 985	10	50	8.46 294	8.46 312	9.99 982	10
30 0	8.41 792	8.41 807	9.99 985	0 **30**	**40** 0	8.46 366	8.46 385	9.99 982	0 **20**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
40 0	8.46 366	8.46 385	9.99 982	0 **20**	**50** 0	8.50 504	8.50 527	9.99 978	0 **10**
10	8.46 439	8.46 457	9.99 982	50	10	8.50 570	8.50 593	9.99 978	50
20	8.46 511	8.46 529	9.99 982	40	20	8.50 636	8.50 658	9.99 978	40
30	8.46 583	8.46 602	9.99 981	30	30	8.50 701	8.50 724	9.99 978	30
40	8.46 655	8.46 674	9.99 981	20	40	8.50 767	8.50 789	9.99 977	20
50	8.46 727	8.46 745	9.99 981	10	50	8.50 832	8.50 855	9.99 977	10
41 0	8.46 799	8.46 817	9.99 981	0 **19**	**51** 0	8.50 897	8.50 920	9.99 977	0 **9**
10	8.46 870	8.46 889	9.99 981	50	10	8.50 963	8.50 985	9.99 977	50
20	8.46 942	8.46 960	9.99 981	40	20	8.51 028	8.51 050	9.99 977	40
30	8.47 013	8.47 032	9.99 981	30	30	8.51 092	8.51 115	9.99 977	30
40	8.47 084	8.47 103	9.99 981	20	40	8.51 157	8.51 180	9.99 977	20
50	8.47 155	8.47 174	9.99 981	10	50	8.51 222	8.51 245	9.99 977	10
42 0	8.47 226	8.47 245	9.99 981	0 **18**	**52** 0	8.51 287	8.51 310	9.99 977	0 **8**
10	8.47 297	8.47 316	9.99 981	50	10	8.51 351	8.51 374	9.99 977	50
20	8.47 368	8.47 387	9.99 981	40	20	8.51 416	8.51 439	9.99 977	40
30	8.47 439	8.47 458	9.99 981	30	30	8.51 480	8.51 503	9.99 977	30
40	8.47 509	8.47 528	9.99 981	20	40	8.51 544	8.51 568	9.99 977	20
50	8.47 580	8.47 599	9.99 981	10	50	8.51 609	8.51 632	9.99 977	10
43 0	8.47 650	8.47 669	9.99 981	0 **17**	**53** 0	8.51 673	8.51 696	9.99 977	0 **7**
10	8.47 720	8.47 740	9.99 980	50	10	8.51 737	8.51 760	9.99 976	50
20	8.47 790	8.47 810	9.99 980	40	20	8.51 801	8.51 824	9.99 976	40
30	8.47 860	8.47 880	9.99 980	30	30	8.51 864	8.51 888	9.99 976	30
40	8.47 930	8.47 950	9.99 980	20	40	8.51 928	8.51 952	9.99 976	20
50	8.48 000	8.48 020	9.99 980	10	50	8.51 992	8.52 015	9.99 976	10
44 0	8.48 069	8.48 090	9.99 980	0 **16**	**54** 0	8.52 055	8.52 079	9.99 976	0 **6**
10	8.48 139	8.48 159	9.99 980	50	10	8.52 119	8.52 143	9.99 976	50
20	8.48 208	8.48 228	9.99 980	40	20	8.52 182	8.52 206	9.99 976	40
30	8.48 278	8.48 298	9.99 980	30	30	8.52 245	8.52 269	9.99 976	30
40	8.48 347	8.48 367	9.99 980	20	40	8.52 308	8.52 332	9.99 976	20
50	8.48 416	8.48 436	9.99 980	10	50	8.52 371	8.52 396	9.99 976	10
45 0	8.48 485	8.48 505	9.99 980	0 **15**	**55** 0	8.52 434	8.52 459	9.99 976	0 **5**
10	8.48 554	8.48 574	9.99 980	50	10	8.52 497	8.52 522	9.99 976	50
20	8.48 622	8.48 643	9.99 980	40	20	8.52 560	8.52 584	9.99 976	40
30	8.48 691	8.48 711	9.99 980	30	30	8.52 623	8.52 647	9.99 975	30
40	8.48 760	8.48 780	9.99 979	20	40	8.52 685	8.52 710	9.99 975	20
50	8.48 828	8.48 849	9.99 979	10	50	8.52 748	8.52 772	9.99 975	10
46 0	8.48 896	8.48 917	9.99 979	0 **14**	**56** 0	8.52 810	8.52 835	9.99 975	0 **4**
10	8.48 965	8.48 985	9.99 979	50	10	8.52 872	8.52 897	9.99 975	50
20	8.49 033	8.49 053	9.99 979	40	20	8.52 935	8.52 960	9.99 975	40
30	8.49 101	8.49 121	9.99 979	30	30	8.52 997	8.53 022	9.99 975	30
40	8.49 169	8.49 189	9.99 979	20	40	8.53 059	8.53 084	9.99 975	20
50	8.49 236	8.49 257	9.99 979	10	50	8.53 121	8.53 146	9.99 975	10
47 0	8.49 304	8.49 325	9.99 979	0 **13**	**57** 0	8.53 183	8.53 208	9.99 975	0 **3**
10	8.49 372	8.49 393	9.99 979	50	10	8.53 245	8.53 270	9.99 975	50
20	8.49 439	8.49 460	9.99 979	40	20	8.53 306	8.53 332	9.99 975	40
30	8.49 506	8.49 528	9.99 979	30	30	8.53 368	8.53 393	9.99 975	30
40	8.49 574	8.49 595	9.99 979	20	40	8.53 429	8.53 455	9.99 975	20
50	8.49 641	8.49 662	9.99 979	10	50	8.53 491	8.53 516	9.99 974	10
48 0	8.49 708	8.49 729	9.99 979	0 **12**	**58** 0	8.53 552	8.53 578	9.99 974	0 **2**
10	8.49 775	8.49 796	9.99 979	50	10	8.53 614	8.53 639	9.99 974	50
20	8.49 842	8.49 863	9.99 978	40	20	8.53 675	8.53 700	9.99 974	40
30	8.49 908	8.49 930	9.99 978	30	30	8.53 736	8.53 762	9.99 974	30
40	8.49 975	8.49 997	9.99 978	20	40	8.53 797	8.53 823	9.99 974	20
50	8.50 042	8.50 063	9.99 978	10	50	8.53 858	8.53 884	9.99 974	10
49 0	8.50 108	8.50 130	9.99 978	0 **11**	**59** 0	8.53 919	8.53 945	9.99 974	0 **1**
10	8.50 174	8.50 196	9.99 978	50	10	8.53 979	8.54 005	9.99 974	50
20	8.50 241	8.50 263	9.99 978	40	20	8.54 040	8.54 066	9.99 974	40
30	8.50 307	8.50 329	9.99 978	30	30	8.54 101	8.54 127	9.99 974	30
40	8.50 373	8.50 395	9.99 978	20	40	8.54 161	8.54 187	9.99 974	[illegible]
50	8.50 439	8.50 461	9.99 978	10	50	8.54 222	8.54 248	9.99 974	[illegible]
50 0	8.50 504	8.50 527	9.99 978	0 **10**	**60** 0	8.54 282	8.54 308	9.99 974	[illegible]
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	[illegible]

1°

′	log sin -8-	log tan -8-	log cot -11-	log cos -9-	′
0	24 186	24 192	75 808	99 993	60
1	24 903	24 910	75 090	99 993	59
2	25 609	25 616	74 384	99 993	58
3	26 304	26 312	73 688	99 993	57
4	26 988	26 996	73 004	99 992	56
5	27 661	27 669	72 331	99 992	55
6	28 324	28 332	71 668	99 992	54
7	28 977	28 986	71 014	99 992	53
8	29 621	29 629	70 371	99 992	52
9	30 255	30 263	69 737	99 991	51
10	30 879	30 888	69 112	99 991	50
11	31 495	31 505	68 495	99 991	49
12	32 103	32 112	67 888	99 990	48
13	32 702	32 711	67 289	99 990	47
14	33 292	33 302	66 698	99 990	46
15	33 875	33 886	66 114	99 990	45
16	34 450	34 461	65 539	99 989	44
17	35 018	35 029	64 971	99 989	43
18	35 578	35 590	64 410	99 989	42
19	36 131	36 143	63 857	99 989	41
20	36 678	36 689	63 311	99 988	40
21	37 217	37 229	62 771	99 988	39
22	37 750	37 762	62 238	99 988	38
23	38 276	38 289	61 711	99 987	37
24	38 796	38 809	61 191	99 987	36
25	39 310	39 323	60 677	99 987	35
26	39 818	39 832	60 168	99 986	34
27	40 320	40 334	59 666	99 986	33
28	40 816	40 830	59 170	99 986	32
29	41 307	41 321	58 679	99 985	31
30	41 792	41 807	58 193	99 985	30
31	42 272	42 287	57 713	99 985	29
32	42 746	42 762	57 238	99 984	28
33	43 216	43 232	56 768	99 984	27
34	43 680	43 696	56 304	99 984	26
35	44 139	44 156	55 844	99 983	25
36	44 594	44 611	55 389	99 983	24
37	45 044	45 061	54 939	99 983	23
38	45 489	45 507	54 493	99 982	22
39	45 930	45 948	54 052	99 982	21
40	46 366	46 385	53 615	99 982	20
41	46 799	46 817	53 183	99 981	19
42	47 226	47 245	52 755	99 981	18
43	47 650	47 669	52 331	99 981	17
44	48 069	48 089	51 911	99 980	16
45	48 485	48 505	51 495	99 980	15
46	48 896	48 917	51 083	99 979	14
47	49 304	49 325	50 675	99 979	13
48	49 708	49 729	50 271	99 979	12
49	50 108	50 130	49 870	99 978	11
50	50 504	50 527	49 473	99 978	10
51	50 897	50 920	49 080	99 977	9
52	51 287	51 310	48 690	99 977	8
53	51 673	51 696	48 304	99 977	7
54	52 055	52 079	47 921	99 976	6
55	52 434	52 459	47 541	99 976	5
56	52 810	52 835	47 165	99 975	4
57	53 183	53 208	46 792	99 975	3
58	53 552	53 578	46 422	99 974	2
59	53 919	53 945	46 055	99 974	1
60	54 282	54 308	45 692	99 974	0
′	-8- log cos	-8- log cot	-11- log tan	-9- log sin	′

88°

2°

′	log sin -8-	log tan -8-	log cot -11-	log cos -9-	′
0	54 282	54 308	45 692	99 974	60
1	54 642	54 669	45 331	99 973	59
2	54 999	55 027	44 973	99 973	58
3	55 354	55 382	44 618	99 972	57
4	55 705	55 734	44 266	99 972	56
5	56 054	56 083	43 917	99 971	55
6	56 400	56 429	43 571	99 971	54
7	56 743	56 773	43 227	99 970	53
8	57 084	57 114	42 886	99 970	52
9	57 421	57 452	42 548	99 969	51
10	57 757	57 788	42 212	99 969	50
11	58 089	58 121	41 879	99 968	49
12	58 419	58 451	41 549	99 968	48
13	58 747	58 779	41 221	99 967	47
14	59 072	59 105	40 895	99 967	46
15	59 395	59 428	40 572	99 967	45
16	59 715	59 749	40 251	99 966	44
17	60 033	60 068	39 932	99 966	43
18	60 349	60 384	39 616	99 965	42
19	60 662	60 698	39 302	99 964	41
20	60 973	61 009	38 991	99 964	40
21	61 282	61 319	38 681	99 963	39
22	61 589	61 626	38 374	99 963	38
23	61 894	61 931	38 069	99 962	37
24	62 196	62 234	37 766	99 962	36
25	62 497	62 535	37 465	99 961	35
26	62 795	62 834	37 166	99 961	34
27	63 091	63 131	36 869	99 960	33
28	63 385	63 426	36 574	99 960	32
29	63 678	63 718	36 282	99 959	31
30	63 968	64 009	35 991	99 959	30
31	64 256	64 298	35 702	99 958	29
32	64 543	64 585	35 415	99 958	28
33	64 827	64 870	35 130	99 957	27
34	65 110	65 154	34 846	99 956	26
35	65 391	65 435	34 565	99 956	25
36	65 670	65 715	34 285	99 955	24
37	65 947	65 993	34 007	99 955	23
38	66 223	66 269	33 731	99 954	22
39	66 497	66 543	33 457	99 954	21
40	66 769	66 816	33 184	99 953	20
41	67 039	67 087	32 913	99 952	19
42	67 308	67 356	32 644	99 952	18
43	67 575	67 624	32 376	99 951	17
44	67 841	67 890	32 110	99 951	16
45	68 104	68 154	31 846	99 950	15
46	68 367	68 417	31 583	99 949	14
47	68 627	68 678	31 322	99 949	13
48	68 886	68 938	31 062	99 948	12
49	69 144	69 196	30 804	99 948	11
50	69 400	69 453	30 547	99 947	10
51	69 654	69 708	30 292	99 946	9
52	69 907	69 962	30 038	99 946	8
53	70 159	70 214	29 786	99 945	7
54	70 409	70 465	29 535	99 944	6
55	70 658	70 714	29 286	99 944	5
56	70 905	70 962	29 038	99 943	4
57	71 151	71 208	28 792	99 942	3
58	71 395	71 453	28 547	99 942	2
59	71 638	71 697	28 303	99 941	1
60	71 880	71 940	28 060	99 940	0
′	-8- log cos	-8- log cot	-11- log tan	-9- log sin	′

87°

3°

′	log sin -8-	log tan -8-	log cot -11-	log cos -9-	′
0	71 880	71 940	28 060	99 940	60
1	72 120	72 181	27 819	99 940	59
2	72 359	72 420	27 580	99 939	58
3	72 597	72 659	27 341	99 938	57
4	72 834	72 896	27 104	99 938	56
5	73 069	73 132	26 868	99 937	55
6	73 303	73 366	26 634	99 936	54
7	73 535	73 600	26 400	99 936	53
8	73 767	73 832	26 168	99 935	52
9	73 997	74 063	25 937	99 934	51
10	74 226	74 292	25 708	99 934	50
11	74 454	74 521	25 479	99 933	49
12	74 680	74 748	25 252	99 932	48
13	74 906	74 974	25 026	99 932	47
14	75 130	75 199	24 801	99 931	46
15	75 353	75 423	24 577	99 930	45
16	75 575	75 645	24 355	99 929	44
17	75 795	75 867	24 133	99 929	43
18	76 015	76 087	23 913	99 928	42
19	76 234	76 306	23 694	99 927	41
20	76 451	76 525	23 475	99 926	40
21	76 667	76 742	23 258	99 926	39
22	76 883	76 958	23 042	99 925	38
23	77 097	77 173	22 827	99 924	37
24	77 310	77 387	22 613	99 923	36
25	77 522	77 600	22 400	99 923	35
26	77 733	77 811	22 189	99 922	34
27	77 943	78 022	21 978	99 921	33
28	78 152	78 232	21 768	99 920	32
29	78 360	78 441	21 559	99 920	31
30	78 568	78 649	21 351	99 919	30
31	78 774	78 855	21 145	99 918	29
32	78 979	79 061	20 939	99 917	28
33	79 183	79 266	20 734	99 917	27
34	79 386	79 470	20 530	99 916	26
35	79 588	79 673	20 327	99 915	25
36	79 789	79 875	20 125	99 914	24
37	79 990	80 076	19 924	99 913	23
38	80 189	80 277	19 723	99 913	22
39	80 388	80 476	19 524	99 912	21
40	80 585	80 674	19 326	99 911	20
41	80 782	80 872	19 128	99 910	19
42	80 978	81 068	18 932	99 909	18
43	81 173	81 264	18 736	99 909	17
44	81 367	81 459	18 541	99 908	16
45	81 560	81 653	18 347	99 907	15
46	81 752	81 846	18 154	99 906	14
47	81 944	82 038	17 962	99 905	13
48	82 134	82 230	17 770	99 904	12
49	82 324	82 420	17 580	99 904	11
50	82 513	82 610	17 390	99 903	10
51	82 701	82 799	17 201	99 902	9
52	82 888	82 987	17 013	99 901	8
53	83 075	83 175	16 825	99 900	7
54	83 261	83 361	16 639	99 899	6
55	83 446	83 547	16 453	99 898	5
56	83 630	83 732	16 268	99 898	4
57	83 813	83 916	16 084	99 897	3
58	83 996	84 100	15 900	99 896	2
59	84 177	84 282	15 718	99 895	1
60	84 358	84 464	15 536	99 894	0
′	log cos -8-	log cot -8-	log tan -11-	log sin -9-	′

86°

4°

′	log sin -8-	log tan -8-	log cot -11-	log cos -9-	′
0	84 358	84 464	15 536	99 894	60
1	84 539	84 646	15 354	99 893	59
2	84 718	84 826	15 174	99 892	58
3	84 897	85 006	14 994	99 891	57
4	85 075	85 185	14 815	99 891	56
5	85 252	85 363	14 637	99 890	55
6	85 429	85 540	14 460	99 889	54
7	85 605	85 717	14 283	99 888	53
8	85 780	85 893	14 107	99 887	52
9	85 955	86 069	13 931	99 886	51
10	86 128	86 243	13 757	99 885	50
11	86 301	86 417	13 583	99 884	49
12	86 474	86 591	13 409	99 883	48
13	86 645	86 763	13 237	99 882	47
14	86 816	86 935	13 065	99 881	46
15	86 987	87 106	12 894	99 880	45
16	87 156	87 277	12 723	99 879	44
17	87 325	87 447	12 553	99 879	43
18	87 494	87 616	12 384	99 878	42
19	87 661	87 785	12 215	99 877	41
20	87 829	87 953	12 047	99 876	40
21	87 995	88 120	11 880	99 875	39
22	88 161	88 287	11 713	99 874	38
23	88 326	88 453	11 547	99 873	37
24	88 490	88 618	11 382	99 872	36
25	88 654	88 783	11 217	99 871	35
26	88 817	88 948	11 052	99 870	34
27	88 980	89 111	10 889	99 869	33
28	89 142	89 274	10 726	99 868	32
29	89 304	89 437	10 563	99 867	31
30	89 464	89 598	10 402	99 866	30
31	89 625	89 760	10 240	99 865	29
32	89 784	89 920	10 080	99 864	28
33	89 943	90 080	09 920	99 863	27
34	90 102	90 240	09 760	99 862	26
35	90 260	90 399	09 601	99 861	25
36	90 417	90 557	09 443	99 860	24
37	90 574	90 715	09 285	99 859	23
38	90 730	90 872	09 128	99 858	22
39	90 885	91 029	08 971	99 857	21
40	91 040	91 185	08 815	99 856	20
41	91 195	91 340	08 660	99 855	19
42	91 349	91 495	08 505	99 854	18
43	91 502	91 650	08 350	99 853	17
44	91 655	91 803	08 197	99 852	16
45	91 807	91 957	08 043	99 851	15
46	91 959	92 110	07 890	99 850	14
47	92 110	92 262	07 738	99 848	13
48	92 261	92 414	07 586	99 847	12
49	92 411	92 565	07 435	99 846	11
50	92 561	92 716	07 284	99 845	10
51	92 710	92 866	07 134	99 844	9
52	92 859	93 016	06 984	99 843	8
53	93 007	93 165	06 835	99 842	7
54	93 154	93 313	06 687	99 841	6
55	93 301	93 462	06 538	99 840	5
56	93 448	93 609	06 391	99 839	4
57	93 594	93 756	06 244	99 838	3
58	93 740	93 903	06 097	99 837	2
59	93 885	94 049	05 951	99 836	1
60	94 030	94 195	05 805	99 834	0
′	log cos -8-	log cot -8-	log tan -11-	log sin -9-	′

85°

5°

′	log sin	log tan	log cot	log cos	′
	—8—	—8—	—11—	—9—	
0	94 030	94 195	05 805	99 834	60
1	94 174	94 340	05 660	99 833	59
2	94 317	94 485	05 515	99 832	58
3	94 461	94 630	05 370	99 831	57
4	94 603	94 773	05 227	99 830	56
5	94 746	94 917	05 083	99 829	55
6	94 887	95 060	04 940	99 828	54
7	95 029	95 202	04 798	99 827	53
8	95 170	95 344	04 656	99 825	52
9	95 310	95 486	04 514	99 824	51
10	95 450	95 627	04 373	99 823	50
11	95 589	95 767	04 233	99 822	49
12	95 728	95 908	04 092	99 821	48
13	95 867	96 047	03 953	99 820	47
14	96 005	96 187	03 813	99 819	46
15	96 143	96 325	03 675	99 817	45
16	96 280	96 464	03 536	99 816	44
17	96 417	96 602	03 398	99 815	43
18	96 553	96 739	03 261	99 814	42
19	96 689	96 877	03 123	99 813	41
20	96 825	97 013	02 987	99 812	40
21	96 960	97 150	02 850	99 810	39
22	97 095	97 285	02 715	99 809	38
23	97 229	97 421	02 579	99 808	37
24	97 363	97 556	02 444	99 807	36
25	97 496	97 691	02 309	99 806	35
26	97 629	97 825	02 175	99 804	34
27	97 762	97 959	02 041	99 803	33
28	97 894	98 092	01 908	99 802	32
29	98 026	98 225	01 775	99 801	31
30	98 157	98 358	01 642	99 800	30
31	98 288	98 490	01 510	99 798	29
32	98 419	98 622	01 378	99 797	28
33	98 549	98 753	01 247	99 796	27
34	98 679	98 884	01 116	99 795	26
35	98 808	99 015	00 985	99 793	25
36	98 937	99 145	00 855	99 792	24
37	99 066	99 275	00 725	99 791	23
38	99 194	99 405	00 595	99 790	22
39	99 322	99 534	00 466	99 788	21
40	99 450	99 662	00 338	99 787	20
41	99 577	99 791	00 209	99 786	19
42	99 704	99 919	00 081	99 785	18
43	99 830	00 046	99 954	99 783	17
44	99 956	00 174	99 826	99 782	16
45	00 082	00 301	99 699	99 781	15
46	00 207	00 427	99 573	99 780	14
47	00 332	00 553	99 447	99 778	13
48	00 456	00 679	99 321	99 777	12
49	00 581	00 805	99 195	99 776	11
50	00 704	00 930	99 070	99 775	10
51	00 828	01 055	98 945	99 773	9
52	00 951	01 179	98 821	99 772	8
53	01 074	01 303	98 697	99 771	7
54	01 196	01 427	98 573	99 769	6
55	01 318	01 550	98 450	99 768	5
56	01 440	01 673	98 327	99 767	4
57	01 561	01 796	98 204	99 765	3
58	01 682	01 918	98 082	99 764	2
59	01 803	02 040	97 960	99 763	1
60	01 923	02 162	97 838	99 761	0
	—9—	—9—	—10—	—9—	
′	log cos	log cot	log tan	log sin	′

6°

′	log sin	log tan	log cot	log cos	′
	—9—	—9—	—10—	—9—	
0	01 923	02 162	97 838	99 761	60
1	02 043	02 283	97 717	99 760	59
2	02 163	02 404	97 596	99 759	58
3	02 283	02 525	97 475	99 757	57
4	02 402	02 645	97 355	99 756	56
5	02 520	02 766	97 234	99 755	55
6	02 639	02 885	97 115	99 753	54
7	02 757	03 005	96 995	99 752	53
8	02 874	03 124	96 876	99 751	52
9	02 992	03 242	96 758	99 749	51
10	03 109	03 361	96 639	99 748	50
11	03 226	03 479	96 521	99 747	49
12	03 342	03 597	96 403	99 745	48
13	03 458	03 714	96 286	99 744	47
14	03 574	03 832	96 168	99 742	46
15	03 690	03 948	96 052	99 741	45
16	03 805	04 065	95 935	99 740	44
17	03 920	04 181	95 819	99 738	43
18	04 034	04 297	95 703	99 737	42
19	04 149	04 413	95 587	99 736	41
20	04 262	04 528	95 472	99 734	40
21	04 376	04 643	95 357	99 733	39
22	04 490	04 758	95 242	99 731	38
23	04 603	04 873	95 127	99 730	37
24	04 715	04 987	95 013	99 728	36
25	04 828	05 101	94 899	99 727	35
26	04 940	05 214	94 786	99 726	34
27	05 052	05 328	94 672	99 724	33
28	05 164	05 441	94 559	99 723	32
29	05 275	05 553	94 447	99 721	31
30	05 386	05 666	94 334	99 720	30
31	05 497	05 778	94 222	99 718	29
32	05 607	05 890	94 110	99 717	28
33	05 717	06 002	93 998	99 716	27
34	05 827	06 113	93 887	99 714	26
35	05 937	06 224	93 776	99 713	25
36	06 046	06 335	93 665	99 711	24
37	06 155	06 445	93 555	99 710	23
38	06 264	06 556	93 444	99 708	22
39	06 372	06 666	93 334	99 707	21
40	06 481	06 775	93 225	99 705	20
41	06 589	06 885	93 115	99 704	19
42	06 696	06 994	93 006	99 702	18
43	06 804	07 103	92 897	99 701	17
44	06 911	07 211	92 789	99 699	16
45	07 018	07 320	92 680	99 698	15
46	07 124	07 428	92 572	99 696	14
47	07 231	07 536	92 464	99 695	13
48	07 337	07 643	92 357	99 693	12
49	07 442	07 751	92 249	99 692	11
50	07 548	07 858	92 142	99 690	10
51	07 653	07 964	92 036	99 689	9
52	07 758	08 071	91 929	99 687	8
53	07 863	08 177	91 823	99 686	7
54	07 968	08 283	91 717	99 684	6
55	08 072	08 389	91 611	99 683	5
56	08 176	08 495	91 505	99 681	4
57	08 280	08 600	91 400	99 680	3
58	08 383	08 705	91 295	99 678	2
59	08 486	08 810	91 190	99 677	1
60	08 589	08 914	91 086	99 675	0
	—9—	—9—	—10—	—9—	
′	log cos	log cot	log tan	log sin	′

83°

7°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	08 589	08 914	91 086	99 675	60
1	08 692	09 019	90 981	99 674	59
2	08 795	09 123	90 877	99 672	58
3	08 897	09 227	90 773	99 670	57
4	08 999	09 330	90 670	99 669	56
5	09 101	09 434	90 566	99 667	55
6	09 202	09 537	90 463	99 666	54
7	09 304	09 640	90 360	99 664	53
8	09 405	09 742	90 258	99 663	52
9	09 506	09 845	90 155	99 661	51
10	09 606	09 947	90 053	99 659	50
11	09 707	10 049	89 951	99 658	49
12	09 807	10 150	89 850	99 656	48
13	09 907	10 252	89 748	99 655	47
14	10 006	10 353	89 647	99 653	46
15	10 106	10 454	89 546	99 651	45
16	10 205	10 555	89 445	99 650	44
17	10 304	10 656	89 344	99 648	43
18	10 402	10 756	89 244	99 647	42
19	10 501	10 856	89 144	99 645	41
20	10 599	10 956	89 044	99 643	40
21	10 697	11 056	88 944	99 642	39
22	10 795	11 155	88 845	99 640	38
23	10 893	11 254	88 746	99 638	37
24	10 990	11 353	88 647	99 637	36
25	11 087	11 452	88 548	99 635	35
26	11 184	11 551	88 449	99 633	34
27	11 281	11 649	88 351	99 632	33
28	11 377	11 747	88 253	99 630	32
29	11 474	11 845	88 155	99 629	31
30	11 570	11 943	88 057	99 627	30
31	11 666	12 040	87 960	99 625	29
32	11 761	12 138	87 862	99 624	28
33	11 857	12 235	87 765	99 622	27
34	11 952	12 332	87 668	99 620	26
35	12 047	12 428	87 572	99 618	25
36	12 142	12 525	87 475	99 617	24
37	12 236	12 621	87 379	99 615	23
38	12 331	12 717	87 283	99 613	22
39	12 425	12 813	87 187	99 612	21
40	12 519	12 909	87 091	99 610	20
41	12 612	13 004	86 996	99 608	19
42	12 706	13 099	86 901	99 607	18
43	12 799	13 194	86 806	99 605	17
44	12 892	13 289	86 711	99 603	16
45	12 985	13 384	86 616	99 601	15
46	13 078	13 478	86 522	99 600	14
47	13 171	13 573	86 427	99 598	13
48	13 263	13 667	86 333	99 596	12
49	13 355	13 761	86 239	99 595	11
50	13 447	13 854	86 146	99 593	10
51	13 539	13 948	86 052	99 591	9
52	13 630	14 041	85 959	99 589	8
53	13 722	14 134	85 866	99 588	7
54	13 813	14 227	85 773	99 586	6
55	13 904	14 320	85 680	99 584	5
56	13 994	14 412	85 588	99 582	4
57	14 085	14 504	85 496	99 581	3
58	14 175	14 597	85 403	99 579	2
59	14 266	14 688	85 312	99 577	1
60	14 356	14 780	85 220	99 575	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

82°

8°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	14 356	14 780	85 220	99 575	60
1	14 445	14 872	85 128	99 574	59
2	14 535	14 963	85 037	99 572	58
3	14 624	15 054	84 946	99 570	57
4	14 714	15 145	84 855	99 568	56
5	14 803	15 236	84 764	99 566	55
6	14 891	15 327	84 673	99 565	54
7	14 980	15 417	84 583	99 563	53
8	15 069	15 508	84 492	99 561	52
9	15 157	15 598	84 402	99 559	51
10	15 245	15 688	84 312	99 557	50
11	15 333	15 777	84 223	99 556	49
12	15 421	15 867	84 133	99 554	48
13	15 508	15 956	84 044	99 552	47
14	15 596	16 046	83 954	99 550	46
15	15 683	16 135	83 865	99 548	45
16	15 770	16 224	83 776	99 546	44
17	15 857	16 312	83 688	99 545	43
18	15 944	16 401	83 599	99 543	42
19	16 030	16 489	83 511	99 541	41
20	16 116	16 577	83 423	99 539	40
21	16 203	16 665	83 335	99 537	39
22	16 289	16 753	83 247	99 535	38
23	16 374	16 841	83 159	99 533	37
24	16 460	16 928	83 072	99 532	36
25	16 545	17 016	82 984	99 530	35
26	16 631	17 103	82 897	99 528	34
27	16 716	17 190	82 810	99 526	33
28	16 801	17 277	82 723	99 524	32
29	16 886	17 363	82 637	99 522	31
30	16 970	17 450	82 550	99 520	30
31	17 055	17 536	82 464	99 518	29
32	17 139	17 622	82 378	99 517	28
33	17 223	17 708	82 292	99 515	27
34	17 307	17 794	82 206	99 513	26
35	17 391	17 880	82 120	99 511	25
36	17 474	17 965	82 035	99 509	24
37	17 558	18 051	81 949	99 507	23
38	17 641	18 136	81 864	99 505	22
39	17 724	18 221	81 779	99 503	21
40	17 807	18 306	81 694	99 501	20
41	17 890	18 391	81 609	99 499	19
42	17 973	18 475	81 525	99 497	18
43	18 055	18 560	81 440	99 495	17
44	18 137	18 644	81 356	99 494	16
45	18 220	18 728	81 272	99 492	15
46	18 302	18 812	81 188	99 490	14
47	18 383	18 896	81 104	99 488	13
48	18 465	18 979	81 021	99 486	12
49	18 547	19 063	80 937	99 484	11
50	18 628	19 146	80 854	99 482	10
51	18 709	19 229	80 771	99 480	9
52	18 790	19 312	80 688	99 478	[illegible]
53	18 871	19 395	80 605	99 476	[illegible]
54	18 952	19 478	80 522	99 474	[illegible]
55	19 033	19 561	80 439	99 472	[illegible]
56	19 113	19 643	80 357	99 470	[illegible]
57	19 193	19 725	80 275	99 468	[illegible]
58	19 273	19 807	80 193	99 466	[illegible]
59	19 353	19 889	80 111	99 464	1
60	19 433	19 971	[illegible]	[illegible]	[illegible]
	9	9	[illegible]	[illegible]	
′	log cos	log cot	[illegible]	[illegible]	[illegible]

81°

9°

′	log sin −9−	log tan −9−	log cot −10−	log cos −9−	′
0	19 433	19 971	80 029	99 462	**60**
1	19 513	20 053	79 947	99 460	59
2	19 592	20 134	79 866	99 458	58
3	19 672	20 216	79 784	99 456	57
4	19 751	20 297	79 703	99 454	56
5	19 830	20 378	79 622	99 452	55
6	19 909	20 459	79 541	99 450	54
7	19 988	20 540	79 460	99 448	53
8	20 067	20 621	79 379	99 446	52
9	20 145	20 701	79 299	99 444	51
10	20 223	20 782	79 218	99 442	**50**
11	20 302	20 862	79 138	99 440	49
12	20 380	20 942	79 058	99 438	48
13	20 458	21 022	78 978	99 436	47
14	20 535	21 102	78 898	99 434	46
15	20 613	21 182	78 818	99 432	45
16	20 691	21 261	78 739	99 429	44
17	20 768	21 341	78 659	99 427	43
18	20 845	21 420	78 580	99 425	42
19	20 922	21 499	78 501	99 423	41
20	20 999	21 578	78 422	99 421	**40**
21	21 076	21 657	78 343	99 419	39
22	21 153	21 736	78 264	99 417	38
23	21 229	21 814	78 186	99 415	37
24	21 306	21 893	78 107	99 413	36
25	21 382	21 971	78 029	99 411	35
26	21 458	22 049	77 951	99 409	34
27	21 534	22 127	77 873	99 407	33
28	21 610	22 205	77 795	99 404	32
29	21 685	22 283	77 717	99 402	31
30	21 761	22 361	77 639	99 400	**30**
31	21 836	22 438	77 562	99 398	29
32	21 912	22 516	77 484	99 396	28
33	21 987	22 593	77 407	99 394	27
34	22 062	22 670	77 330	99 392	26
35	22 137	22 747	77 253	99 390	25
36	22 211	22 824	77 176	99 388	24
37	22 286	22 901	77 099	99 385	23
38	22 361	22 977	77 023	99 383	22
39	22 435	23 054	76 946	99 381	21
40	22 509	23 130	76 870	99 379	**20**
41	22 583	23 206	76 794	99 377	19
42	22 657	23 283	76 717	99 375	18
43	22 731	23 359	76 641	99 372	17
44	22 805	23 435	76 565	99 370	16
45	22 878	23 510	76 490	99 368	15
46	22 952	23 586	76 414	99 366	14
47	23 025	23 661	76 339	99 364	13
48	23 098	23 737	76 263	99 362	12
49	23 171	23 812	76 188	99 359	11
50	23 244	23 887	76 113	99 357	**10**
51	23 317	23 962	76 038	99 355	9
52	23 390	24 037	75 963	99 353	8
53	23 462	24 112	75 888	99 351	7
54	23 535	24 186	75 814	99 348	6
55	23 607	24 261	75 739	99 346	5
56	23 679	24 335	75 665	99 344	4
57	23 752	24 410	75 590	99 342	3
58	23 823	24 484	75 516	99 340	2
59	23 895	24 558	75 442	99 337	1
60	23 967	24 632	75 368	99 335	**0**
′	−9− log cos	−9− log cot	−10− log tan	−9− log sin	′

80°

10°

′	log sin −9−	log tan −9−	log cot −10−	log cos −9−	′
0	23 967	24 632	75 368	99 335	**60**
1	24 039	24 706	75 294	99 333	59
2	24 110	24 779	75 221	99 331	58
3	24 181	24 853	75 147	99 328	57
4	24 253	24 926	75 074	99 326	56
5	24 324	25 000	75 000	99 324	55
6	24 395	25 073	74 927	99 322	54
7	24 466	25 146	74 854	99 319	53
8	24 536	25 219	74 781	99 317	52
9	24 607	25 292	74 708	99 315	51
10	24 677	25 365	74 635	99 313	**50**
11	24 748	25 437	74 563	99 310	49
12	24 818	25 510	74 490	99 308	48
13	24 888	25 582	74 418	99 306	47
14	24 958	25 655	74 345	99 304	46
15	25 028	25 727	74 273	99 301	45
16	25 098	25 799	74 201	99 299	44
17	25 168	25 871	74 129	99 297	43
18	25 237	25 943	74 057	99 294	42
19	25 307	26 015	73 985	99 292	41
20	25 376	26 086	73 914	99 290	**40**
21	25 445	26 158	73 842	99 288	39
22	25 514	26 229	73 771	99 285	38
23	25 583	26 301	73 699	99 283	37
24	25 652	26 372	73 628	99 281	36
25	25 721	26 443	73 557	99 278	35
26	25 790	26 514	73 486	99 276	34
27	25 858	26 585	73 415	99 274	33
28	25 927	26 655	73 345	99 271	32
29	25 995	26 726	73 274	99 269	31
30	26 063	26 797	73 203	99 267	**30**
31	26 131	26 867	73 133	99 264	29
32	26 199	26 937	73 063	99 262	28
33	26 267	27 008	72 992	99 260	27
34	26 335	27 078	72 922	99 257	26
35	26 403	27 148	72 852	99 255	25
36	26 470	27 218	72 782	99 252	24
37	26 538	27 288	72 712	99 250	23
38	26 605	27 357	72 643	99 248	22
39	26 672	27 427	72 573	99 245	21
40	26 739	27 496	72 504	99 243	**20**
41	26 806	27 566	72 434	99 241	19
42	26 873	27 635	72 365	99 238	18
43	26 940	27 704	72 296	99 236	17
44	27 007	27 773	72 227	99 233	16
45	27 073	27 842	72 158	99 231	15
46	27 140	27 911	72 089	99 229	14
47	27 206	27 980	72 020	99 226	13
48	27 273	28 049	71 951	99 224	12
49	27 339	28 117	71 883	99 221	11
50	27 405	28 186	71 814	99 219	**10**
51	27 471	28 254	71 746	99 217	9
52	27 537	28 323	71 677	99 214	8
53	27 602	28 391	71 609	99 212	7
54	27 668	28 459	71 541	99 209	6
55	27 734	28 527	71 473	99 207	5
56	27 799	28 595	71 405	99 204	4
57	27 864	28 662	71 338	99 202	3
58	27 930	28 730	71 270	99 200	2
59	27 995	28 798	71 202	99 197	1
60	28 060	28 865	71 135	99 195	**0**
′	−9− log cos	−9− log cot	−10− log tan	−9− log sin	′

79°

11°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	28 060	28 865	71 135	99 195	60
1	28 125	28 933	71 067	99 192	59
2	28 190	29 000	71 000	99 190	58
3	28 254	29 067	70 933	99 187	57
4	28 319	29 134	70 866	99 185	56
5	28 384	29 201	70 799	99 182	55
6	28 448	29 268	70 732	99 180	54
7	28 512	29 335	70 665	99 177	53
8	28 577	29 402	70 598	99 175	52
9	28 641	29 468	70 532	99 172	51
10	28 705	29 535	70 465	99 170	50
11	28 769	29 601	70 399	99 167	49
12	28 833	29 668	70 332	99 165	48
13	28 896	29 734	70 266	99 162	47
14	28 960	29 800	70 200	99 160	46
15	29 024	29 866	70 134	99 157	45
16	29 087	29 932	70 068	99 155	44
17	29 150	29 998	70 002	99 152	43
18	29 214	30 064	69 936	99 150	42
19	29 277	30 130	69 870	99 147	41
20	29 340	30 195	69 805	99 145	40
21	29 403	30 261	69 739	99 142	39
22	29 466	30 326	69 674	99 140	38
23	29 529	30 391	69 609	99 137	37
24	29 591	30 457	69 543	99 135	36
25	29 654	30 522	69 478	99 132	35
26	29 716	30 587	69 413	99 130	34
27	29 779	30 652	69 348	99 127	33
28	29 841	30 717	69 283	99 124	32
29	29 903	30 782	69 218	99 122	31
30	29 966	30 846	69 154	99 119	30
31	30 028	30 911	69 089	99 117	29
32	30 090	30 975	69 025	99 114	28
33	30 151	31 040	68 960	99 112	27
34	30 213	31 104	68 896	99 109	26
35	30 275	31 168	68 832	99 106	25
36	30 336	31 233	68 767	99 104	24
37	30 398	31 297	68 703	99 101	23
38	30 459	31 361	68 639	99 099	22
39	30 521	31 425	68 575	99 096	21
40	30 582	31 489	68 511	99 093	20
41	30 643	31 552	68 448	99 091	19
42	30 704	31 616	68 384	99 088	18
43	30 765	31 679	68 321	99 086	17
44	30 826	31 743	68 257	99 083	16
45	30 887	31 806	68 194	99 080	15
46	30 947	31 870	68 130	99 078	14
47	31 008	31 933	68 067	99 075	13
48	31 068	31 996	68 004	99 072	12
49	31 129	32 059	67 941	99 070	11
50	31 189	32 122	67 878	99 067	10
51	31 250	32 185	67 815	99 064	9
52	31 310	32 248	67 752	99 062	8
53	31 370	32 311	67 689	99 059	7
54	31 430	32 373	67 627	99 056	6
55	31 490	32 436	67 564	99 054	5
56	31 549	32 498	67 502	99 051	4
57	31 609	32 561	67 439	99 048	3
58	31 669	32 623	67 377	99 046	2
59	31 728	32 685	67 315	99 043	1
60	31 788	32 747	67 253	99 040	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

78°

12°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	31 788	32 747	67 253	99 040	60
1	31 847	32 810	67 190	99 038	59
2	31 907	32 872	67 128	99 035	58
3	31 966	32 933	67 067	99 032	57
4	32 025	32 995	67 005	99 030	56
5	32 084	33 057	66 943	99 027	55
6	32 143	33 119	66 881	99 024	54
7	32 202	33 180	66 820	99 022	53
8	32 261	33 242	66 758	99 019	52
9	32 319	33 303	66 697	99 016	51
10	32 378	33 365	66 635	99 013	50
11	32 437	33 426	66 574	99 011	49
12	32 495	33 487	66 513	99 008	48
13	32 553	33 548	66 452	99 005	47
14	32 612	33 609	66 391	99 002	46
15	32 670	33 670	66 330	99 000	45
16	32 728	33 731	66 269	98 997	44
17	32 786	33 792	66 208	98 994	43
18	32 844	33 853	66 147	98 991	42
19	32 902	33 913	66 087	98 989	41
20	32 960	33 974	66 026	98 986	40
21	33 018	34 034	65 966	98 983	39
22	33 075	34 095	65 905	98 980	38
23	33 133	34 155	65 845	98 978	37
24	33 190	34 215	65 785	98 975	36
25	33 248	34 276	65 724	98 972	35
26	33 305	34 336	65 664	98 969	34
27	33 362	34 396	65 604	98 967	33
28	33 420	34 456	65 544	98 964	32
29	33 477	34 516	65 484	98 961	31
30	33 534	34 576	65 424	98 958	30
31	33 591	34 635	65 365	98 955	29
32	33 647	34 695	65 305	98 953	28
33	33 704	34 755	65 245	98 950	27
34	33 761	34 814	65 186	98 947	26
35	33 818	34 874	65 126	98 944	25
36	33 874	34 933	65 067	98 941	24
37	33 931	34 992	65 008	98 938	23
38	33 987	35 051	64 949	98 936	22
39	34 043	35 111	64 889	98 933	21
40	34 100	35 170	64 830	98 930	20
41	34 156	35 229	64 771	98 927	19
42	34 212	35 288	64 712	98 924	18
43	34 268	35 347	64 653	98 921	17
44	34 324	35 405	64 595	98 919	16
45	34 380	35 464	64 536	98 916	15
46	34 436	35 523	64 477	98 913	14
47	34 491	35 581	64 419	98 910	13
48	34 547	35 640	64 360	98 907	12
49	34 602	35 698	64 302	98 904	11
50	34 658	35 757	64 243	98 901	10
51	34 713	35 815	64 185	98 898	9
52	34 769	35 873	64 127	98 896	8
53	34 824	35 931	64 069	98 893	7
54	34 879	35 989	64 011	98 890	[illegible]
55	34 934	36 047	63 953	98 887	[illegible]
56	34 989	36 105	63 895	98 884	[illegible]
57	35 044	36 163	63 837	98 881	[illegible]
58	35 099	36 221	63 779	98 878	[illegible]
59	35 154	36 279	63 721	98 875	1
60	35 209	36 336	63 664	98 872	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

77°

13°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	35 209	36 336	63 664	98 872	60
1	35 263	36 394	63 606	98 869	59
2	35 318	36 452	63 548	98 867	58
3	35 373	36 509	63 491	98 864	57
4	35 427	36 566	63 434	98 861	56
5	35 481	36 624	63 376	98 858	55
6	35 536	36 681	63 319	98 855	54
7	35 590	36 738	63 262	98 852	53
8	35 644	36 795	63 205	98 849	52
9	35 698	36 852	63 148	98 846	51
10	35 752	36 909	63 091	98 843	50
11	35 806	36 966	63 034	98 840	49
12	35 860	37 023	62 977	98 837	48
13	35 914	37 080	62 920	98 834	47
14	35 968	37 137	62 863	98 831	46
15	36 022	37 193	62 807	98 828	45
16	36 075	37 250	62 750	98 825	44
17	36 129	37 306	62 694	98 822	43
18	36 182	37 363	62 637	98 819	42
19	36 236	37 419	62 581	98 816	41
20	36 289	37 476	62 524	98 813	40
21	36 342	37 532	62 468	98 810	39
22	36 395	37 588	62 412	98 807	38
23	36 449	37 644	62 356	98 804	37
24	36 502	37 700	62 300	98 801	36
25	36 555	37 756	62 244	98 798	35
26	36 608	37 812	62 188	98 795	34
27	36 660	37 868	62 132	98 792	33
28	36 713	37 924	62 076	98 789	32
29	36 766	37 980	62 020	98 786	31
30	36 819	38 035	61 965	98 783	30
31	36 871	38 091	61 909	98 780	29
32	36 924	38 147	61 853	98 777	28
33	36 976	38 202	61 798	98 774	27
34	37 028	38 257	61 743	98 771	26
35	37 081	38 313	61 687	98 768	25
36	37 133	38 368	61 632	98 765	24
37	37 185	38 423	61 577	98 762	23
38	37 237	38 479	61 521	98 759	22
39	37 289	38 534	61 466	98 756	21
40	37 341	38 589	61 411	98 753	20
41	37 393	38 644	61 356	98 750	19
42	37 445	38 699	61 301	98 746	18
43	37 497	38 754	61 246	98 743	17
44	37 549	38 808	61 192	98 740	16
45	37 600	38 863	61 137	98 737	15
46	37 652	38 918	61 082	98 734	14
47	37 703	38 972	61 028	98 731	13
48	37 755	39 027	60 973	98 728	12
49	37 806	39 082	60 918	98 725	11
50	37 858	39 136	60 864	98 722	10
51	37 909	39 190	60 810	98 719	9
52	37 960	39 245	60 755	98 715	8
53	38 011	39 299	60 701	98 712	7
54	38 062	39 353	60 647	98 709	6
55	38 113	39 407	60 593	98 706	5
56	38 164	39 461	60 539	98 703	4
57	38 215	39 515	60 485	98 700	3
58	38 266	39 569	60 431	98 697	2
59	38 317	39 623	60 377	98 694	1
60	38 368	39 677	60 323	98 690	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

76°

14°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	38 368	39 677	60 323	98 690	60
1	38 418	39 731	60 269	98 687	59
2	38 469	39 785	60 215	98 684	58
3	38 519	39 838	60 162	98 681	57
4	38 570	39 892	60 108	98 678	56
5	38 620	39 945	60 055	98 675	55
6	38 670	39 999	60 001	98 671	54
7	38 721	40 052	59 948	98 668	53
8	38 771	40 106	59 894	98 665	52
9	38 821	40 159	59 841	98 662	51
10	38 871	40 212	59 788	98 659	50
11	38 921	40 266	59 734	98 656	49
12	38 971	40 319	59 681	98 652	48
13	39 021	40 372	59 628	98 649	47
14	39 071	40 425	59 575	98 646	46
15	39 121	40 478	59 522	98 643	45
16	39 170	40 531	59 469	98 640	44
17	39 220	40 584	59 416	98 636	43
18	39 270	40 636	59 364	98 633	42
19	39 319	40 689	59 311	98 630	41
20	39 369	40 742	59 258	98 627	40
21	39 418	40 795	59 205	98 623	39
22	39 467	40 847	59 153	98 620	38
23	39 517	40 900	59 100	98 617	37
24	39 566	40 952	59 048	98 614	36
25	39 615	41 005	58 995	98 610	35
26	39 664	41 057	58 943	98 607	34
27	39 713	41 109	58 891	98 604	33
28	39 762	41 161	58 839	98 601	32
29	39 811	41 214	58 786	98 597	31
30	39 860	41 266	58 734	98 594	30
31	39 909	41 318	58 682	98 591	29
32	39 958	41 370	58 630	98 588	28
33	40 006	41 422	58 578	98 584	27
34	40 055	41 474	58 526	98 581	26
35	40 103	41 526	58 474	98 578	25
36	40 152	41 578	58 422	98 574	24
37	40 200	41 629	58 371	98 571	23
38	40 249	41 681	58 319	98 568	22
39	40 297	41 733	58 267	98 565	21
40	40 346	41 784	58 216	98 561	20
41	40 394	41 836	58 164	98 558	19
42	40 442	41 887	58 113	98 555	18
43	40 490	41 939	58 061	98 551	17
44	40 538	41 990	58 010	98 548	16
45	40 586	42 041	57 959	98 545	15
46	40 634	42 093	57 907	98 541	14
47	40 682	42 144	57 856	98 538	13
48	40 730	42 195	57 805	98 535	12
49	40 778	42 246	57 754	98 531	11
50	40 825	42 297	57 703	98 528	10
51	40 873	42 348	57 652	98 525	9
52	40 921	42 399	57 601	98 521	8
53	40 968	42 450	57 550	98 518	7
54	41 016	42 501	57 499	98 515	6
55	41 063	42 552	57 448	98 511	5
56	41 111	42 603	57 397	98 508	4
57	41 158	42 653	57 347	98 505	3
58	41 205	42 704	57 296	98 501	2
59	41 252	42 755	57 245	98 498	1
60	41 300	42 805	57 195	98 494	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

75°

15°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	41 300	42 805	57 195	98 494	60
1	41 347	42 856	57 144	98 491	59
2	41 394	42 906	57 094	98 488	58
3	41 441	42 957	57 043	98 484	57
4	41 488	43 007	56 993	98 481	56
5	41 535	43 057	56 943	98 477	55
6	41 582	43 108	56 892	98 474	54
7	41 628	43 158	56 842	98 471	53
8	41 675	43 208	56 792	98 467	52
9	41 722	43 258	56 742	98 464	51
10	41 768	43 308	56 692	98 460	50
11	41 815	43 358	56 642	98 457	49
12	41 861	43 408	56 592	98 453	48
13	41 908	43 458	56 542	98 450	47
14	41 954	43 508	56 492	98 447	46
15	42 001	43 558	56 442	98 443	45
16	42 047	43 607	56 393	98 440	44
17	42 093	43 657	56 343	98 436	43
18	42 140	43 707	56 293	98 433	42
19	42 186	43 756	56 244	98 429	41
20	42 232	43 806	56 194	98 426	40
21	42 278	43 855	56 145	98 422	39
22	42 324	43 905	56 095	98 419	38
23	42 370	43 954	56 046	98 415	37
24	42 416	44 004	55 996	98 412	36
25	42 461	44 053	55 947	98 409	35
26	42 507	44 102	55 898	98 405	34
27	42 553	44 151	55 849	98 402	33
28	42 599	44 201	55 799	98 398	32
29	42 644	44 250	55 750	98 395	31
30	42 690	44 299	55 701	98 391	30
31	42 735	44 348	55 652	98 388	29
32	42 781	44 397	55 603	98 384	28
33	42 826	44 446	55 554	98 381	27
34	42 872	44 495	55 505	98 377	26
35	42 917	44 544	55 456	98 373	25
36	42 962	44 592	55 408	98 370	24
37	43 008	44 641	55 359	98 366	23
38	43 053	44 690	55 310	98 363	22
39	43 098	44 738	55 262	98 359	21
40	43 143	44 787	55 213	98 356	20
41	43 188	44 836	55 164	98 352	19
42	43 233	44 884	55 116	98 349	18
43	43 278	44 933	55 067	98 345	17
44	43 323	44 981	55 019	98 342	16
45	43 367	45 029	54 971	98 338	15
46	43 412	45 078	54 922	98 334	14
47	43 457	45 126	54 874	98 331	13
48	43 502	45 174	54 826	98 327	12
49	43 546	45 222	54 778	98 324	11
50	43 591	45 271	54 729	98 320	10
51	43 635	45 319	54 681	98 317	9
52	43 680	45 367	54 633	98 313	8
53	43 724	45 415	54 585	98 309	7
54	43 769	45 463	54 537	98 306	6
55	43 813	45 511	54 489	98 302	5
56	43 857	45 559	54 441	98 299	4
57	43 901	45 606	54 394	98 295	3
58	43 946	45 654	54 346	98 291	2
59	43 990	45 702	54 298	98 288	1
60	44 034	45 750	54 250	98 284	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

74°

16°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	44 034	45 750	54 250	98 284	60
1	44 078	45 797	54 203	98 281	59
2	44 122	45 845	54 155	98 277	58
3	44 166	45 892	54 108	98 273	57
4	44 210	45 940	54 060	98 270	56
5	44 253	45 987	54 013	98 266	55
6	44 297	46 035	53 965	98 262	54
7	44 341	46 082	53 918	98 259	53
8	44 385	46 130	53 870	98 255	52
9	44 428	46 177	53 823	98 251	51
10	44 472	46 224	53 776	98 248	50
11	44 516	46 271	53 729	98 244	49
12	44 559	46 319	53 681	98 240	48
13	44 602	46 366	53 634	98 237	47
14	44 646	46 413	53 587	98 233	46
15	44 689	46 460	53 540	98 229	45
16	44 733	46 507	53 493	98 226	44
17	44 776	46 554	53 446	98 222	43
18	44 819	46 601	53 399	98 218	42
19	44 862	46 648	53 352	98 215	41
20	44 905	46 694	53 306	98 211	40
21	44 948	46 741	53 259	98 207	39
22	44 992	46 788	53 212	98 204	38
23	45 035	46 835	53 165	98 200	37
24	45 077	46 881	53 119	98 196	36
25	45 120	46 928	53 072	98 192	35
26	45 163	46 975	53 025	98 189	34
27	45 206	47 021	52 979	98 185	33
28	45 249	47 068	52 932	98 181	32
29	45 292	47 114	52 886	98 177	31
30	45 334	47 160	52 840	98 174	30
31	45 377	47 207	52 793	98 170	29
32	45 419	47 253	52 747	98 166	28
33	45 462	47 299	52 701	98 162	27
34	45 504	47 346	52 654	98 159	26
35	45 547	47 392	52 608	98 155	25
36	45 589	47 438	52 562	98 151	24
37	45 632	47 484	52 516	98 147	23
38	45 674	47 530	52 470	98 144	22
39	45 716	47 576	52 424	98 140	21
40	45 758	47 622	52 378	98 136	20
41	45 801	47 668	52 332	98 132	19
42	45 843	47 714	52 286	98 129	18
43	45 885	47 760	52 240	98 125	17
44	45 927	47 806	52 194	98 121	16
45	45 969	47 852	52 148	98 117	15
46	46 011	47 897	52 103	98 113	14
47	46 053	47 943	52 057	98 110	13
48	46 095	47 989	52 011	98 106	12
49	46 136	48 035	51 965	98 102	11
50	46 178	48 080	51 920	98 098	10
51	46 220	48 126	51 874	98 094	9
52	46 262	48 171	51 829	98 090	8
53	46 303	48 217	51 783	98 087	7
54	46 345	48 262	51 738	98 083	6
55	46 386	48 307	51 693	98 079	5
56	46 428	48 353	51 647	98 075	4
57	46 469	48 398	51 602	98 071	3
58	46 511	48 443	51 557	98 067	2
59	46 552	48 489	51 511	98 063	1
60	46 594	48 534	51 466	98 06[illegible]	[illegible]
	9	9	10	9	
′	log cos	log cot	log tan	log s[illegible]	′

73°

17°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	46 594	48 534	51 466	98 060	60
1	46 635	48 579	51 421	98 056	59
2	46 676	48 624	51 376	98 052	58
3	46 717	48 669	51 331	98 048	57
4	46 758	48 714	51 286	98 044	56
5	46 800	48 759	51 241	98 040	55
6	46 841	48 804	51 196	98 036	54
7	46 882	48 849	51 151	98 032	53
8	46 923	48 894	51 106	98 029	52
9	46 964	48 939	51 061	98 025	51
10	47 005	48 984	51 016	98 021	50
11	47 045	49 029	50 971	98 017	49
12	47 086	49 073	50 927	98 013	48
13	47 127	49 118	50 882	98 009	47
14	47 168	49 163	50 837	98 005	46
15	47 209	49 207	50 793	98 001	45
16	47 249	49 252	50 748	97 997	44
17	47 290	49 296	50 704	97 993	43
18	47 330	49 341	50 659	97 989	42
19	47 371	49 385	50 615	97 986	41
20	47 411	49 430	50 570	97 982	40
21	47 452	49 474	50 526	97 978	39
22	47 492	49 519	50 481	97 974	38
23	47 533	49 563	50 437	97 970	37
24	47 573	49 607	50 393	97 966	36
25	47 613	49 652	50 348	97 962	35
26	47 654	49 696	50 304	97 958	34
27	47 694	49 740	50 260	97 954	33
28	47 734	49 784	50 216	97 950	32
29	47 774	49 828	50 172	97 946	31
30	47 814	49 872	50 128	97 942	30
31	47 854	49 916	50 084	97 938	29
32	47 894	49 960	50 040	97 934	28
33	47 934	50 004	49 996	97 930	27
34	47 974	50 048	49 952	97 926	26
35	48 014	50 092	49 908	97 922	25
36	48 054	50 136	49 864	97 918	24
37	48 094	50 180	49 820	97 914	23
38	48 133	50 223	49 777	97 910	22
39	48 173	50 267	49 733	97 906	21
40	48 213	50 311	49 689	97 902	20
41	48 252	50 355	49 645	97 898	19
42	48 292	50 398	49 602	97 894	18
43	48 332	50 442	49 558	97 890	17
44	48 371	50 485	49 515	97 886	16
45	48 411	50 529	49 471	97 882	15
46	48 450	50 572	49 428	97 878	14
47	48 490	50 616	49 384	97 874	13
48	48 529	50 659	49 341	97 870	12
49	48 568	50 703	49 297	97 866	11
50	48 607	50 746	49 254	97 861	10
51	48 647	50 789	49 211	97 857	9
52	48 686	50 833	49 167	97 853	8
53	48 725	50 876	49 124	97 849	7
54	48 764	50 919	49 081	97 845	6
55	48 803	50 962	49 038	97 841	5
56	48 842	51 005	48 995	97 837	4
57	48 881	51 048	48 952	97 833	3
58	48 920	51 092	48 908	97 829	2
59	48 959	51 135	48 865	97 825	1
60	48 998	51 178	48 822	97 821	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

18°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	48 998	51 178	48 822	97 821	60
1	49 037	51 221	48 779	97 817	59
2	49 076	51 264	48 736	97 812	58
3	49 115	51 306	48 694	97 808	57
4	49 153	51 349	48 651	97 804	56
5	49 192	51 392	48 608	97 800	55
6	49 231	51 435	48 565	97 796	54
7	49 269	51 478	48 522	97 792	53
8	49 308	51 520	48 480	97 788	52
9	49 347	51 563	48 437	97 784	51
10	49 385	51 606	48 394	97 779	50
11	49 424	51 648	48 352	97 775	49
12	49 462	51 691	48 309	97 771	48
13	49 500	51 734	48 266	97 767	47
14	49 539	51 776	48 224	97 763	46
15	49 577	51 819	48 181	97 759	45
16	49 615	51 861	48 139	97 754	44
17	49 654	51 903	48 097	97 750	43
18	49 692	51 946	48 054	97 746	42
19	49 730	51 988	48 012	97 742	41
20	49 768	52 031	47 969	97 738	40
21	49 806	52 073	47 927	97 734	39
22	49 844	52 115	47 885	97 729	38
23	49 882	52 157	47 843	97 725	37
24	49 920	52 200	47 800	97 721	36
25	49 958	52 242	47 758	97 717	35
26	49 996	52 284	47 716	97 713	34
27	50 034	52 326	47 674	97 708	33
28	50 072	52 368	47 632	97 704	32
29	50 110	52 410	47 590	97 700	31
30	50 148	52 452	47 548	97 696	30
31	50 185	52 494	47 506	97 691	29
32	50 223	52 536	47 464	97 687	28
33	50 261	52 578	47 422	97 683	27
34	50 298	52 620	47 380	97 679	26
35	50 336	52 661	47 339	97 674	25
36	50 374	52 703	47 297	97 670	24
37	50 411	52 745	47 255	97 666	23
38	50 449	52 787	47 213	97 662	22
39	50 486	52 829	47 171	97 657	21
40	50 523	52 870	47 130	97 653	20
41	50 561	52 912	47 088	97 649	19
42	50 598	52 953	47 047	97 645	18
43	50 635	52 995	47 005	97 640	17
44	50 673	53 037	46 963	97 636	16
45	50 710	53 078	46 922	97 632	15
46	50 747	53 120	46 880	97 628	14
47	50 784	53 161	46 839	97 623	13
48	50 821	53 202	46 798	97 619	12
49	50 858	53 244	46 756	97 615	11
50	50 896	53 285	46 715	97 610	10
51	50 933	53 327	46 673	97 606	9
52	50 970	53 368	46 632	97 602	8
53	51 007	53 409	46 591	97 597	7
54	51 043	53 450	46 550	97 593	6
55	51 080	53 492	46 508	97 589	5
56	51 117	53 533	46 467	97 584	4
57	51 154	53 574	46 426	97 580	3
58	51 191	53 615	46 385	97 576	2
59	51 227	53 656	46 344	97 571	1
60	51 264	53 697	46 303	97 567	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

71°

19°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	51 264	53 697	46 303	97 567	60
1	51 301	53 738	46 262	97 563	59
2	51 338	53 779	46 221	97 558	58
3	51 374	53 820	46 180	97 554	57
4	51 411	53 861	46 139	97 555	56
5	51 447	53 902	46 098	97 545	55
6	51 484	53 943	46 057	97 541	54
7	51 520	53 984	46 016	97 536	53
8	51 557	54 025	45 975	97 532	52
9	51 593	54 065	45 935	97 528	51
10	51 629	54 106	45 894	97 523	50
11	51 666	54 147	45 853	97 519	49
12	51 702	54 187	45 813	97 515	48
13	51 738	54 228	45 772	97 510	47
14	51 774	54 269	45 731	97 506	46
15	51 811	54 309	45 691	97 501	45
16	51 847	54 350	45 650	97 497	44
17	51 883	54 390	45 610	97 492	43
18	51 919	54 431	45 569	97 488	42
19	51 955	54 471	45 529	97 484	41
20	51 991	54 512	45 488	97 479	40
21	52 027	54 552	45 448	97 475	39
22	52 063	54 593	45 407	97 470	38
23	52 099	54 633	45 367	97 466	37
24	52 135	54 673	45 327	97 461	36
25	52 171	54 714	45 286	97 457	35
26	52 207	54 754	45 246	97 453	34
27	52 242	54 794	45 206	97 448	33
28	52 278	54 835	45 165	97 444	32
29	52 314	54 875	45 125	97 439	31
30	52 350	54 915	45 085	97 435	30
31	52 385	54 955	45 045	97 430	29
32	52 421	54 995	45 005	97 426	28
33	52 456	55 035	44 965	97 421	27
34	52 492	55 075	44 925	97 417	26
35	52 527	55 115	44 885	97 412	25
36	52 563	55 155	44 845	97 408	24
37	52 598	55 195	44 805	97 403	23
38	52 634	55 235	44 765	97 399	22
39	52 669	55 275	44 725	97 394	21
40	52 705	55 315	44 685	97 390	20
41	52 740	55 355	44 645	97 385	19
42	52 775	55 395	44 605	97 381	18
43	52 811	55 434	44 566	97 376	17
44	52 846	55 474	44 526	97 372	16
45	52 881	55 514	44 486	97 367	15
46	52 916	55 554	44 446	97 363	14
47	52 951	55 593	44 407	97 358	13
48	52 986	55 633	44 367	97 353	12
49	53 021	55 673	44 327	97 349	11
50	53 056	55 712	44 288	97 344	10
51	53 092	55 752	44 248	97 340	9
52	53 126	55 791	44 209	97 335	8
53	53 161	55 831	44 169	97 331	7
54	53 196	55 870	44 130	97 326	6
55	53 231	55 910	44 090	97 322	5
56	53 266	55 949	44 051	97 317	4
57	53 301	55 989	44 011	97 312	3
58	53 336	56 028	43 972	97 308	2
59	53 370	56 067	43 933	97 303	1
60	53 405	56 107	43 893	97 299	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

70°

20°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	53 405	56 107	43 893	97 299	60
1	53 440	56 146	43 854	97 294	59
2	53 475	56 185	43 815	97 289	58
3	53 509	56 224	43 776	97 285	57
4	53 544	56 264	43 736	97 280	56
5	53 578	56 303	43 697	97 276	55
6	53 613	56 342	43 658	97 271	54
7	53 647	56 381	43 619	97 266	53
8	53 682	56 420	43 580	97 262	52
9	53 716	56 459	43 541	97 257	51
10	53 751	56 498	43 502	97 252	50
11	53 785	56 537	43 463	97 248	49
12	53 819	56 576	43 424	97 243	48
13	53 854	56 615	43 385	97 238	47
14	53 888	56 654	43 346	97 234	46
15	53 922	56 693	43 307	97 229	45
16	53 957	56 732	43 268	97 224	44
17	53 991	56 771	43 229	97 220	43
18	54 025	56 810	43 190	97 215	42
19	54 059	56 849	43 151	97 210	41
20	54 093	56 887	43 113	97 206	40
21	54 127	56 926	43 074	97 201	39
22	54 161	56 965	43 035	97 196	38
23	54 195	57 004	42 996	97 192	37
24	54 229	57 042	42 958	97 187	36
25	54 263	57 081	42 919	97 182	35
26	54 297	57 120	42 880	97 178	34
27	54 331	57 158	42 842	97 173	33
28	54 365	57 197	42 803	97 168	32
29	54 399	57 235	42 765	97 163	31
30	54 433	57 274	42 726	97 159	30
31	54 466	57 312	42 688	97 154	29
32	54 500	57 351	42 649	97 149	28
33	54 534	57 389	42 611	97 145	27
34	54 567	57 428	42 572	97 140	26
35	54 601	57 466	42 534	97 135	25
36	54 635	57 504	42 496	97 130	24
37	54 668	57 543	42 457	97 126	23
38	54 702	57 581	42 419	97 121	22
39	54 735	57 619	42 381	97 116	21
40	54 769	57 658	42 342	97 111	20
41	54 802	57 696	42 304	97 107	19
42	54 836	57 734	42 266	97 102	18
43	54 869	57 772	42 228	97 097	17
44	54 903	57 810	42 190	97 092	16
45	54 936	57 849	42 151	97 087	15
46	54 969	57 887	42 113	97 083	14
47	55 003	57 925	42 075	97 078	13
48	55 036	57 963	42 037	97 073	12
49	55 069	58 001	41 999	97 068	11
50	55 102	58 039	41 961	97 063	10
51	55 136	58 077	41 923	97 059	9
52	55 169	58 115	41 885	97 054	8
53	55 202	58 153	41 847	97 049	7
54	55 235	58 191	41 809	97 044	6
55	55 268	58 229	41 771	97 039	5
56	55 301	58 267	41 733	97 035	4
57	55 334	58 304	41 696	[illegible]	[illegible]
58	55 367	58 342	41 658	[illegible]	[illegible]
59	55 400	58 380	41 620	[illegible]	[illegible]
60	55 433	58 418	41 582	[illegible]	[illegible]
′	log cos 9	log cot 9	log tan 10	[illegible]	[illegible]

69°

21°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	55 433	58 418	41 582	97 015	60
1	55 466	58 455	41 545	97 010	59
2	55 499	58 493	41 507	97 005	58
3	55 532	58 531	41 469	97 001	57
4	55 564	58 569	41 431	96 996	56
5	55 597	58 606	41 394	96 991	55
6	55 630	58 644	41 356	96 986	54
7	55 663	58 681	41 319	96 981	53
8	55 695	58 719	41 281	96 976	52
9	55 728	58 757	41 243	96 971	51
10	55 761	58 794	41 206	96 966	50
11	55 793	58 832	41 168	96 962	49
12	55 826	58 869	41 131	96 957	48
13	55 858	58 907	41 093	96 952	47
14	55 891	58 944	41 056	96 947	46
15	55 923	58 981	41 019	96 942	45
16	55 956	59 019	40 981	96 937	44
17	55 988	59 056	40 944	96 932	43
18	56 021	59 094	40 906	96 927	42
19	56 053	59 131	40 869	96 922	41
20	56 085	59 168	40 832	96 917	40
21	56 118	59 205	40 795	96 912	39
22	56 150	59 243	40 757	96 907	38
23	56 182	59 280	40 720	96 903	37
24	56 215	59 317	40 683	96 898	36
25	56 247	59 354	40 646	96 893	35
26	56 279	59 391	40 609	96 888	34
27	56 311	59 429	40 571	96 883	33
28	56 343	59 466	40 534	96 878	32
29	56 375	59 503	40 497	96 873	31
30	56 408	59 540	40 460	96 868	30
31	56 440	59 577	40 423	96 863	29
32	56 472	59 614	40 386	96 858	28
33	56 504	59 651	40 349	96 853	27
34	56 536	59 688	40 312	96 848	26
35	56 568	59 725	40 275	96 843	25
36	56 599	59 762	40 238	96 838	24
37	56 631	59 799	40 201	96 833	23
38	56 663	59 835	40 165	96 828	22
39	56 695	59 872	40 128	96 823	21
40	56 727	59 909	40 091	96 818	20
41	56 759	59 946	40 054	96 813	19
42	56 790	59 983	40 017	96 808	18
43	56 822	60 019	39 981	96 803	17
44	56 854	60 056	39 944	96 798	16
45	56 886	60 093	39 907	96 793	15
46	56 917	60 130	39 870	96 788	14
47	56 949	60 166	39 834	96 783	13
48	56 980	60 203	39 797	96 778	12
49	57 012	60 240	39 760	96 772	11
50	57 044	60 276	39 724	96 767	10
51	57 075	60 313	39 687	96 762	9
52	57 107	60 349	39 651	96 757	8
53	57 138	60 386	39 614	96 752	7
54	57 169	60 422	39 578	96 747	6
55	57 201	60 459	39 541	96 742	5
56	57 232	60 495	39 505	96 737	4
57	57 264	60 532	39 468	96 732	3
58	57 295	60 568	39 432	96 727	2
59	57 326	60 605	39 395	96 722	1
60	57 358	60 641	39 359	96 717	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

22°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	57 358	60 641	39 359	96 717	60
1	57 389	60 677	39 323	96 711	59
2	57 420	60 714	39 286	96 706	58
3	57 451	60 750	39 250	96 701	57
4	57 482	60 786	39 214	96 696	56
5	57 514	60 823	39 177	96 691	55
6	57 545	60 859	39 141	96 686	54
7	57 576	60 895	39 105	96 681	53
8	57 607	60 931	39 069	96 676	52
9	57 638	60 967	39 033	96 670	51
10	57 669	61 004	38 996	96 665	50
11	57 700	61 040	38 960	96 660	49
12	57 731	61 076	38 924	96 655	48
13	57 762	61 112	38 888	96 650	47
14	57 793	61 148	38 852	96 645	46
15	57 824	61 184	38 816	96 640	45
16	57 855	61 220	38 780	96 634	44
17	57 885	61 256	38 744	96 629	43
18	57 916	61 292	38 708	96 624	42
19	57 947	61 328	38 672	96 619	41
20	57 978	61 364	38 636	96 614	40
21	58 008	61 400	38 600	96 608	39
22	58 039	61 436	38 564	96 603	38
23	58 070	61 472	38 528	96 598	37
24	58 101	61 508	38 492	96 593	36
25	58 131	61 544	38 456	96 588	35
26	58 162	61 579	38 421	96 582	34
27	58 192	61 615	38 385	96 577	33
28	58 223	61 651	38 349	96 572	32
29	58 253	61 687	38 313	96 567	31
30	58 284	61 722	38 278	96 562	30
31	58 314	61 758	38 242	96 556	29
32	58 345	61 794	38 206	96 551	28
33	58 375	61 830	38 170	96 546	27
34	58 406	61 865	38 135	96 541	26
35	58 436	61 901	38 099	96 535	25
36	58 467	61 936	38 064	96 530	24
37	58 497	61 972	38 028	96 525	23
38	58 527	62 008	37 992	96 520	22
39	58 557	62 043	37 957	96 514	21
40	58 588	62 079	37 921	96 509	20
41	58 618	62 114	37 886	96 504	19
42	58 648	62 150	37 850	96 498	18
43	58 678	62 185	37 815	96 493	17
44	58 709	62 221	37 779	96 488	16
45	58 739	62 256	37 744	96 483	15
46	58 769	62 292	37 708	96 477	14
47	58 799	62 327	37 673	96 472	13
48	58 829	62 362	37 638	96 467	12
49	58 859	62 398	37 602	96 461	11
50	58 889	62 433	37 567	96 456	10
51	58 919	62 468	37 532	96 451	9
52	58 949	62 504	37 496	96 445	8
53	58 979	62 539	37 461	96 440	7
54	59 009	62 574	37 426	96 435	6
55	59 039	62 609	37 391	96 429	5
56	59 069	62 645	37 355	96 424	4
57	59 098	62 680	37 320	96 419	3
58	59 128	62 715	37 285	96 413	2
59	59 158	62 750	37 250	96 408	1
60	59 188	62 785	37 215	96 403	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

67°

23°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	59 188	62 785	37 215	96 403	**60**
1	59 218	62 820	37 180	96 397	59
2	59 247	62 855	37 145	96 392	58
3	59 277	62 890	37 110	96 387	57
4	59 307	62 926	37 074	96 381	56
5	59 336	62 961	37 039	96 376	55
6	59 366	62 996	37 004	96 370	54
7	59 396	63 031	36 969	96 365	53
8	59 425	63 066	36 934	96 360	52
9	59 455	63 101	36 899	96 354	51
10	59 484	63 135	36 865	96 349	**50**
11	59 514	63 170	36 830	96 343	49
12	59 543	63 205	36 795	96 338	48
13	59 573	63 240	36 760	96 333	47
14	59 602	63 275	36 725	96 327	46
15	59 632	63 310	36 690	96 322	45
16	59 661	63 345	36 655	96 316	44
17	59 690	63 379	36 621	96 311	43
18	59 720	63 414	36 586	96 305	42
19	59 749	63 449	36 551	96 300	41
20	59 778	63 484	36 516	96 294	**40**
21	59 808	63 519	36 481	96 289	39
22	59 837	63 553	36 447	96 284	38
23	59 866	63 588	36 412	96 278	37
24	59 895	63 623	36 377	96 273	36
25	59 924	63 657	36 343	96 267	35
26	59 954	63 692	36 308	96 262	34
27	59 983	63 726	36 274	96 256	33
28	60 012	63 761	36 239	96 251	32
29	60 041	63 796	36 204	96 245	31
30	60 070	63 830	36 170	96 240	**30**
31	60 099	63 865	36 135	96 234	29
32	60 128	63 899	36 101	96 229	28
33	60 157	63 934	36 066	96 223	27
34	60 186	63 968	36 032	96 218	26
35	60 215	64 003	35 997	96 212	25
36	60 244	64 037	35 963	96 207	24
37	60 273	64 072	35 928	96 201	23
38	60 302	64 106	35 894	96 196	22
39	60 331	64 140	35 860	96 190	21
40	60 359	64 175	35 825	96 185	**20**
41	60 388	64 209	35 791	96 179	19
42	60 417	64 243	35 757	96 174	18
43	60 446	64 278	35 722	96 168	17
44	60 474	64 312	35 688	96 162	16
45	60 503	64 346	35 654	96 157	15
46	60 532	64 381	35 619	96 151	14
47	60 561	64 415	35 585	96 146	13
48	60 589	64 449	35 551	96 140	12
49	60 618	64 483	35 517	96 135	11
50	60 646	64 517	35 483	96 129	**10**
51	60 675	64 552	35 448	96 123	9
52	60 704	64 586	35 414	96 118	8
53	60 732	64 620	35 380	96 112	7
54	60 761	64 654	35 346	96 107	6
55	60 789	64 688	35 312	96 101	5
56	60 818	64 722	35 278	96 095	4
57	60 846	64 756	35 244	96 090	3
58	60 875	64 790	35 210	96 084	2
59	60 903	64 824	35 176	96 079	1
60	60 931	64 858	35 142	96 073	**0**
′	log cos −9	log cot −9	log tan −10	log sin −9	′

66°

24°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	60 931	64 858	35 142	96 073	**60**
1	60 960	64 892	35 108	96 067	59
2	60 988	64 926	35 074	96 062	58
3	61 016	64 960	35 040	96 056	57
4	61 045	64 994	35 006	96 050	56
5	61 073	65 028	34 972	96 045	55
6	61 101	65 062	34 938	96 039	54
7	61 129	65 096	34 904	96 034	53
8	61 158	65 130	34 870	96 028	52
9	61 186	65 164	34 836	96 022	51
10	61 214	65 197	34 803	96 017	**50**
11	61 242	65 231	34 769	96 011	49
12	61 270	65 265	34 735	96 005	48
13	61 298	65 299	34 701	96 000	47
14	61 326	65 333	34 667	95 994	46
15	61 354	65 366	34 634	95 988	45
16	61 382	65 400	34 600	95 982	44
17	61 411	65 434	34 566	95 977	43
18	61 438	65 467	34 533	95 971	42
19	61 466	65 501	34 499	95 965	41
20	61 494	65 535	34 465	95 960	**40**
21	61 522	65 568	34 432	95 954	39
22	61 550	65 602	34 398	95 948	38
23	61 578	65 636	34 364	95 942	37
24	61 606	65 669	34 331	95 937	36
25	61 634	65 703	34 297	95 931	35
26	61 662	65 736	34 264	95 925	34
27	61 689	65 770	34 230	95 920	33
28	61 717	65 803	34 197	95 914	32
29	61 745	65 837	34 163	95 908	31
30	61 773	65 870	34 130	95 902	**30**
31	61 800	65 904	34 096	95 897	29
32	61 828	65 937	34 063	95 891	28
33	61 856	65 971	34 029	95 885	27
34	61 883	66 004	33 996	95 879	26
35	61 911	66 038	33 962	95 873	25
36	61 939	66 071	33 929	95 868	24
37	61 966	66 104	33 896	95 862	23
38	61 994	66 138	33 862	95 856	22
39	62 021	66 171	33 829	95 850	21
40	62 049	66 204	33 796	95 844	**20**
41	62 076	66 238	33 762	95 839	19
42	62 104	66 271	33 729	95 833	18
43	62 131	66 304	33 696	95 827	17
44	62 159	66 337	33 663	95 821	16
45	62 186	66 371	33 629	95 815	15
46	62 214	66 404	33 596	95 810	14
47	62 241	66 437	33 563	95 804	13
48	62 268	66 470	33 530	95 798	12
49	62 296	66 503	33 497	95 792	11
50	62 323	66 537	33 463	95 786	**10**
51	62 350	66 570	33 430	95 780	9
52	62 377	66 603	33 397	95 775	8
53	62 405	66 636	33 364	95 769	7
54	62 432	66 669	33 331	95 763	6
55	62 459	66 702	33 298	95 757	5
56	62 486	66 735	33 265	95 751	4
57	62 513	66 768	33 232	95 745	3
58	62 541	66 801	33 199	95 739	2
59	62 568	66 834	33 166	95 733	1
60	62 595	66 867	33 133	95 728	**0**
′	log cos −9	log cot −9	log tan −10	log sin −9	′

65°

25°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	62 595	66 867	33 133	95 728	60
1	62 622	66 900	33 100	95 722	59
2	62 649	66 933	33 067	95 716	58
3	62 676	66 966	33 034	95 710	57
4	62 703	66 999	33 001	95 704	56
5	62 730	67 032	32 968	95 698	55
6	62 757	67 065	32 935	95 692	54
7	62 784	67 098	32 902	95 686	53
8	62 811	67 131	32 869	95 680	52
9	62 838	67 163	32 837	95 674	51
10	62 865	67 196	32 804	95 668	50
11	62 892	67 229	32 771	95 663	49
12	62 918	67 262	32 738	95 657	48
13	62 945	67 295	32 705	95 651	47
14	62 972	67 327	32 673	95 645	46
15	62 999	67 360	32 640	95 639	45
16	63 026	67 393	32 607	95 633	44
17	63 052	67 426	32 574	95 627	43
18	63 079	67 458	32 542	95 621	42
19	63 106	67 491	32 509	95 615	41
20	63 133	67 524	32 476	95 609	40
21	63 159	67 556	32 444	95 603	39
22	63 186	67 589	32 411	95 597	38
23	63 213	67 622	32 378	95 591	37
24	63 239	67 654	32 346	95 585	36
25	63 266	67 687	32 313	95 579	35
26	63 292	67 719	32 281	95 573	34
27	63 319	67 752	32 248	95 567	33
28	63 345	67 785	32 215	95 561	32
29	63 372	67 817	32 183	95 555	31
30	63 398	67 850	32 150	95 549	30
31	63 425	67 882	32 118	95 543	29
32	63 451	67 915	32 085	95 537	28
33	63 478	67 947	32 053	95 531	27
34	63 504	67 980	32 020	95 525	26
35	63 531	68 012	31 988	95 519	25
36	63 557	68 044	31 956	95 513	24
37	63 583	68 077	31 923	95 507	23
38	63 610	68 109	31 891	95 500	22
39	63 636	68 142	31 858	95 494	21
40	63 662	68 174	31 826	95 488	20
41	63 689	68 206	31 794	95 482	19
42	63 715	68 239	31 761	95 476	18
43	63 741	68 271	31 729	95 470	17
44	63 767	68 303	31 697	95 464	16
45	63 794	68 336	31 664	95 458	15
46	63 820	68 368	31 632	95 452	14
47	63 846	68 400	31 600	95 446	13
48	63 872	68 432	31 568	95 440	12
49	63 898	68 465	31 535	95 434	11
50	63 924	68 497	31 503	95 427	10
51	63 950	68 529	31 471	95 421	9
52	63 976	68 561	31 439	95 415	8
53	64 002	68 593	31 407	95 409	7
54	64 028	68 626	31 374	95 403	6
55	64 054	68 658	31 342	95 397	5
56	64 080	68 690	31 310	95 391	4
57	64 106	68 722	31 278	95 384	3
58	64 132	68 754	31 246	95 378	2
59	64 158	68 786	31 214	95 372	1
60	64 184	68 818	31 182	95 366	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

64°

26°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	64 184	68 818	31 182	95 366	60
1	64 210	68 850	31 150	95 360	59
2	64 236	68 882	31 118	95 354	58
3	64 262	68 914	31 086	95 348	57
4	64 288	68 946	31 054	95 341	56
5	64 313	68 978	31 022	95 335	55
6	64 339	69 010	30 990	95 329	54
7	64 365	69 042	30 958	95 323	53
8	64 391	69 074	30 926	95 317	52
9	64 417	69 106	30 894	95 310	51
10	64 442	69 138	30 862	95 304	50
11	64 468	69 170	30 830	95 298	49
12	64 494	69 202	30 798	95 292	48
13	64 519	69 234	30 766	95 286	47
14	64 545	69 266	30 734	95 279	46
15	64 571	69 298	30 702	95 273	45
16	64 596	69 329	30 671	95 267	44
17	64 622	69 361	30 639	95 261	43
18	64 647	69 393	30 607	95 254	42
19	64 673	69 425	30 575	95 248	41
20	64 698	69 457	30 543	95 242	40
21	64 724	69 488	30 512	95 236	39
22	64 749	69 520	30 480	95 229	38
23	64 775	69 552	30 448	95 223	37
24	64 800	69 584	30 416	95 217	36
25	64 826	69 615	30 385	95 211	35
26	64 851	69 647	30 353	95 204	34
27	64 877	69 679	30 321	95 198	33
28	64 902	69 710	30 290	95 192	32
29	64 927	69 742	30 258	95 185	31
30	64 953	69 774	30 226	95 179	30
31	64 978	69 805	30 195	95 173	29
32	65 003	69 837	30 163	95 167	28
33	65 029	69 868	30 132	95 160	27
34	65 054	69 900	30 100	95 154	26
35	65 079	69 932	30 068	95 148	25
36	65 104	69 963	30 037	95 141	24
37	65 130	69 995	30 005	95 135	23
38	65 155	70 026	29 974	95 129	22
39	65 180	70 058	29 942	95 122	21
40	65 205	70 089	29 911	95 116	20
41	65 230	70 121	29 879	95 110	19
42	65 255	70 152	29 848	95 103	18
43	65 281	70 184	29 816	95 097	17
44	65 306	70 215	29 785	95 090	16
45	65 331	70 247	29 753	95 084	15
46	65 356	70 278	29 722	95 078	14
47	65 381	70 309	29 691	95 071	13
48	65 406	70 341	29 659	95 065	12
49	65 431	70 372	29 628	95 059	11
50	65 456	70 404	29 596	95 052	10
51	65 481	70 435	29 565	95 046	9
52	65 506	70 466	29 534	95 039	8
53	65 531	70 498	29 502	95 033	7
54	65 556	70 529	29 471	95 027	6
55	65 580	70 560	29 440	95 020	5
56	65 605	70 592	29 408	95 014	4
57	65 630	70 623	29 377	95 007	3
58	65 655	70 654	29 346	95 001	2
59	65 680	70 685	29 315	94 995	1
60	65 705	70 717	29 283	94 988	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

63°

27°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	65 705	70 717	29 283	94 988	**60**
1	65 729	70 748	29 252	94 982	59
2	65 754	70 779	29 221	94 975	58
3	65 779	70 810	29 190	94 969	57
4	65 804	70 841	29 159	94 962	56
5	65 828	70 873	29 127	94 956	55
6	65 853	70 904	29 096	94 949	54
7	65 878	70 935	29 065	94 943	53
8	65 902	70 966	29 034	94 936	52
9	65 927	70 997	29 003	94 930	51
10	65 952	71 028	28 972	94 923	**50**
11	65 976	71 059	28 941	94 917	49
12	66 001	71 090	28 910	94 911	48
13	66 025	71 121	28 879	94 904	47
14	66 050	71 153	28 847	94 898	46
15	66 075	71 184	28 816	94 891	45
16	66 099	71 215	28 785	94 885	44
17	66 124	71 246	28 754	94 878	43
18	66 148	71 277	28 723	94 871	42
19	66 173	71 308	28 692	94 865	41
20	66 197	71 339	28 661	94 858	**40**
21	66 221	71 370	28 630	94 852	39
22	66 246	71 401	28 599	94 845	38
23	66 270	71 431	28 569	94 839	37
24	66 295	71 462	28 538	94 832	36
25	66 319	71 493	28 507	94 826	35
26	66 343	71 524	28 476	94 819	34
27	66 368	71 555	28 445	94 813	33
28	66 392	71 586	28 414	94 806	32
29	66 416	71 617	28 383	94 799	31
30	66 441	71 648	28 352	94 793	**30**
31	66 465	71 679	28 321	94 786	29
32	66 489	71 709	28 291	94 780	28
33	66 513	71 740	28 260	94 773	27
34	66 537	71 771	28 229	94 767	26
35	66 562	71 802	28 198	94 760	25
36	66 586	71 833	28 167	94 753	24
37	66 610	71 863	28 137	94 747	23
38	66 634	71 894	28 106	94 740	22
39	66 658	71 925	28 075	94 734	21
40	66 682	71 955	28 045	94 727	**20**
41	66 706	71 986	28 014	94 720	19
42	66 731	72 017	27 983	94 714	18
43	66 755	72 048	27 952	94 707	17
44	66 779	72 078	27 922	94 700	16
45	66 803	72 109	27 891	94 694	15
46	66 827	72 140	27 860	94 687	14
47	66 851	72 170	27 830	94 680	13
48	66 875	72 201	27 799	94 674	12
49	66 899	72 231	27 769	94 667	11
50	66 922	72 262	27 738	94 660	**10**
51	66 946	72 293	27 707	94 654	9
52	66 970	72 323	27 677	94 647	8
53	66 994	72 354	27 646	94 640	7
54	67 018	72 384	27 616	94 634	6
55	67 042	72 415	27 585	94 627	5
56	67 066	72 445	27 555	94 620	4
57	67 090	72 476	27 524	94 614	3
58	67 113	72 506	27 494	94 607	2
59	67 137	72 537	27 463	94 600	1
60	67 161	72 567	27 433	94 593	**0**
′	log cos −9	log cot −9	log tan −10	log sin −9	′

62°

28°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	67 161	72 567	27 433	94 593	**60**
1	67 185	72 598	27 402	94 587	59
2	67 208	72 628	27 372	94 580	58
3	67 232	72 659	27 341	94 573	57
4	67 256	72 689	27 311	94 567	56
5	67 280	72 720	27 280	94 560	55
6	67 303	72 750	27 250	94 553	54
7	67 327	72 780	27 220	94 546	53
8	67 350	72 811	27 189	94 540	52
9	67 374	72 841	27 159	94 533	51
10	67 398	72 872	27 128	94 526	**50**
11	67 421	72 902	27 098	94 519	49
12	67 445	72 932	27 068	94 513	48
13	67 468	72 963	27 037	94 506	47
14	67 492	72 993	27 007	94 499	46
15	67 515	73 023	26 977	94 492	45
16	67 539	73 054	26 946	94 485	44
17	67 562	73 084	26 916	94 479	43
18	67 586	73 114	26 886	94 472	42
19	67 609	73 144	26 856	94 465	41
20	67 633	73 175	26 825	94 458	**40**
21	67 656	73 205	26 795	94 451	39
22	67 680	73 235	26 765	94 445	38
23	67 703	73 265	26 735	94 438	37
24	67 726	73 295	26 705	94 431	36
25	67 750	73 326	26 674	94 424	35
26	67 773	73 356	26 644	94 417	34
27	67 796	73 386	26 614	94 410	33
28	67 820	73 416	26 584	94 404	32
29	67 843	73 446	26 554	94 397	31
30	67 866	73 476	26 524	94 390	**30**
31	67 890	73 507	26 493	94 383	29
32	67 913	73 537	26 463	94 376	28
33	67 936	73 567	26 433	94 369	27
34	67 959	73 597	26 403	94 362	26
35	67 982	73 627	26 373	94 355	25
36	68 006	73 657	26 343	94 349	24
37	68 029	73 687	26 313	94 342	23
38	68 052	73 717	26 283	94 335	22
39	68 075	73 747	26 253	94 328	21
40	68 098	73 777	26 223	94 321	**20**
41	68 121	73 807	26 193	94 314	19
42	68 144	73 837	26 163	94 307	18
43	68 167	73 867	26 133	94 300	17
44	68 190	73 897	26 103	94 293	16
45	68 213	73 927	26 073	94 286	15
46	68 237	73 957	26 043	94 279	14
47	68 260	73 987	26 013	94 273	13
48	68 283	74 017	25 983	94 266	12
49	68 305	74 047	25 953	94 259	11
50	68 328	74 077	25 923	94 252	**10**
51	68 351	74 107	25 893	94 245	9
52	68 374	74 137	25 863	94 238	8
53	68 397	74 166	25 834	94 231	7
54	68 420	74 196	25 804	94 224	6
55	68 443	74 226	25 774	94 217	5
56	68 466	74 256	25 744	94 210	4
57	68 489	74 286	25 714	94 203	3
58	68 512	74 316	25 684	94 196	2
59	68 534	74 345	25 655	94 189	1
60	68 557	74 375	25 625	94 182	**0**
′	log cos −9	log cot −9	log tan −10	log sin −9	′

61°

29°

′	log sin (9)	log tan (9)	log cot (10)	log cos (9)	′
0	68 557	74 375	25 625	94 182	60
1	68 580	74 405	25 595	94 175	59
2	68 603	74 435	25 565	94 168	58
3	68 625	74 465	25 535	94 161	57
4	68 648	74 494	25 506	94 154	56
5	68 671	74 524	25 476	94 147	55
6	68 694	74 554	25 446	94 140	54
7	68 716	74 583	25 417	94 133	53
8	68 739	74 613	25 387	94 126	52
9	68 762	74 643	25 357	94 119	51
10	68 784	74 673	25 327	94 112	50
11	68 807	74 702	25 298	94 105	49
12	68 829	74 732	25 268	94 098	48
13	68 852	74 762	25 238	94 090	47
14	68 875	74 791	25 209	94 083	46
15	68 897	74 821	25 179	94 076	45
16	68 920	74 851	25 149	94 069	44
17	68 942	74 880	25 120	94 062	43
18	68 965	74 910	25 090	94 055	42
19	68 987	74 939	25 061	94 048	41
20	69 010	74 969	25 031	94 041	40
21	69 032	74 998	25 002	94 034	39
22	69 055	75 028	24 972	94 027	38
23	69 077	75 058	24 942	94 020	37
24	69 100	75 087	24 913	94 012	36
25	69 122	75 117	24 883	94 005	35
26	69 144	75 146	24 854	93 998	34
27	69 167	75 176	24 824	93 991	33
28	69 189	75 205	24 795	93 984	32
29	69 212	75 235	24 765	93 977	31
30	69 234	75 264	24 736	93 970	30
31	69 256	75 294	24 706	93 963	29
32	69 279	75 323	24 677	93 955	28
33	69 301	75 353	24 647	93 948	27
34	69 323	75 382	24 618	93 941	26
35	69 345	75 411	24 589	93 934	25
36	69 368	75 441	24 559	93 927	24
37	69 390	75 470	24 530	93 920	23
38	69 412	75 500	24 500	93 912	22
39	69 434	75 529	24 471	93 905	21
40	69 456	75 558	24 442	93 898	20
41	69 479	75 588	24 412	93 891	19
42	69 501	75 617	24 383	93 884	18
43	69 523	75 647	24 353	93 876	17
44	69 545	75 676	24 324	93 869	16
45	69 567	75 705	24 295	93 862	15
46	69 589	75 735	24 265	93 855	14
47	69 611	75 764	24 236	93 847	13
48	69 633	75 793	24 207	93 840	12
49	69 655	75 822	24 178	93 833	11
50	69 677	75 852	24 148	93 826	10
51	69 699	75 881	24 119	93 819	9
52	69 721	75 910	24 090	93 811	8
53	69 743	75 939	24 061	93 804	7
54	69 765	75 969	24 031	93 797	6
55	69 787	75 998	24 002	93 789	5
56	69 809	76 027	23 973	93 782	4
57	69 831	76 056	23 944	93 775	3
58	69 853	76 086	23 914	93 768	2
59	69 875	76 115	23 885	93 760	1
60	69 897	76 144	23 856	93 753	0
′	log cos (9)	log cot (9)	log tan (10)	log sin (9)	′

60°

30°

′	log sin (9)	log tan (9)	log cot (10)	log cos (9)	′
0	69 897	76 144	23 856	93 753	60
1	69 919	76 173	23 827	93 746	59
2	69 941	76 202	23 798	93 738	58
3	69 963	76 231	23 769	93 731	57
4	69 984	76 261	23 739	93 724	56
5	70 006	76 290	23 710	93 717	55
6	70 028	76 319	23 681	93 709	54
7	70 050	76 348	23 652	93 702	53
8	70 072	76 377	23 623	93 695	52
9	70 093	76 406	23 594	93 687	51
10	70 115	76 435	23 565	93 680	50
11	70 137	76 464	23 536	93 673	49
12	70 159	76 493	23 507	93 665	48
13	70 180	76 522	23 478	93 658	47
14	70 202	76 551	23 449	93 650	46
15	70 224	76 580	23 420	93 643	45
16	70 245	76 609	23 391	93 636	44
17	70 267	76 639	23 361	93 628	43
18	70 288	76 668	23 332	93 621	42
19	70 310	76 697	23 303	93 614	41
20	70 332	76 725	23 275	93 606	40
21	70 353	76 754	23 246	93 599	39
22	70 375	76 783	23 217	93 591	38
23	70 396	76 812	23 188	93 584	37
24	70 418	76 841	23 159	93 577	36
25	70 439	76 870	23 130	93 569	35
26	70 461	76 899	23 101	93 562	34
27	70 482	76 928	23 072	93 554	33
28	70 504	76 957	23 043	93 547	32
29	70 525	76 986	23 014	93 539	31
30	70 547	77 015	22 985	93 532	30
31	70 568	77 044	22 956	93 525	29
32	70 590	77 073	22 927	93 517	28
33	70 611	77 101	22 899	93 510	27
34	70 633	77 130	22 870	93 502	26
35	70 654	77 159	22 841	93 495	25
36	70 675	77 188	22 812	93 487	24
37	70 697	77 217	22 783	93 480	23
38	70 718	77 246	22 754	93 472	22
39	70 739	77 274	22 726	93 465	21
40	70 761	77 303	22 697	93 457	20
41	70 782	77 332	22 668	93 450	19
42	70 803	77 361	22 639	93 442	18
43	70 824	77 390	22 610	93 435	17
44	70 846	77 418	22 582	93 427	16
45	70 867	77 447	22 553	93 420	15
46	70 888	77 476	22 524	93 412	14
47	70 909	77 505	22 495	93 405	13
48	70 931	77 533	22 467	93 397	12
49	70 952	77 562	22 438	93 390	11
50	70 973	77 591	22 409	93 382	10
51	70 994	77 619	22 381	93 375	9
52	71 015	77 648	22 352	93 367	8
53	71 036	77 677	22 323	93 360	7
54	71 058	77 706	22 294	93 352	6
55	71 079	77 734	22 266	93 344	5
56	71 100	77 763	22 237	93 337	4
57	71 121	77 791	22 209	93 329	3
58	71 142	77 820	22 180	93 322	2
59	71 163	77 849	22 151	93 314	1
60	71 184	77 877	22 123	93 307	0
′	log cos (9)	log cot (9)	log tan (10)	log sin (9)	′

59°

31°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	71 184	77 877	22 123	93 307	60
1	71 205	77 906	22 094	93 299	59
2	71 226	77 935	22 065	93 291	58
3	71 247	77 963	22 037	93 284	57
4	71 268	77 992	22 008	93 276	56
5	71 289	78 020	21 980	93 269	55
6	71 310	78 049	21 951	93 261	54
7	71 331	78 077	21 923	93 253	53
8	71 352	78 106	21 894	93 246	52
9	71 373	78 135	21 865	93 238	51
10	71 393	78 163	21 837	93 230	50
11	71 414	78 192	21 808	93 223	49
12	71 435	78 220	21 780	93 215	48
13	71 456	78 249	21 751	93 207	47
14	71 477	78 277	21 723	93 200	46
15	71 498	78 306	21 694	93 192	45
16	71 519	78 334	21 666	93 184	44
17	71 539	78 363	21 637	93 177	43
18	71 560	78 391	21 609	93 169	42
19	71 581	78 419	21 581	93 161	41
20	71 602	78 448	21 552	93 154	40
21	71 622	78 476	21 524	93 146	39
22	71 643	78 505	21 495	93 138	38
23	71 664	78 533	21 467	93 131	37
24	71 685	78 562	21 438	93 123	36
25	71 705	78 590	21 410	93 115	35
26	71 726	78 618	21 382	93 108	34
27	71 747	78 647	21 353	93 100	33
28	71 767	78 675	21 325	93 092	32
29	71 788	78 704	21 296	93 084	31
30	71 809	78 732	21 268	93 077	30
31	71 829	78 760	21 240	93 069	29
32	71 850	78 789	21 211	93 061	28
33	71 870	78 817	21 183	93 053	27
34	71 891	78 845	21 155	93 046	26
35	71 911	78 874	21 126	93 038	25
36	71 932	78 902	21 098	93 030	24
37	71 952	78 930	21 070	93 022	23
38	71 973	78 959	21 041	93 014	22
39	71 994	78 987	21 013	93 007	21
40	72 014	79 015	20 985	92 999	20
41	72 034	79 043	20 957	92 991	19
42	72 055	79 072	20 928	92 983	18
43	72 075	79 100	20 900	92 976	17
44	72 096	79 128	20 872	92 968	16
45	72 116	79 156	20 844	92 960	15
46	72 137	79 185	20 815	92 952	14
47	72 157	79 213	20 787	92 944	13
48	72 177	79 241	20 759	92 936	12
49	72 198	79 269	20 731	92 929	11
50	72 218	79 297	20 703	92 921	10
51	72 238	79 326	20 674	92 913	9
52	72 259	79 354	20 646	92 905	8
53	72 279	79 382	20 618	92 897	7
54	72 299	79 410	20 590	92 889	6
55	72 320	79 438	20 562	92 881	5
56	72 340	79 466	20 534	92 874	4
57	72 360	79 495	20 505	92 866	3
58	72 381	79 523	20 477	92 858	2
59	72 401	79 551	20 449	92 850	1
60	72 421	79 579	20 421	92 842	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

58°

32°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	72 421	79 579	20 421	92 842	60
1	72 441	79 607	20 393	92 834	59
2	72 461	79 635	20 365	92 826	58
3	72 482	79 663	20 337	92 818	57
4	72 502	79 691	20 309	92 810	56
5	72 522	79 719	20 281	92 803	55
6	72 542	79 747	20 253	92 795	54
7	72 562	79 776	20 224	92 787	53
8	72 582	79 804	20 196	92 779	52
9	72 602	79 832	20 168	92 771	51
10	72 622	79 860	20 140	92 763	50
11	72 643	79 888	20 112	92 755	49
12	72 663	79 916	20 084	92 747	48
13	72 683	79 944	20 056	92 739	47
14	72 703	79 972	20 028	92 731	46
15	72 723	80 000	20 000	92 723	45
16	72 743	80 028	19 972	92 715	44
17	72 763	80 056	19 944	92 707	43
18	72 783	80 084	19 916	92 699	42
19	72 803	80 112	19 888	92 691	41
20	72 823	80 140	19 860	92 683	40
21	72 843	80 168	19 832	92 675	39
22	72 863	80 195	19 805	92 667	38
23	72 883	80 223	19 777	92 659	37
24	72 902	80 251	19 749	92 651	36
25	72 922	80 279	19 721	92 643	35
26	72 942	80 307	19 693	92 635	34
27	72 962	80 335	19 665	92 627	33
28	72 982	80 363	19 637	92 619	32
29	73 002	80 391	19 609	92 611	31
30	73 022	80 419	19 581	92 603	30
31	73 041	80 447	19 553	92 595	29
32	73 061	80 474	19 526	92 587	28
33	73 081	80 502	19 498	92 579	27
34	73 101	80 530	19 470	92 571	26
35	73 121	80 558	19 442	92 563	25
36	73 140	80 586	19 414	92 555	24
37	73 160	80 614	19 386	92 546	23
38	73 180	80 642	19 358	92 538	22
39	73 200	80 669	19 331	92 530	21
40	73 219	80 697	19 303	92 522	20
41	73 239	80 725	19 275	92 514	19
42	73 259	80 753	19 247	92 506	18
43	73 278	80 781	19 219	92 498	17
44	73 298	80 808	19 192	92 490	16
45	73 318	80 836	19 164	92 482	15
46	73 337	80 864	19 136	92 473	14
47	73 357	80 892	19 108	92 465	13
48	73 377	80 919	19 081	92 457	12
49	73 396	80 947	19 053	92 449	11
50	73 416	80 975	19 025	92 441	10
51	73 435	81 003	18 997	92 433	9
52	73 455	81 030	18 970	92 425	8
53	73 474	81 058	18 942	92 416	7
54	73 494	81 086	18 914	92 408	6
55	73 513	81 113	18 887	92 400	5
56	73 533	81 141	18 859	92 392	4
57	73 552	81 169	18 831	92 384	3
58	73 572	81 196	18 804	92 376	2
59	73 591	81 224	18 776	92 367	1
60	73 611	81 252	18 748	92 359	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

57°

33°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	73 611	81 252	18 748	92 359	60
1	73 630	81 279	18 721	92 351	59
2	73 650	81 307	18 693	92 343	58
3	73 669	81 335	18 665	92 335	57
4	73 689	81 362	18 638	92 326	56
5	73 708	81 390	18 610	92 318	55
6	73 727	81 418	18 582	92 310	54
7	73 747	81 445	18 555	92 302	53
8	73 766	81 473	18 527	92 293	52
9	73 785	81 500	18 500	92 285	51
10	73 805	81 528	18 472	92 277	50
11	73 824	81 556	18 444	92 269	49
12	73 843	81 583	18 417	92 260	48
13	73 863	81 611	18 389	92 252	47
14	73 882	81 638	18 362	92 244	46
15	73 901	81 666	18 334	92 235	45
16	73 921	81 693	18 307	92 227	44
17	73 940	81 721	18 279	92 219	43
18	73 959	81 748	18 252	92 211	42
19	73 978	81 776	18 224	92 202	41
20	73 997	81 803	18 197	92 194	40
21	74 017	81 831	18 169	92 186	39
22	74 036	81 858	18 142	92 177	38
23	74 055	81 886	18 114	92 169	37
24	74 074	81 913	18 087	92 161	36
25	74 093	81 941	18 059	92 152	35
26	74 113	81 968	18 032	92 144	34
27	74 132	81 996	18 004	92 136	33
28	74 151	82 023	17 977	92 127	32
29	74 170	82 051	17 949	92 119	31
30	74 189	82 078	17 922	92 111	30
31	74 208	82 106	17 894	92 102	29
32	74 227	82 133	17 867	92 094	28
33	74 246	82 161	17 839	92 086	27
34	74 265	82 188	17 812	92 077	26
35	74 284	82 215	17 785	92 069	25
36	74 303	82 243	17 757	92 060	24
37	74 322	82 270	17 730	92 052	23
38	74 341	82 298	17 702	92 044	22
39	74 360	82 325	17 675	92 035	21
40	74 379	82 352	17 648	92 027	20
41	74 398	82 380	17 620	92 018	19
42	74 417	82 407	17 593	92 010	18
43	74 436	82 435	17 565	92 002	17
44	74 455	82 462	17 538	91 993	16
45	74 474	82 489	17 511	91 985	15
46	74 493	82 517	17 483	91 976	14
47	74 512	82 544	17 456	91 968	13
48	74 531	82 571	17 429	91 959	12
49	74 549	82 599	17 401	91 951	11
50	74 568	82 626	17 374	91 942	10
51	74 587	82 653	17 347	91 934	9
52	74 606	82 681	17 319	91 925	8
53	74 625	82 708	17 292	91 917	7
54	74 644	82 735	17 265	91 908	6
55	74 662	82 762	17 238	91 900	5
56	74 681	82 790	17 210	91 891	4
57	74 700	82 817	17 183	91 883	3
58	74 719	82 844	17 156	91 874	2
59	74 737	82 871	17 129	91 866	1
60	74 756	82 899	17 101	91 857	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

56°

34°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	74 756	82 899	17 101	91 857	60
1	74 775	82 926	17 074	91 849	59
2	74 794	82 953	17 047	91 840	58
3	74 812	82 980	17 020	91 832	57
4	74 831	83 008	16 992	91 823	56
5	74 850	83 035	16 965	91 815	55
6	74 868	83 062	16 938	91 806	54
7	74 887	83 089	16 911	91 798	53
8	74 906	83 117	16 883	91 789	52
9	74 924	83 144	16 856	91 781	51
10	74 943	83 171	16 829	91 772	50
11	74 961	83 198	16 802	91 763	49
12	74 980	83 225	16 775	91 755	48
13	74 999	83 252	16 748	91 746	47
14	75 017	83 280	16 720	91 738	46
15	75 036	83 307	16 693	91 729	45
16	75 054	83 334	16 666	91 720	44
17	75 073	83 361	16 639	91 712	43
18	75 091	83 388	16 612	91 703	42
19	75 110	83 415	16 585	91 695	41
20	75 128	83 442	16 558	91 686	40
21	75 147	83 470	16 530	91 677	39
22	75 165	83 497	16 503	91 669	38
23	75 184	83 524	16 476	91 660	37
24	75 202	83 551	16 449	91 651	36
25	75 221	83 578	16 422	91 643	35
26	75 239	83 605	16 395	91 634	34
27	75 258	83 632	16 368	91 625	33
28	75 276	83 659	16 341	91 617	32
29	75 294	83 686	16 314	91 608	31
30	75 313	83 713	16 287	91 599	30
31	75 331	83 740	16 260	91 591	29
32	75 350	83 768	16 232	91 582	28
33	75 368	83 795	16 205	91 573	27
34	75 386	83 822	16 178	91 565	26
35	75 405	83 849	16 151	91 556	25
36	75 423	83 876	16 124	91 547	24
37	75 441	83 903	16 097	91 538	23
38	75 459	83 930	16 070	91 530	22
39	75 478	83 957	16 043	91 521	21
40	75 496	83 984	16 016	91 512	20
41	75 514	84 011	15 989	91 504	19
42	75 533	84 038	15 962	91 495	18
43	75 551	84 065	15 935	91 486	17
44	75 569	84 092	15 908	91 477	16
45	75 587	84 119	15 881	91 469	15
46	75 605	84 146	15 854	91 460	14
47	75 624	84 173	15 827	91 451	13
48	75 642	84 200	15 800	91 442	12
49	75 660	84 227	15 773	91 433	11
50	75 678	84 254	15 746	91 425	10
51	75 696	84 280	15 720	91 416	9
52	75 714	84 307	15 693	91 407	8
53	75 733	84 334	15 666	91 398	7
54	75 751	84 361	15 639	91 389	6
55	75 769	84 388	15 612	91 381	5
56	75 787	84 415	15 585	91 372	4
57	75 805	84 442	15 558	91 363	3
58	75 823	84 469	15 531	91 354	2
59	75 841	84 496	15 504	91 345	1
60	75 859	84 523	15 477	91 336	0
′	log cos 9	log cot 9	log tan 10	log sin 9	′

55°

35°

′	log sin (9)	log tan (9)	log cot (10)	log cos (9)	′
0	75 859	84 523	15 477	91 336	60
1	75 877	84 550	15 450	91 328	59
2	75 895	84 576	15 424	91 319	58
3	75 913	84 603	15 397	91 310	57
4	75 931	84 630	15 370	91 301	56
5	75 949	84 657	15 343	91 292	55
6	75 967	84 684	15 316	91 283	54
7	75 985	84 711	15 289	91 274	53
8	76 003	84 738	15 262	91 266	52
9	76 021	84 764	15 236	91 257	51
10	76 039	84 791	15 209	91 248	50
11	76 057	84 818	15 182	91 239	49
12	76 075	84 845	15 155	91 230	48
13	76 093	84 872	15 128	91 221	47
14	76 111	84 899	15 101	91 212	46
15	76 129	84 925	15 075	91 203	45
16	76 146	84 952	15 048	91 194	44
17	76 164	84 979	15 021	91 185	43
18	76 182	85 006	14 994	91 176	42
19	76 200	85 033	14 967	91 167	41
20	76 218	85 059	14 941	91 158	40
21	76 236	85 086	14 914	91 149	39
22	76 253	85 113	14 887	91 141	38
23	76 271	85 140	14 860	91 132	37
24	76 289	85 166	14 834	91 123	36
25	76 307	85 193	14 807	91 114	35
26	76 324	85 220	14 780	91 105	34
27	76 342	85 247	14 753	91 096	33
28	76 360	85 273	14 727	91 087	32
29	76 378	85 300	14 700	91 078	31
30	76 395	85 327	14 673	91 069	30
31	76 413	85 354	14 646	91 060	29
32	76 431	85 380	14 620	91 051	28
33	76 448	85 407	14 593	91 042	27
34	76 466	85 434	14 566	91 033	26
35	76 484	85 460	14 540	91 023	25
36	76 501	85 487	14 513	91 014	24
37	76 519	85 514	14 486	91 005	23
38	76 537	85 540	14 460	90 996	22
39	76 554	85 567	14 433	90 987	21
40	76 572	85 594	14 406	90 978	20
41	76 590	85 620	14 380	90 969	19
42	76 607	85 647	14 353	90 960	18
43	76 625	85 674	14 326	90 951	17
44	76 642	85 700	14 300	90 942	16
45	76 660	85 727	14 273	90 933	15
46	76 677	85 754	14 246	90 924	14
47	76 695	85 780	14 220	90 915	13
48	76 712	85 807	14 193	90 906	12
49	76 730	85 834	14 166	90 896	11
50	76 747	85 860	14 140	90 887	10
51	76 765	85 887	14 113	90 878	9
52	76 782	85 913	14 087	90 869	8
53	76 800	85 940	14 060	90 860	7
54	76 817	85 967	14 033	90 851	6
55	76 835	85 993	14 007	90 842	5
56	76 852	86 020	13 980	90 832	4
57	76 870	86 046	13 954	90 823	3
58	76 887	86 073	13 927	90 814	2
59	76 904	86 100	13 900	90 805	1
60	76 922	86 126	13 874	90 796	0
′	log cos (9)	log cot (9)	log tan (10)	log sin (9)	′

54°

36°

′	log sin (9)	log tan (9)	log cot (10)	log cos (9)	′
0	76 922	86 126	13 874	90 796	60
1	76 939	86 153	13 847	90 787	59
2	76 957	86 179	13 821	90 777	58
3	76 974	86 206	13 794	90 768	57
4	76 991	86 232	13 768	90 759	56
5	77 009	86 259	13 741	90 750	55
6	77 026	86 285	13 715	90 741	54
7	77 043	86 312	13 688	90 731	53
8	77 061	86 338	13 662	90 722	52
9	77 078	86 365	13 635	90 713	51
10	77 095	86 392	13 608	90 704	50
11	77 112	86 418	13 582	90 694	49
12	77 130	86 445	13 555	90 685	48
13	77 147	86 471	13 529	90 676	47
14	77 164	86 498	13 502	90 667	46
15	77 181	86 524	13 476	90 657	45
16	77 199	86 551	13 449	90 648	44
17	77 216	86 577	13 423	90 639	43
18	77 233	86 603	13 397	90 630	42
19	77 250	86 630	13 370	90 620	41
20	77 268	86 656	13 344	90 611	40
21	77 285	86 683	13 317	90 602	39
22	77 302	86 709	13 291	90 592	38
23	77 319	86 736	13 264	90 583	37
24	77 336	86 762	13 238	90 574	36
25	77 353	86 789	13 211	90 565	35
26	77 370	86 815	13 185	90 555	34
27	77 387	86 842	13 158	90 546	33
28	77 405	86 868	13 132	90 537	32
29	77 422	86 894	13 106	90 527	31
30	77 439	86 921	13 079	90 518	30
31	77 456	86 947	13 053	90 509	29
32	77 473	86 974	13 026	90 499	28
33	77 490	87 000	13 000	90 490	27
34	77 507	87 027	12 973	90 480	26
35	77 524	87 053	12 947	90 471	25
36	77 541	87 079	12 921	90 462	24
37	77 558	87 106	12 894	90 452	23
38	77 575	87 132	12 868	90 443	22
39	77 592	87 158	12 842	90 434	21
40	77 609	87 185	12 815	90 424	20
41	77 626	87 211	12 789	90 415	19
42	77 643	87 238	12 762	90 405	18
43	77 660	87 264	12 736	90 396	17
44	77 677	87 290	12 710	90 386	16
45	77 694	87 317	12 683	90 377	15
46	77 711	87 343	12 657	90 368	14
47	77 728	87 369	12 631	90 358	13
48	77 744	87 396	12 604	90 349	12
49	77 761	87 422	12 578	90 339	11
50	77 778	87 448	12 552	90 330	10
51	77 795	87 475	12 525	90 320	9
52	77 812	87 501	12 499	90 311	8
53	77 829	87 527	12 473	90 301	7
54	77 846	87 554	12 446	90 292	6
55	77 862	87 580	12 420	90 282	5
56	77 879	87 606	12 394	90 273	4
57	77 896	87 633	12 367	90 263	3
58	77 913	87 659	12 341	90 254	2
59	77 930	87 685	12 315	90 244	1
60	77 946	87 711	12 289	90 235	0
′	log cos (9)	log cot (9)	log tan (10)	log sin (9)	′

53°

37°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	77 946	87 711	12 289	90 235	60
1	77 963	87 738	12 262	90 225	59
2	77 980	87 764	12 236	90 216	58
3	77 997	87 790	12 210	90 206	57
4	78 013	87 817	12 183	90 197	56
5	78 030	87 843	12 157	90 187	55
6	78 047	87 869	12 131	90 178	54
7	78 063	87 895	12 105	90 168	53
8	78 080	87 922	12 078	90 159	52
9	78 097	87 948	12 052	90 149	51
10	78 113	87 974	12 026	90 139	50
11	78 130	88 000	12 000	90 130	49
12	78 147	88 027	11 973	90 120	48
13	78 163	88 053	11 947	90 111	47
14	78 180	88 079	11 921	90 101	46
15	78 197	88 105	11 895	90 091	45
16	78 213	88 131	11 869	90 082	44
17	78 230	88 158	11 842	90 072	43
18	78 246	88 184	11 816	90 063	42
19	78 263	88 210	11 790	90 053	41
20	78 280	88 236	11 764	90 043	40
21	78 296	88 262	11 738	90 034	39
22	78 313	88 289	11 711	90 024	38
23	78 329	88 315	11 685	90 014	37
24	78 346	88 341	11 659	90 005	36
25	78 362	88 367	11 633	89 995	35
26	78 379	88 393	11 607	89 985	34
27	78 395	88 420	11 580	89 976	33
28	78 412	88 446	11 554	89 966	32
29	78 428	88 472	11 528	89 956	31
30	78 445	88 498	11 502	89 947	30
31	78 461	88 524	11 476	89 937	29
32	78 478	88 550	11 450	89 927	28
33	78 494	88 577	11 423	89 918	27
34	78 510	88 603	11 397	89 908	26
35	78 527	88 629	11 371	89 898	25
36	78 543	88 655	11 345	89 888	24
37	78 560	88 681	11 319	89 879	23
38	78 576	88 707	11 293	89 869	22
39	78 592	88 733	11 267	89 859	21
40	78 609	88 759	11 241	89 849	20
41	78 625	88 786	11 214	89 840	19
42	78 642	88 812	11 188	89 830	18
43	78 658	88 838	11 162	89 820	17
44	78 674	88 864	11 136	89 810	16
45	78 691	88 890	11 110	89 801	15
46	78 707	88 916	11 084	89 791	14
47	78 723	88 942	11 058	89 781	13
48	78 739	88 968	11 032	89 771	12
49	78 756	88 994	11 006	89 761	11
50	78 772	89 020	10 980	89 752	10
51	78 788	89 046	10 954	89 742	9
52	78 805	89 073	10 927	89 732	8
53	78 821	89 099	10 901	89 722	7
54	78 837	89 125	10 875	89 712	6
55	78 853	89 151	10 849	89 702	5
56	78 869	89 177	10 823	89 693	4
57	78 886	89 203	10 797	89 683	3
58	78 902	89 229	10 771	89 673	2
59	78 918	89 255	10 745	89 663	1
60	78 934	89 281	10 719	89 653	0
′	log cos −9	log cot −9	log tan −10	log sin −9	′

52°

38°

′	log sin −9	log tan −9	log cot −10	log cos −9	′
0	78 934	89 281	10 719	89 653	60
1	78 950	89 307	10 693	89 643	59
2	78 967	89 333	10 667	89 633	58
3	78 983	89 359	10 641	89 624	57
4	78 999	89 385	10 615	89 614	56
5	79 015	89 411	10 589	89 604	55
6	79 031	89 437	10 563	89 594	54
7	79 047	89 463	10 537	89 584	53
8	79 063	89 489	10 511	89 574	52
9	79 079	89 515	10 485	89 564	51
10	79 095	89 541	10 459	89 554	50
11	79 111	89 567	10 433	89 544	49
12	79 128	89 593	10 407	89 534	48
13	79 144	89 619	10 381	89 524	47
14	79 160	89 645	10 355	89 514	46
15	79 176	89 671	10 329	89 504	45
16	79 192	89 697	10 303	89 495	44
17	79 208	89 723	10 277	89 485	43
18	79 224	89 749	10 251	89 475	42
19	79 240	89 775	10 225	89 465	41
20	79 256	89 801	10 199	89 455	40
21	79 272	89 827	10 173	89 445	39
22	79 288	89 853	10 147	89 435	38
23	79 304	89 879	10 121	89 425	37
24	79 319	89 905	10 095	89 415	36
25	79 335	89 931	10 069	89 405	35
26	79 351	89 957	10 043	89 395	34
27	79 367	89 983	10 017	89 385	33
28	79 383	90 009	09 991	89 375	32
29	79 399	90 035	09 965	89 364	31
30	79 415	90 061	09 939	89 354	30
31	79 431	90 086	09 914	89 344	29
32	79 447	90 112	09 888	89 334	28
33	79 463	90 138	09 862	89 324	27
34	79 478	90 164	09 836	89 314	26
35	79 494	90 190	09 810	89 304	25
36	79 510	90 216	09 784	89 294	24
37	79 526	90 242	09 758	89 284	23
38	79 542	90 268	09 732	89 274	22
39	79 558	90 294	09 706	89 264	21
40	79 573	90 320	09 680	89 254	20
41	79 589	90 346	09 654	89 244	19
42	79 605	90 371	09 629	89 233	18
43	79 621	90 397	09 603	89 223	17
44	79 636	90 423	09 577	89 213	16
45	79 652	90 449	09 551	89 203	15
46	79 668	90 475	09 525	89 193	14
47	79 684	90 501	09 499	89 183	13
48	79 699	90 527	09 473	89 173	12
49	79 715	90 553	09 447	89 162	11
50	79 731	90 578	09 422	89 152	10
51	79 746	90 604	09 396	89 142	9
52	79 762	90 630	09 370	89 132	8
53	79 778	90 656	09 344	89 122	7
54	79 793	90 682	09 318	89 112	6
55	79 809	90 708	09 292	89 101	5
56	79 825	90 734	09 266	89 091	4
57	79 840	90 759	09 241	89 081	3
58	79 856	90 785	09 215	89 071	2
59	79 872	90 811	09 189	89 060	1
60	79 887	90 837	09 163	89 050	0
′	log cos −9	log cot −9	log tan −10	log sin −9	′

51°

39°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	79 887	90 837	09 163	89 050	60
1	79 903	90 863	09 137	89 040	59
2	79 918	90 889	09 111	89 030	58
3	79 934	90 914	09 086	89 020	57
4	79 950	90 940	09 060	89 009	56
5	79 965	90 966	09 034	88 999	55
6	79 981	90 992	09 008	88 989	54
7	79 996	91 018	08 982	88 978	53
8	80 012	91 043	08 957	88 968	52
9	80 027	91 069	08 931	88 958	51
10	80 043	91 095	08 905	88 948	50
11	80 058	91 121	08 879	88 937	49
12	80 074	91 147	08 853	88 927	48
13	80 089	91 172	08 828	88 917	47
14	80 105	91 198	08 802	88 906	46
15	80 120	91 224	08 776	88 896	45
16	80 136	91 250	08 750	88 886	44
17	80 151	91 276	08 724	88 875	43
18	80 166	91 301	08 699	88 865	42
19	80 182	91 327	08 673	88 855	41
20	80 197	91 353	08 647	88 844	40
21	80 213	91 379	08 621	88 834	39
22	80 228	91 404	08 596	88 824	38
23	80 244	91 430	08 570	88 813	37
24	80 259	91 456	08 544	88 803	36
25	80 274	91 482	08 518	88 793	35
26	80 290	91 507	08 493	88 782	34
27	80 305	91 533	08 467	88 772	33
28	80 320	91 559	08 441	88 761	32
29	80 336	91 585	08 415	88 751	31
30	80 351	91 610	08 390	88 741	30
31	80 366	91 636	08 364	88 730	29
32	80 382	91 662	08 338	88 720	28
33	80 397	91 688	08 312	88 709	27
34	80 412	91 713	08 287	88 699	26
35	80 428	91 739	08 261	88 688	25
36	80 443	91 765	08 235	88 678	24
37	80 458	91 791	08 209	88 668	23
38	80 473	91 816	08 184	88 657	22
39	80 489	91 842	08 158	88 647	21
40	80 504	91 868	08 132	88 636	20
41	80 519	91 893	08 107	88 626	19
42	80 534	91 919	08 081	88 615	18
43	80 550	91 945	08 055	88 605	17
44	80 565	91 971	08 029	88 594	16
45	80 580	91 996	08 004	88 584	15
46	80 595	92 022	07 978	88 573	14
47	80 610	92 048	07 952	88 563	13
48	80 625	92 073	07 927	88 552	12
49	80 641	92 099	07 901	88 542	11
50	80 656	92 125	07 875	88 531	10
51	80 671	92 150	07 850	88 521	9
52	80 686	92 176	07 824	88 510	8
53	80 701	92 202	07 798	88 499	7
54	80 716	92 227	07 773	88 489	6
55	80 731	92 253	07 747	88 478	5
56	80 746	92 279	07 721	88 468	4
57	80 762	92 304	07 696	88 457	3
58	80 777	92 330	07 670	88 447	2
59	80 792	92 356	07 644	88 436	1
60	80 807	92 381	07 619	88 425	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

50°

40°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	80 807	92 381	07 619	88 425	60
1	80 822	92 407	07 593	88 415	59
2	80 837	92 433	07 567	88 404	58
3	80 852	92 458	07 542	88 394	57
4	80 867	92 484	07 516	88 383	56
5	80 882	92 510	07 490	88 372	55
6	80 897	92 535	07 465	88 362	54
7	80 912	92 561	07 439	88 351	53
8	80 927	92 587	07 413	88 340	52
9	80 942	92 612	07 388	88 330	51
10	80 957	92 638	07 362	88 319	50
11	80 972	92 663	07 337	88 308	49
12	80 987	92 689	07 311	88 298	48
13	81 002	92 715	07 285	88 287	47
14	81 017	92 740	07 260	88 276	46
15	81 032	92 766	07 234	88 266	45
16	81 047	92 792	07 208	88 255	44
17	81 061	92 817	07 183	88 244	43
18	81 076	92 843	07 157	88 234	42
19	81 091	92 868	07 132	88 223	41
20	81 106	92 894	07 106	88 212	40
21	81 121	92 920	07 080	88 201	39
22	81 136	92 945	07 055	88 191	38
23	81 151	92 971	07 029	88 180	37
24	81 166	92 996	07 004	88 169	36
25	81 180	93 022	06 978	88 158	35
26	81 195	93 048	06 952	88 148	34
27	81 210	93 073	06 927	88 137	33
28	81 225	93 099	06 901	88 126	32
29	81 240	93 124	06 876	88 115	31
30	81 254	93 150	06 850	88 105	30
31	81 269	93 175	06 825	88 094	29
32	81 284	93 201	06 799	88 083	28
33	81 299	93 227	06 773	88 072	27
34	81 314	93 252	06 748	88 061	26
35	81 328	93 278	06 722	88 051	25
36	81 343	93 303	06 697	88 040	24
37	81 358	93 329	06 671	88 029	23
38	81 372	93 354	06 646	88 018	22
39	81 387	93 380	06 620	88 007	21
40	81 402	93 406	06 594	87 996	20
41	81 417	93 431	06 569	87 985	19
42	81 431	93 457	06 543	87 975	18
43	81 446	93 482	06 518	87 964	17
44	81 461	93 508	06 492	87 953	16
45	81 475	93 533	06 467	87 942	15
46	81 490	93 559	06 441	87 931	14
47	81 505	93 584	06 416	87 920	13
48	81 519	93 610	06 390	87 909	12
49	81 534	93 636	06 364	87 898	11
50	81 549	93 661	06 339	87 887	10
51	81 563	93 687	06 313	87 877	9
52	81 578	93 712	06 288	87 866	8
53	81 592	93 738	06 262	87 855	7
54	81 607	93 763	06 237	87 844	6
55	81 622	93 789	06 211	87 833	5
56	81 636	93 814	06 186	87 822	4
57	81 651	93 840	06 160	87 811	3
58	81 665	93 865	06 135	87 800	2
59	81 680	93 891	06 109	87 789	1
60	81 694	93 916	06 084	87 778	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

49°

41°

′	log sin -9-	log tan -9-	log cot -10-	log cos -9-	′
0	81 694	93 916	06 084	87 778	60
1	81 709	93 942	06 058	87 767	59
2	81 723	93 967	06 033	87 756	58
3	81 738	93 993	06 007	87 745	57
4	81 752	94 018	05 982	87 734	56
5	81 767	94 044	05 956	87 723	55
6	81 781	94 069	05 931	87 712	54
7	81 796	94 095	05 905	87 701	53
8	81 810	94 120	05 880	87 690	52
9	81 825	94 146	05 854	87 679	51
10	81 839	94 171	05 829	87 668	50
11	81 854	94 197	05 803	87 657	49
12	81 868	94 222	05 778	87 646	48
13	81 882	94 248	05 752	87 635	47
14	81 897	94 273	05 727	87 624	46
15	81 911	94 299	05 701	87 613	45
16	81 926	94 324	05 676	87 601	44
17	81 940	94 350	05 650	87 590	43
18	81 955	94 375	05 625	87 579	42
19	81 969	94 401	05 599	87 568	41
20	81 983	94 426	05 574	87 557	40
21	81 998	94 452	05 548	87 546	39
22	82 012	94 477	05 523	87 535	38
23	82 026	94 503	05 497	87 524	37
24	82 041	94 528	05 472	87 513	36
25	82 055	94 554	05 446	87 501	35
26	82 069	94 579	05 421	87 490	34
27	82 084	94 604	05 396	87 479	33
28	82 098	94 630	05 370	87 468	32
29	82 112	94 655	05 345	87 457	31
30	82 126	94 681	05 319	87 446	30
31	82 141	94 706	05 294	87 434	29
32	82 155	94 732	05 268	87 423	28
33	82 169	94 757	05 243	87 412	27
34	82 184	94 783	05 217	87 401	26
35	82 198	94 808	05 192	87 390	25
36	82 212	94 834	05 166	87 378	24
37	82 226	94 859	05 141	87 367	23
38	82 240	94 884	05 116	87 356	22
39	82 255	94 910	05 090	87 345	21
40	82 269	94 935	05 065	87 334	20
41	82 283	94 961	05 039	87 322	19
42	82 297	94 986	05 014	87 311	18
43	82 311	95 012	04 988	87 300	17
44	82 326	95 037	04 963	87 288	16
45	82 340	95 062	04 938	87 277	15
46	82 354	95 088	04 912	87 266	14
47	82 368	95 113	04 887	87 255	13
48	82 382	95 139	04 861	87 243	12
49	82 396	95 164	04 836	87 232	11
50	82 410	95 190	04 810	87 221	10
51	82 424	95 215	04 785	87 209	9
52	82 439	95 240	04 760	87 198	8
53	82 453	95 266	04 734	87 187	7
54	82 467	95 291	04 709	87 175	6
55	82 481	95 317	04 683	87 164	5
56	82 495	95 342	04 658	87 153	4
57	82 509	95 368	04 632	87 141	3
58	82 523	95 393	04 607	87 130	2
59	82 537	95 418	04 582	87 119	1
60	82 551	95 444	04 556	87 107	0
′	log cos -9-	log cot -9-	log tan -10-	log sin -9-	′

48°

42°

′	log sin -9-	log tan -9-	log cot -10-	log cos -9-	′
0	82 551	95 444	04 556	87 107	60
1	82 565	95 469	04 531	87 096	59
2	82 579	95 495	04 505	87 085	58
3	82 593	95 520	04 480	87 073	57
4	82 607	95 545	04 455	87 062	56
5	82 621	95 571	04 429	87 050	55
6	82 635	95 596	04 404	87 039	54
7	82 649	95 622	04 378	87 028	53
8	82 663	95 647	04 353	87 016	52
9	82 677	95 672	04 328	87 005	51
10	82 691	95 698	04 302	86 993	50
11	82 705	95 723	04 277	86 982	49
12	82 719	95 748	04 252	86 970	48
13	82 733	95 774	04 226	86 959	47
14	82 747	95 799	04 201	86 947	46
15	82 761	95 825	04 175	86 936	45
16	82 775	95 850	04 150	86 924	44
17	82 788	95 875	04 125	86 913	43
18	82 802	95 901	04 099	86 902	42
19	82 816	95 926	04 074	86 890	41
20	82 830	95 952	04 048	86 879	40
21	82 844	95 977	04 023	86 867	39
22	82 858	96 002	03 998	86 855	38
23	82 872	96 028	03 972	86 844	37
24	82 885	96 053	03 947	86 832	36
25	82 899	96 078	03 922	86 821	35
26	82 913	96 104	03 896	86 809	34
27	82 927	96 129	03 871	86 798	33
28	82 941	96 155	03 845	86 786	32
29	82 955	96 180	03 820	86 775	31
30	82 968	96 205	03 795	86 763	30
31	82 982	96 231	03 769	86 752	29
32	82 996	96 256	03 744	86 740	28
33	83 010	96 281	03 719	86 728	27
34	83 023	96 307	03 693	86 717	26
35	83 037	96 332	03 668	86 705	25
36	83 051	96 357	03 643	86 694	24
37	83 065	96 383	03 617	86 682	23
38	83 078	96 408	03 592	86 670	22
39	83 092	96 433	03 567	86 659	21
40	83 106	96 459	03 541	86 647	20
41	83 120	96 484	03 516	86 635	19
42	83 133	96 510	03 490	86 624	18
43	83 147	96 535	03 465	86 612	17
44	83 161	96 560	03 440	86 600	16
45	83 174	96 586	03 414	86 589	15
46	83 188	96 611	03 389	86 577	14
47	83 202	96 636	03 364	86 565	13
48	83 215	96 662	03 338	86 554	12
49	83 229	96 687	03 313	86 542	11
50	83 242	96 712	03 288	86 530	10
51	83 256	96 738	03 262	86 518	9
52	83 270	96 763	03 237	86 507	8
53	83 283	96 788	03 212	86 495	7
54	83 297	96 814	03 186	86 483	6
55	83 310	96 839	03 161	86 472	5
56	83 324	96 864	03 136	86 460	4
57	83 338	96 890	03 110	86 448	3
58	83 351	96 915	03 085	86 436	2
59	83 365	96 940	03 060	86 425	1
60	83 378	96 966	03 034	86 413	0
′	log cos -9-	log cot -9-	log tan -10-	log sin -9-	′

47°

43°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	83 378	96 966	03 034	86 413	60
1	83 392	96 991	03 009	86 401	59
2	83 405	97 016	02 984	86 389	58
3	83 419	97 042	02 958	86 377	57
4	83 432	97 067	02 933	86 366	56
5	83 446	97 092	02 908	86 354	55
6	83 459	97 118	02 882	86 342	54
7	83 473	97 143	02 857	86 330	53
8	83 486	97 168	02 832	86 318	52
9	83 500	97 193	02 807	86 306	51
10	83 513	97 219	02 781	86 295	50
11	83 527	97 244	02 756	86 283	49
12	83 540	97 269	02 731	86 271	48
13	83 554	97 295	02 705	86 259	47
14	83 567	97 320	02 680	86 247	46
15	83 581	97 345	02 655	86 235	45
16	83 594	97 371	02 629	86 223	44
17	83 608	97 396	02 604	86 211	43
18	83 621	97 421	02 579	86 200	42
19	83 634	97 447	02 553	86 188	41
20	83 648	97 472	02 528	86 176	40
21	83 661	97 497	02 503	86 164	39
22	83 674	97 523	02 477	86 152	38
23	83 688	97 548	02 452	86 140	37
24	83 701	97 573	02 427	86 128	36
25	83 715	97 598	02 402	86 116	35
26	83 728	97 624	02 376	86 104	34
27	83 741	97 649	02 351	86 092	33
28	83 755	97 674	02 326	86 080	32
29	83 768	97 700	02 300	86 068	31
30	83 781	97 725	02 275	86 056	30
31	83 795	97 750	02 250	86 044	29
32	83 808	97 776	02 224	86 032	28
33	83 821	97 801	02 199	86 020	27
34	83 834	97 826	02 174	86 008	26
35	83 848	97 851	02 149	85 996	25
36	83 861	97 877	02 123	85 984	24
37	83 874	97 902	02 098	85 972	23
38	83 887	97 927	02 073	85 960	22
39	83 901	97 953	02 047	85 948	21
40	83 914	97 978	02 022	85 936	20
41	83 927	98 003	01 997	85 924	19
42	83 940	98 029	01 971	85 912	18
43	83 954	98 054	01 946	85 900	17
44	83 967	98 079	01 921	85 888	16
45	83 980	98 104	01 896	85 876	15
46	83 993	98 130	01 870	85 864	14
47	84 006	98 155	01 845	85 851	13
48	84 020	98 180	01 820	85 839	12
49	84 033	98 206	01 794	85 827	11
50	84 046	98 231	01 769	85 815	10
51	84 059	98 256	01 744	85 803	9
52	84 072	98 281	01 719	85 791	8
53	84 085	98 307	01 693	85 779	7
54	84 098	98 332	01 668	85 766	6
55	84 112	98 357	01 643	85 754	5
56	84 125	98 383	01 617	85 742	4
57	84 138	98 408	01 592	85 730	3
58	84 151	98 433	01 567	85 718	2
59	84 164	98 458	01 542	85 706	1
60	84 177	98 484	01 516	85 693	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

46°

44°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	84 177	98 484	01 516	85 693	60
1	84 190	98 509	01 491	85 681	59
2	84 203	98 534	01 466	85 669	58
3	84 216	98 560	01 440	85 657	57
4	84 229	98 585	01 415	85 645	56
5	84 242	98 610	01 390	85 632	55
6	84 255	98 635	01 365	85 620	54
7	84 269	98 661	01 339	85 608	53
8	84 282	98 686	01 314	85 596	52
9	84 295	98 711	01 289	85 583	51
10	84 308	98 737	01 263	85 571	50
11	84 321	98 762	01 238	85 559	49
12	84 334	98 787	01 213	85 547	48
13	84 347	98 812	01 188	85 534	47
14	84 360	98 838	01 162	85 522	46
15	84 373	98 863	01 137	85 510	45
16	84 385	98 888	01 112	85 497	44
17	84 398	98 913	01 087	85 485	43
18	84 411	98 939	01 061	85 473	42
19	84 424	98 964	01 036	85 460	41
20	84 437	98 989	01 011	85 448	40
21	84 450	99 015	00 985	85 436	39
22	84 463	99 040	00 960	85 423	38
23	84 476	99 065	00 935	85 411	37
24	84 489	99 090	00 910	85 399	36
25	84 502	99 116	00 884	85 386	35
26	84 515	99 141	00 859	85 374	34
27	84 528	99 166	00 834	85 361	33
28	84 540	99 191	00 809	85 349	32
29	84 553	99 217	00 783	85 337	31
30	84 566	99 242	00 758	85 324	30
31	84 579	99 267	00 733	85 312	29
32	84 592	99 293	00 707	85 299	28
33	84 605	99 318	00 682	85 287	27
34	84 618	99 343	00 657	85 274	26
35	84 630	99 368	00 632	85 262	25
36	84 643	99 394	00 606	85 250	24
37	84 656	99 419	00 581	85 237	23
38	84 669	99 444	00 556	85 225	22
39	84 682	99 469	00 531	85 212	21
40	84 694	99 495	00 505	85 200	20
41	84 707	99 520	00 480	85 187	19
42	84 720	99 545	00 455	85 175	18
43	84 733	99 570	00 430	85 162	17
44	84 745	99 596	00 404	85 150	16
45	84 758	99 621	00 379	85 137	15
46	84 771	99 646	00 354	85 125	14
47	84 784	99 672	00 328	85 112	13
48	84 796	99 697	00 303	85 100	12
49	84 809	99 722	00 278	85 087	11
50	84 822	99 747	00 253	85 074	10
51	84 835	99 773	00 227	85 062	9
52	84 847	99 798	00 202	85 049	8
53	84 860	99 823	00 177	85 037	7
54	84 873	99 848	00 152	85 024	6
55	84 885	99 874	00 126	85 012	5
56	84 898	99 899	00 101	84 999	4
57	84 911	99 924	00 076	84 986	3
58	84 923	99 949	00 051	84 974	2
59	84 936	99 975	00 025	84 961	1
60	84 949	00 000	00 000	84 949	0
	9	10	10	9	
′	log cos	log cot	log tan	log sin	′

45°

TABLE IV.

FOR DETERMINING WITH GREATER ACCURACY THAN CAN BE DONE BY MEANS OF TABLE III.:

1. *log sin*, *log tan*, and *log cot*, when the angle is between 0° and 2°;
2. *log cos*, *log tan*, and *log cot*, when the angle is between 88° and 90°;
3. The value of the angle when the logarithm of the function does *not* lie between the limits **8. 54 684** and **11. 45 316.**

FORMULAS FOR THE USE OF THE NUMBERS S AND T.

I. When the angle α is between 0° and 2°:

$$\log \sin \alpha = \log \alpha'' + S.$$
$$\log \tan \alpha = \log \alpha'' + T.$$
$$\log \cot \alpha = \operatorname{colog} \tan \alpha.$$

$$\log \alpha'' = \log \sin \alpha - S,$$
$$= \log \tan \alpha - T,$$
$$= \operatorname{colog} \cot \alpha - T.$$

II. When the angle α is between 88° and 90°:

$$\log \cos \alpha = \log (90° - \alpha)'' + S.$$
$$\log \cot \alpha = \log (90° - \alpha)'' + T.$$
$$\log \tan \alpha = \operatorname{colog} \cot \alpha.$$

$$\log (90° - \alpha)'' = \log \cos \alpha - S,$$
$$= \log \cot \alpha - T,$$
$$= \operatorname{colog} \tan \alpha - T,$$
$$\text{and } \alpha = 90° - (90° - \alpha).$$

VALUES OF S AND T.

α''	S	log sin α
0		—
	4. 68 557	
2 409		8. 06 740
	4. 68 556	
3 417		8. 21 920
	4. 68 555	
3 823		8. 26 795
	4. 68 55$\underline{5}$	
4 190		8. 30 776
	4. 68 554	
4 840		8. 37 038
	4. 68 553	
5 414		8. 41 904
	4. 68 552	
5 932		8. 45 872
	4. 68 551	
6 408		8. 49 223
	4. 68 550	
6 633		8. 50 721
	4. 68 5$\underline{5}$0	
6 851		8. 52 125
	4. 68 549	
7 267		8. 54 684
α''	S	log sin α

α''	T	log tan α	α	T	log tan α
0		—	5 146		8. 39 713
	4. 68 557			4. 68 567	
200		6. 98 660	5 424		8. 41 999
	4. 68 558			4. 68 568	
1 726		7. 92 263	5 689		8. 44 072
	4. 68 559			4. 68 569	
2 432		8. 07 156	5 941		8. 45 955
	4. 68 560			4. 68 570	
2 976		8. 15 924	6 184		8. 47 697
	4. 68 561			4. 68 571	
3 434		8. 22 142	6 417		8. 49 305
	4. 68 562			4. 68 572	
3 838		8. 26 973	6 642		8. 50 802
	4. 68 563			4. 68 573	
4 204		8. 30 930	6 859		8. 52 200
	4. 68 564			4. 68 574	
4 540		8. 34 270	7 070		8. 53 516
	4. 68 56$\underline{5}$			4. 68 57$\underline{5}$	
4 699		8. 35 766	7 173		8. 54 145
	4. 68 565			4. 68 575	
4 853		8. 37 167	7 274		8. 54 753
	4. 68 566				
5 146		8. 39 713			
α''	T	log tan α	α	T	log tan α

TABLE IV.

This table (page 50) must be used when great accuracy is desired in working with angles between 0° and 2°, or between 88° and 90°.

The values of **S** and **T** are such that when the angle a is expressed in seconds,

$$S = \log \sin a - \log a'',$$
$$T = \log \tan a - \log a''.$$

Hence, follow the formulas given on the page containing the table.

The values of **S** and **T** are printed with the characteristic 10 too large, and in using them -10 must always be annexed.

Find log sin 0° 58′ 17″.

$$\begin{aligned} 0^\circ\, 58'\, 17'' &= 3497.'' \\ \log 3497 &= 3.54370 \\ S &= \underline{4.68555 - 10} \\ \log \sin 0^\circ\, 58'\, 17'' &= 8.22925 - 10 \end{aligned}$$

Find log cos 88° 26′ 41.2″.

$$\begin{aligned} 90^\circ - 88^\circ\, 26'\, 41.2'' &= 1^\circ\, 33'\, 18.8'' \\ &= 5598.8.'' \\ \log 5598.8 &= 3.74809 \\ S &= \underline{4.68552 - 10} \\ \log \cos 88^\circ\, 26'\, 41.2'' &= 8.43361 - 10 \end{aligned}$$

Find log tan 0° 52′ 47.5″.

$$\begin{aligned} 0^\circ\, 52'\, 47.5'' &= 3167.5.'' \\ \log 3167.5 &= 3.50072 \\ T &= \underline{4.68561 - 10} \\ \log \tan 0^\circ\, 52'\, 47.5'' &= 8.18633 - 10 \end{aligned}$$

Find log tan 89° 54′ 37.362″.

$$\begin{aligned} 90^\circ - 89^\circ\, 54'\, 37.362'' &= 322.638''. \\ \log 322.638 &= 2.50871 \\ T &= \underline{5.68558 - 10} \\ \log \cot 89^\circ\, 54'\, 37.362'' &= 7.19429 - 10 \\ \log \tan 89^\circ\, 54'\, 37.362'' &= 2.80571. \end{aligned}$$

Find the angle, if log sin = 6.72306 − 10.

$$\begin{aligned} & 6.72306 - 10 \\ S = {} & \underline{4.68557 - 10} \\ \text{Subtract,}\quad & 2.03749 \quad = \log 109.015. \\ & 109.015'' = 0^\circ\, 1'\, 49.015''. \end{aligned}$$

Find the angle for which log cot = 1.67604.

$$\begin{aligned} \text{colog cot} = {} & 8.32396 - 10 \\ T = {} & \underline{4.68564 - 10} \\ \text{Subtract,}\quad & 3.63832 \quad = \log 4348.3. \\ & 4348.3'' = 1^\circ\, 12'\, 28.3''. \end{aligned}$$

Find the angle for which log tan = 1.55407.

$$\begin{aligned} \text{colog tan} = {} & 8.44593 - 10 \\ T = {} & \underline{4.68569 - 10} \\ \text{Subtract,}\quad & 3.76024 \quad = \log 5757.6. \\ & 5757.6'' = 1^\circ\, 35'\, 57.6'', \\ \text{and}\quad & 90^\circ - 1^\circ\, 35'\, 57.6'' = 88^\circ\, 24'\, 2.4'' = \text{angle required.} \end{aligned}$$

TABLE V.

SHOWING LENGTHS IN NAUTICAL MILES AND STATUTE MILES OF DEGREES OF LATITUDE AND LONGITUDE IN DIFFERENT LATITUDES.

Degree of the Parallel.			Degree of the Meridian.		
Latitude of Parallel.	Nautical Miles.	Statute Miles.	Latitude of Middle Point.	Nautical Miles.	Statute Miles.
20°	56.404	65.018	20°	59.664	68.777
21°	56.039	64.598			
22°	55.657	64.158			
23°	55.258	63.698			
24°	54.843	63.219			
25°	54.411	62.721	25°	59.706	68.825
26°	53.962	62.204			
27°	53.497	61.668			
28°	53.016	61.113			
29°	52.518	60.540			
30°	52.005	59.948	30°	59.749	68.875
31°	51.476	59.338			
32°	50.931	58.709			
33°	50.370	58.063			
34°	49.794	57.399			
35°	49.203	56.718	35°	59.796	68.929
36°	48.597	56.019			
37°	47.976	55.304			
38°	47.341	54.571			
39°	46.960	53.822			
40°	46.026	53.056	40°	59.847	68.987
41°	45.348	52.274			
42°	44.654	51.476			
43°	43.949	50.662			
44°	43.230	49.833			
45°	42.497	48.988	45°	59.899	69.048
46°	41.752	48.128			
47°	40.993	47.254			
48°	40.222	46.365			
49°	39.439	45.462			
50°	38.643	44.545	50°	59.951	69.108

TABLE VI.

MISCELLANEOUS FORMULÆ, AND EQUIVALENTS OF METRES, CHAINS, AND FEET.

	Logarithm.		Logarithm.
π . . . $= 3.14159265$	0.4971499	$\sqrt{\pi}$ . . $= 1.77245385$	0.2485749
$\frac{1}{\pi}$ . . . $= 0.31830989$	9.5028501 − 10	$\frac{1}{\sqrt{\pi}}$ . . $= 0.56418958$	9.7514251 − 10
π^2 . . . $= 9.86960440$	0.9942997	$\sqrt[3]{\pi}$ . . $= 1.46459189$	0.1657166
$\frac{1}{\pi^2}$ . . . $= 0.10132118$	9.0057006 − 10		

		Logarithm.
Circumference of circle, diameter being unity, Area of circle, radius being unity . . . . Surface of sphere, diameter being unity	$= \pi = 3.14159265$	0.4971499
Area of a circle, diameter being unity	$= \frac{\pi}{4} = 0.7853982$	9.8950899 − 10
Volume of sphere, diameter being unity	$= \frac{\pi}{6} = 0.52359878$	9.7189986 − 10
Volume of sphere, radius being unity	$= \frac{4\pi}{3} = 4.1887902$	0.6220886
Arc whose length is equal to the radius:		
Expressed in degrees	$= \frac{180}{\pi} = 57.2957795°$	1.7581226
Expressed in minutes	$= \frac{10800}{\pi} = 3437.74677'$	3.5362739
Expressed in seconds	$= \frac{648000}{\pi} = 206264.806$	5.3144251
If radius is unity:		
Length of arc for one degree	$= \frac{\pi}{180} = 0.0174533$	8.2418774 − 10
Length of arc for one minute	$= \frac{\pi}{10800} = 0.0002909$	6.4637261 − 10
Length of arc for one second	$= \frac{\pi}{648000} = 0.00000485$	4.68557487 − 10
Sine of one second	$= 0.00000485$	4.68557487 − 10
Base of Hyperbolic or Napier's System of Logarithms .	$= 2.7182818$	0.4342945
Modulus of Common or Briggs' System of Logarithms .	$= 0.4342945$	9.6377843 − 10
Equatorial radius of the earth in feet	$= 20923600$	
Polar radius of the earth in feet	$= 20853657$	
Length of degree of latitude at the equator, in feet . .	$= 362748.33$	
Length of degree of latitude at 45°, in feet	$= 364571.77$	

FEET.	METRES.	CHAINS.	METRES.	FEET.
1	0.3048	0.0151	1	3.2809
2	0.6096	0.0303	2	6.5617
3	0.9144	0.0455	3	9.8426
4	1.2192	0.0606	4	13.1235
5	1.5240	0.0758	5	16.4044
6	1.8288	0.0909	6	19.6852
7	2.1336	0.1061	7	22.9661
8	2.4384	0.1212	8	26.2470
9	2.7432	0.1364	9	29.5278
10	3.0480	0.1515	10	32.8087

TABLE VII.—TRAVERSE TABLE.

This table gives the latitude and departure to three places of decimals for distances from 1 to 10, corresponding to bearings from 0° to 90°, at intervals of 15′.

If the bearing does not exceed 45°, it is found in the *left-hand* column, and the designations of the columns under "Distance" are taken from the *top* of the page; but if the bearing exceeds 45°, it is found in the *right-hand* column, and the designations of the columns under "Distance" are taken from the *bottom* of the page.

The method of using the table will be made plain by the following examples:

1. Let it be required to find the latitude and departure of a line running N. 35° 15′ E. 6 chains.

On page 60, left-hand column, look for 35° 15′; opposite this bearing, in the vertical column headed "Distance 6," are found 4.900 and 3.463, under the headings "Latitude" and "Departure" respectively. Hence latitude, or northing, = 4.900 chains, and departure, or easting, = 3.463 chains.

2. Let it be required to find the latitude and departure of a line running S. 87° W. 2 chains.

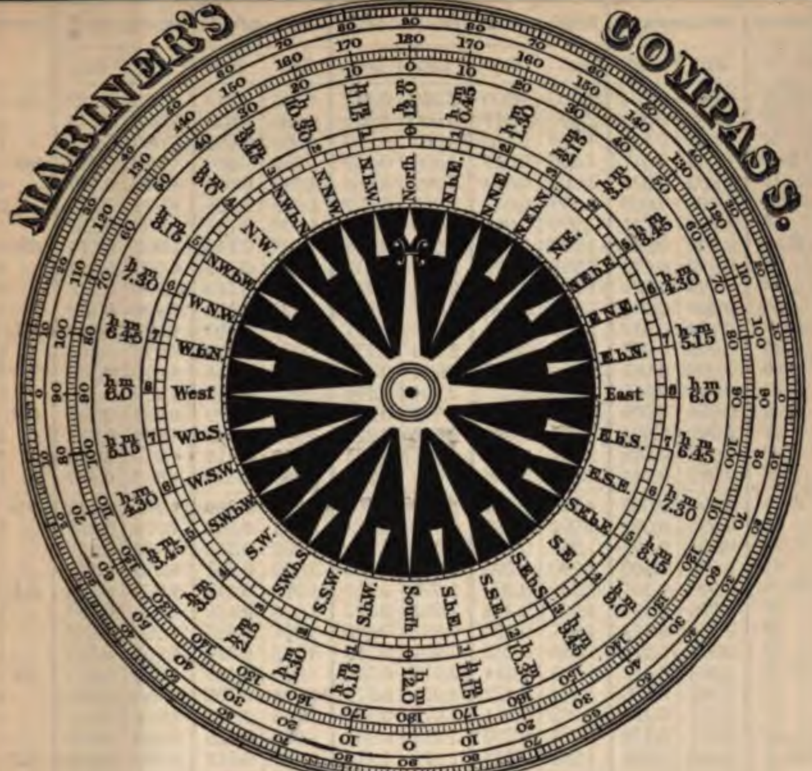
As the bearing exceeds 45°, we look in the right-hand column on page 55, and opposite 87°, in the column marked "Distance 2," we find (taking the designations of the columns from the bottom of the page) latitude = 0.105 chains, and departure = 1.997 chains. Hence latitude, or southing, = 0.105 chains, and departure, or westing, = 1.997 chains.

3. Let it be required to find the latitude and departure of a line running N. 15° 45′ W. 27.36 chains.

In this case, we find the required number for each figure of the distance separately, arranging the work as in the following table. In practice, only the last columns under "Latitude" and "Departure" are written.

Distance.	Latitude.	Departure.
20 $= 2 \times 10$	$1.925 \times 10 = 19.25$	$0.543 \times 10 = 5.43$
7	6.737	1.90
0.3 $= 3 \div 10$	$2.887 \div 10 = 0.289$	$0.814 \div 10 = 0.081$
0.06 $= 6 \div 100$	$5.775 \div 100 = 0.058$	$1.628 \div 100 = 0.016$
27.36	26.334	7.427

Hence latitude = 26.334 chains, and departure = 7.427 chains.



A TABLE OF THE ANGLES

Which every Point and Quarter Point of the Compass makes with the Meridian.

North.		Points.	° ′ ″	Points.	South.	
		0–¼	2 48 45	0–¼		
		0–½	5 37 30	0–½		
		0–¾	8 26 15	0–¾		
N. by E.	N. by W.	1	11 15 0	1	S. by E.	S. by W.
		1–¼	14 3 45	1–¼		
		1–½	16 52 30	1–½		
		1–¾	19 41 15	1–¾		
N.N.E.	N.N.W.	2	22 30 0	2	S.S.E.	S.S.W.
		2–¼	25 18 45	2–¼		
		2–½	28 7 30	2–½		
		2–¾	30 56 15	2–¾		
N.E. by N.	N.W. by N.	3	33 45 0	3	S.E. by S.	S.W. by S.
		3–¼	36 33 45	3–¼		
		3–½	39 22 30	3–½		
		3–¾	42 11 15	3–¾		
N.E.	N.W.	4	45 0 0	4	S.E.	S.W.
		4–¼	47 48 45	4–¼		
		4–½	50 37 30	4–½		
		4–¾	53 26 15	4–¾		
N.E. by E.	N.W. by W.	5	56 15 0	5	S.E. by E.	S.W. by W.
		5–¼	59 3 45	5–¼		
		5–½	61 52 30	5–½		
		5–¾	64 41 15	5–¾		
E.N.E.	W.N.W.	6	67 30 0	6	E.S.E.	W.S.W.
		6–¼	70 18 45	6–¼		
		6–½	73 7 30	6–½		
		6–¾	75 56 15	6–¾		
E. by N.	W. by N.	7	78 45 0	7	E. by S.	W. by S.
		7–¼	81 33 45	7–¼		
		7–½	84 22 30	7–½		
		7–¾	87 11 15	7–¾		
East.	West.	8	90 0 0	8	East.	West.

TABLE VII.—TRAVERSE TABLE.

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	1.000	0.004	2.000	0.009	3.000	0.013	4.000	0.017	5.000	0.022	89 45
30	1.000	0.009	2.000	0.017	3.000	0.026	4.000	0.035	5.000	0.044	30
45	1.000	0.013	2.000	0.026	3.000	0.039	4.000	0.052	5.000	0.065	15
1 0	1.000	0.017	2.000	0.035	3.000	0.052	3.999	0.070	4.999	0.087	89 0
15	1.000	0.022	2.000	0.044	2.999	0.065	3.999	0.087	4.999	0.109	45
30	1.000	0.026	1.999	0.052	2.999	0.079	3.999	0.105	4.998	0.131	30
45	1.000	0.031	1.999	0.061	2.999	0.092	3.998	0.122	4.998	0.153	15
2 0	0.999	0.035	1.999	0.070	2.998	0.105	3.998	0.140	4.997	0.174	88 0
15	0.999	0.039	1.998	0.079	2.998	0.118	3.997	0.157	4.996	0.196	45
30	0.999	0.044	1.998	0.087	2.997	0.131	3.996	0.174	4.995	0.218	30
45	0.999	0.048	1.998	0.096	2.997	0.144	3.995	0.192	4.994	0.240	15
3 0	0.999	0.052	1.997	0.105	2.996	0.157	3.995	0.209	4.993	0.262	87 0
15	0.998	0.057	1.997	0.113	2.995	0.170	3.994	0.227	4.992	0.283	45
30	0.998	0.061	1.996	0.122	2.994	0.183	3.993	0.244	4.991	0.305	30
45	0.998	0.065	1.996	0.131	2.994	0.196	3.991	0.262	4.989	0.327	15
4 0	0.998	0.070	1.995	0.140	2.993	0.209	3.990	0.279	4.988	0.349	86 0
15	0.997	0.074	1.995	0.148	2.992	0.222	3.989	0.296	4.986	0.371	45
30	0.997	0.078	1.994	0.157	2.991	0.235	3.988	0.314	4.985	0.392	30
45	0.997	0.083	1.993	0.166	2.990	0.248	3.986	0.331	4.983	0.414	15
5 0	0.996	0.087	1.992	0.174	2.989	0.261	3.985	0.349	4.981	0.436	85 0
15	0.996	0.092	1.992	0.183	2.987	0.275	3.983	0.366	4.979	0.458	45
30	0.995	0.096	1.991	0.192	2.986	0.288	3.982	0.383	4.977	0.479	30
45	0.995	0.100	1.990	0.200	2.985	0.301	3.980	0.401	4.975	0.501	15
6 0	0.995	0.105	1.989	0.209	2.984	0.314	3.978	0.418	4.973	0.523	84 0
15	0.994	0.109	1.988	0.218	2.982	0.327	3.976	0.435	4.970	0.544	45
30	0.994	0.113	1.987	0.226	2.981	0.340	3.974	0.453	4.968	0.566	30
45	0.993	0.118	1.986	0.235	2.979	0.353	3.972	0.470	4.965	0.588	15
7 0	0.993	0.122	1.985	0.244	2.978	0.366	3.970	0.487	4.963	0.609	83 0
15	0.992	0.126	1.984	0.252	2.976	0.379	3.968	0.505	4.960	0.631	45
30	0.991	0.131	1.983	0.261	2.974	0.392	3.966	0.522	4.957	0.653	30
45	0.991	0.135	1.982	0.270	2.973	0.405	3.963	0.539	4.954	0.674	15
8 0	0.990	0.139	1.981	0.278	2.971	0.418	3.961	0.557	4.951	0.696	82 0
15	0.990	0.143	1.979	0.287	2.969	0.430	3.959	0.574	4.948	0.717	45
30	0.989	0.148	1.978	0.296	2.967	0.443	3.956	0.591	4.945	0.739	30
45	0.988	0.152	1.977	0.304	2.965	0.456	3.953	0.608	4.942	0.761	15
9 0	0.988	0.156	1.975	0.313	2.963	0.469	3.951	0.626	4.938	0.782	81 0
15	0.987	0.161	1.974	0.321	2.961	0.482	3.948	0.643	4.935	0.804	45
30	0.986	0.165	1.973	0.330	2.959	0.495	3.945	0.660	4.931	0.825	30
45	0.986	0.169	1.971	0.339	2.957	0.508	3.942	0.677	4.928	0.847	15
10 0	0.985	0.174	1.970	0.347	2.954	0.521	3.939	0.695	4.924	0.868	80 0
15	0.984	0.178	1.968	0.356	2.952	0.534	3.936	0.712	4.920	0.890	45
30	0.983	0.182	1.967	0.364	2.950	0.547	3.933	0.729	4.916	0.911	30
45	0.982	0.187	1.965	0.373	2.947	0.560	3.930	0.746	4.912	0.933	15
11 0	0.982	0.191	1.963	0.382	2.945	0.572	3.927	0.763	4.908	0.954	79 0
15	0.981	0.195	1.962	0.390	2.942	0.585	3.923	0.780	4.904	0.975	45
30	0.980	0.199	1.960	0.399	2.940	0.598	3.920	0.797	4.900	0.997	30
45	0.979	0.204	1.958	0.407	2.937	0.611	3.916	0.815	4.895	1.018	15
12 0	0.978	0.208	1.956	0.416	2.934	0.624	3.913	0.832	4.891	1.040	78 0
15	0.977	0.212	1.954	0.424	2.932	0.637	3.909	0.849	4.886	1.061	45
30	0.976	0.216	1.953	0.433	2.929	0.649	3.905	0.866	4.881	1.082	30
45	0.975	0.221	1.951	0.441	2.926	0.662	3.901	0.883	4.877	1.103	15
13 0	0.974	0.225	1.949	0.450	2.923	0.675	3.897	0.900	4.872	1.125	77 0
15	0.973	0.229	1.947	0.458	2.920	0.688	3.894	0.917	4.867	1.146	45
30	0.972	0.233	1.945	0.467	2.917	0.700	3.889	0.934	4.862	1.167	30
45	0.971	0.238	1.943	0.475	2.914	0.713	3.885	0.951	4.857	1.188	15
14 0	0.970	0.242	1.941	0.484	2.911	0.726	3.881	0.968	4.851	1.210	76 0
15	0.969	0.246	1.938	0.492	2.908	0.738	3.877	0.985	4.846	1.231	45
30	0.968	0.250	1.936	0.501	2.904	0.751	3.873	1.002	4.841	1.252	30
45	0.967	0.255	1.934	0.509	2.901	0.764	3.868	1.018	4.835	1.273	15
15 0	0.966	0.259	1.932	0.518	2.898	0.776	3.864	1.035	4.830	1.294	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.

75° — 90°

Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	6.000	0.026	7.000	0.031	8.000	0.035	9.000	0.039	10.000	0.044	89 45
30	6.000	0.052	7.000	0.061	8.000	0.070	9.000	0.079	10.000	0.087	30
45	5.999	0.079	6.999	0.092	7.999	0.105	8.999	0.118	9.999	0.131	15
1 0	5.999	0.105	6.999	0.122	7.999	0.140	8.999	0.157	9.999	0.175	89 0
15	5.999	0.131	6.998	0.153	7.998	0.175	8.998	0.196	9.998	0.218	45
30	5.998	0.157	6.998	0.183	7.997	0.209	8.997	0.236	9.997	0.262	30
45	5.997	0.183	6.997	0.214	7.996	0.244	8.996	0.275	9.995	0.305	15
2 0	5.996	0.209	6.996	0.244	7.995	0.279	8.995	0.314	9.994	0.349	88 0
15	5.995	0.236	6.995	0.275	7.994	0.314	8.993	0.353	9.992	0.393	45
30	5.994	0.262	6.993	0.305	7.992	0.349	8.991	0.393	9.991	0.436	30
45	5.993	0.288	6.992	0.336	7.991	0.384	8.990	0.432	9.989	0.480	15
3 0	5.992	0.314	6.990	0.366	7.989	0.419	8.988	0.471	9.986	0.523	87 0
15	5.990	0.340	6.989	0.397	7.987	0.454	8.986	0.510	9.984	0.567	45
30	5.989	0.366	6.987	0.427	7.985	0.488	8.983	0·549	9.981	0.611	30
45	5.987	0.392	6.985	0.458	7.983	0.523	8.981	0.589	9.979	0.654	15
4 0	5.985	0.419	6.983	0.488	7.981	0.558	8.978	0.628	9.976	0.698	86 0
15	5.984	0.445	6.981	0.519	7.978	0.593	8.975	0.667	9.973	0.741	45
30	5.982	0.471	6.978	0.549	7.975	0.628	8.972	0.706	9.969	0.785	30
45	5.979	0.497	6.976	0.580	7.973	0.662	8.969	0.745	9.966	0.828	15
5 0	5.977	0.523	6.973	0.610	7.970	0.697	8.966	0.784	9.962	0.872	85 0
15	5.975	0.549	6.971	0.641	7.966	0.732	8.962	0.824	9.958	0.915	45
30	5.972	0.575	6.968	0.671	7.963	0.767	8.959	0.863	9.954	0.959	30
45	5.970	0.601	6.965	0.701	7.960	0.802	8.955	0.902	9.950	1.002	15
6 0	5.967	0.627	6.962	0.732	7.956	0.836	8.951	0.941	9.945	1.045	84 0
15	5.964	0.653	6.958	0.762	7.952	0.871	8.947	0.980	9.941	1.089	45
30	5.961	0.679	6.955	0.792	7.949	0.906	8.942	1.019	9.936	1.132	30
45	5.958	0.705	6.951	0.823	7.945	0.940	8.938	1.058	9.931	1.175	15
7 0	5.955	0.731	6.948	0.853	7.940	0.975	8.933	1.097	9.926	1.219	83 0
15	5.952	0.757	6.944	0.883	7.936	1.010	8.928	1.136	9.920	1.262	45
30	5.949	0.783	6.940	0.914	7.932	1.044	8.923	1.175	9.914	1.305	30
45	5.945	0.809	6.936	0.944	7.927	1.079	8.918	1.214	9.909	1.349	15
8 0	5.942	0.835	6.932	0.974	7.922	1.113	8.912	1.253	9.903	1.392	82 0
15	5.938	0.861	6.928	1.004	7.917	1.148	8.907	1.291	9.897	1.435	45
30	5.934	0.887	6.923	1.035	7.912	1.182	8.901	1.330	9.890	1.478	30
45	5.930	0.913	6.919	1.065	7.907	1.217	8.895	1.369	9.884	1.521	15
9 0	5.926	0.939	6.914	1.095	7.902	1.251	8.889	1.408	9.877	1.564	81 0
15	5.922	0.964	6.909	1.125	7.896	1.286	8.883	1.447	9.870	1.607	45
30	5.918	0.990	6.904	1.155	7.890	1.320	8.877	1.485	9.863	1.651	30
45	5.913	1.016	6.899	1.185	7.884	1.355	8.870	1.524	9.856	1.694	15
10 0	5.909	1.042	6.894	1.216	7.878	1.389	8.863	1.563	9.848	1.737	80 0
15	5.904	1.068	6.888	1.246	7.872	1.424	8.856	1.601	9.840	1.779	45
30	5.900	1.093	6.883	1.276	7.866	1.458	8.849	1.640	9.833	1.822	30
45	5.895	1.119	6.877	1.306	7.860	1.492	8.842	1.679	9.825	1.865	15
11 0	5.890	1.145	6.871	1.336	7.853	1.526	8.835	1.717	9.816	1.908	79 0
15	5.885	1.171	6.866	1.366	7.846	1.561	8.827	1.756	9.808	1.951	45
30	5.880	1.196	6.859	1.396	7.839	1.595	8.819	1.794	9.799	1.994	30
45	5.874	1.222	6.853	1.425	7.832	1.629	8.811	1.833	9.791	2.036	15
12 0	5.869	1.247	6.847	1.455	7.825	1.663	8.803	1.871	9.782	2.079	78 0
15	5.863	1.273	6.841	1.485	7.818	1.697	8.795	1.910	9.772	2.122	45
30	5.858	1.299	6.834	1.515	7.810	1.732	8.787	1.948	9.763	2.164	30
45	5.852	1.324	6.827	1.545	7.803	1.766	8.778	1.986	9.753	2.207	15
13 0	5.846	1.350	6.821	1.575	7.795	1.800	8.769	2.025	9.744	2.250	77 0
15	5.840	1.375	6.814	1.604	7.787	1.834	8.760	2.063	9.734	2.292	45
30	5.834	1.401	6.807	1.634	7.779	1.868	8.751	2.101	9.724	2.335	30
45	5.828	1.426	6.799	1.664	7.771	1.902	8.742	2.139	9.713	2.377	15
14 0	5.822	1.452	6.792	1.693	7.762	1.935	8.733	2.177	9.703	2.419	76 0
15	5.815	1.477	6.785	1.723	7.754	1.969	8.723	2.215	9.692	2.462	45
30	5.809	1.502	6.777	1.753	7.745	2.003	8.713	2.253	9.682	2.504	30
45	5.802	1.528	6.769	1.782	7.736	2.037	8.703	2.291	9.671	2.546	15
15 0	5.796	1.553	6.761	1.812	7.727	2.071	8.693	2.329	9.659	2.588	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	1.000	0.004	2.000	0.009	3.000	0.013	4.000	0.017	5.000	0.022	89 45
30	1.000	0.009	2.000	0.017	3.000	0.026	4.000	0.035	5.000	0.044	30
45	1.000	0.013	2.000	0.026	3.000	0.039	4.000	0.052	5.000	0.065	15
1 0	1.000	0.017	2.000	0.035	3.000	0.052	3.999	0.070	4.999	0.087	89 0
15	1.000	0.022	2.000	0.044	2.999	0.065	3.999	0.087	4.999	0.109	45
30	1.000	0.026	1.999	0.052	2.999	0.079	3.999	0.105	4.998	0.131	30
45	1.000	0.031	1.999	0.061	2.999	0.092	3.998	0.122	4.998	0.153	15
2 0	0.999	0.035	1.999	0.070	2.998	0.105	3.998	0.140	4.997	0.174	88 0
15	0.999	0.039	1.998	0.079	2.998	0.118	3.997	0.157	4.996	0.196	45
30	0.999	0.044	1.998	0.087	2.997	0.131	3.996	0.174	4.995	0.218	30
45	0.999	0.048	1.998	0.096	2.997	0.144	3.995	0.192	4.994	0.240	15
3 0	0.999	0.052	1.997	0.105	2.996	0.157	3.995	0.209	4.993	0.262	87 0
15	0.998	0.057	1.997	0.113	2.995	0.170	3.994	0.227	4.992	0.283	45
30	0.998	0.061	1.996	0.122	2.994	0.183	3.993	0.244	4.991	0.305	30
45	0.998	0.065	1.996	0.131	2.994	0.196	3.991	0.262	4.989	0.327	15
4 0	0.998	0.070	1.995	0.140	2.993	0.209	3.990	0.279	4.988	0.349	86 0
15	0.997	0.074	1.995	0.148	2.992	0.222	3.989	0.296	4.986	0.371	45
30	0.997	0.078	1.994	0.157	2.991	0.235	3.988	0.314	4.985	0.392	30
45	0.997	0.083	1.993	0.166	2.990	0.248	3.986	0.331	4.983	0.414	15
5 0	0.996	0.087	1.992	0.174	2.989	0.261	3.985	0.349	4.981	0.436	85 0
15	0.996	0.092	1.992	0.183	2.987	0.275	3.983	0.366	4.979	0.458	45
30	0.995	0.096	1.991	0.192	2.986	0.288	3.982	0.383	4.977	0.479	30
45	0.995	0.100	1.990	0.200	2.985	0.301	3.980	0.401	4.975	0.501	15
6 0	0.995	0.105	1.989	0.209	2.984	0.314	3.978	0.418	4.973	0.523	84 0
15	0.994	0.109	1.988	0.218	2.982	0.327	3.976	0.435	4.970	0.544	45
30	0.994	0.113	1.987	0.226	2.981	0.340	3.974	0.453	4.968	0.566	30
45	0.993	0.118	1.986	0.235	2.979	0.353	3.972	0.470	4.965	0.588	15
7 0	0.993	0.122	1.985	0.244	2.978	0.366	3.970	0.487	4.963	0.609	83 0
15	0.992	0.126	1.984	0.252	2.976	0.379	3.968	0.505	4.960	0.631	45
30	0.991	0.131	1.983	0.261	2.974	0.392	3.966	0.522	4.957	0.653	30
45	0.991	0.135	1.982	0.270	2.973	0.405	3.963	0.539	4.954	0.674	15
8 0	0.990	0.139	1.981	0.278	2.971	0.418	3.961	0.557	4.951	0.696	82 0
15	0.990	0.143	1.979	0.287	2.969	0.430	3.959	0.574	4.948	0.717	45
30	0.989	0.148	1.978	0.296	2.967	0.443	3.956	0.591	4.945	0.739	30
45	0.988	0.152	1.977	0.304	2.965	0.456	3.953	0.608	4.942	0.761	15
9 0	0.988	0.156	1.975	0.313	2.963	0.469	3.951	0.626	4.938	0.782	81 0
15	0.987	0.161	1.974	0.321	2.961	0.482	3.948	0.643	4.935	0.804	45
30	0.986	0.165	1.973	0.330	2.959	0.495	3.945	0.660	4.931	0.825	30
45	0.986	0.169	1.971	0.339	2.957	0.508	3.942	0.677	4.928	0.847	15
10 0	0.985	0.174	1.970	0.347	2.954	0.521	3.939	0.695	4.924	0.868	80 0
15	0.984	0.178	1.968	0.356	2.952	0.534	3.936	0.712	4.920	0.890	45
30	0.983	0.182	1.967	0.364	2.950	0.547	3.933	0.729	4.916	0.911	30
45	0.982	0.187	1.965	0.373	2.947	0.560	3.930	0.746	4.912	0.933	15
11 0	0.982	0.191	1.963	0.382	2.945	0.572	3.927	0.763	4.908	0.954	79 0
15	0.981	0.195	1.962	0.390	2.942	0.585	3.923	0.780	4.904	0.975	45
30	0.980	0.199	1.960	0.399	2.940	0.598	3.920	0.797	4.900	0.997	30
45	0.979	0.204	1.958	0.407	2.937	0.611	3.916	0.815	4.895	1.018	15
12 0	0.978	0.208	1.956	0.416	2.934	0.624	3.913	0.832	4.891	1.040	78 0
15	0.977	0.212	1.954	0.424	2.932	0.637	3.909	0.849	4.886	1.061	45
30	0.976	0.216	1.953	0.433	2.929	0.649	3.905	0.866	4.881	1.082	30
45	0.975	0.221	1.951	0.441	2.926	0.662	3.901	0.883	4.877	1.103	15
13 0	0.974	0.225	1.949	0.450	2.923	0.675	3.897	0.900	4.872	1.125	77 0
15	0.973	0.229	1.947	0.458	2.920	0.688	3.894	0.917	4.867	1.146	45
30	0.972	0.233	1.945	0.467	2.917	0.700	3.889	0.934	4.862	1.167	30
45	0.971	0.238	1.943	0.475	2.914	0.713	3.885	0.951	4.857	1.188	15
14 0	0.970	0.242	1.941	0.484	2.911	0.726	3.881	0.968	4.851	1.210	76 0
15	0.969	0.246	1.938	0.492	2.908	0.738	3.877	0.985	4.846	1.231	45
30	0.968	0.250	1.936	0.501	2.904	0.751	3.873	1.002	4.841	1.252	30
45	0.967	0.255	1.934	0.509	2.901	0.764	3.868	1.018	4.835	1.273	15
15 0	0.966	0.259	1.932	0.518	2.898	0.776	3.864	1.035	4.830	1.294	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.

Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	6.000	0.026	7.000	0.031	8.000	0.035	9.000	0.039	10.000	0.044	89 45
30	6.000	0.052	7.000	0.061	8.000	0.070	9.000	0.079	10.000	0.087	30
45	5.999	0.079	6.999	0.092	7.999	0.105	8.999	0.118	9.999	0.131	15
1 0	5.999	0.105	6.999	0.122	7.999	0.140	8.999	0.157	9.999	0.175	89 0
15	5.999	0.131	6.998	0.153	7.998	0.175	8.998	0.196	9.998	0.218	45
30	5.998	0.157	6.998	0.183	7.997	0.209	8.997	0.236	9.997	0.262	30
45	5.997	0.183	6.997	0.214	7.996	0.244	8.996	0.275	9.995	0.305	15
2 0	5.996	0.209	6.996	0.244	7.995	0.279	8.995	0.314	9.994	0.349	88 0
15	5.995	0.236	6.995	0.275	7.994	0.314	8.993	0.353	9.992	0.393	45
30	5.994	0.262	6.993	0.305	7.992	0.349	8.991	0.393	9.991	0.436	30
45	5.993	0.288	6.992	0.336	7.991	0.384	8.990	0.432	9.989	0.480	15
3 0	5.992	0.314	6.990	0.366	7.989	0.419	8.988	0.471	9.986	0.523	87 0
15	5.990	0.340	6.989	0.397	7.987	0.454	8.986	0.510	9.984	0.567	45
30	5.989	0.366	6.987	0.427	7.985	0.488	8.983	0·549	9.981	0.611	30
45	5.987	0.392	6.985	0.458	7.983	0.523	8.981	0.589	9.979	0.654	15
4 0	5.985	0.419	6.983	0.488	7.981	0.558	8.978	0.628	9.976	0.698	86 0
15	5.984	0.445	6.981	0.519	7.978	0.593	8.975	0.667	9.973	0.741	45
30	5.982	0.471	6.978	0.549	7.975	0.628	8.972	0.706	9.969	0.785	30
45	5.979	0.497	6.976	0.580	7.973	0.662	8.969	0.745	9.966	0.828	15
5 0	5.977	0.523	6.973	0.610	7.970	0.697	8.966	0.784	9.962	0.872	85 0
15	5.975	0.549	6.971	0.641	7.966	0.732	8.962	0.824	9.958	0.915	45
30	5.972	0.575	6.968	0.671	7.963	0.767	8.959	0.863	9.954	0.959	30
45	5.970	0.601	6.965	0.701	7.960	0.802	8.955	0.902	9.950	1.002	15
6 0	5.967	0.627	6.962	0.732	7.956	0.836	8.951	0.941	9.945	1.045	84 0
15	5.964	0.653	6.958	0.762	7.952	0.871	8.947	0.980	9.941	1.089	45
30	5.961	0.679	6.955	0.792	7.949	0.906	8.942	1.019	9.936	1.132	30
45	5.958	0.705	6.951	0.823	7.945	0.940	8.938	1.058	9.931	1.175	15
7 0	5.955	0.731	6.948	0.853	7.940	0.975	8.933	1.097	9.926	1.219	83 0
15	5.952	0.757	6.944	0.883	7.936	1.010	8.928	1.136	9.920	1.262	45
30	5.949	0.783	6.940	0.914	7.932	1.044	8.923	1.175	9.914	1.305	30
45	5.945	0.809	6.936	0.944	7.927	1.079	8.918	1.214	9.909	1.349	15
8 0	5.942	0.835	6.932	0.974	7.922	1.113	8.912	1.253	9.903	1.392	82 0
15	5.938	0.861	6.928	1.004	7.917	1.148	8.907	1.291	9.897	1.435	45
30	5.934	0.887	6.923	1.035	7.912	1.182	8.901	1.330	9.890	1.478	30
45	5.930	0.913	6.919	1.065	7.907	1.217	8.895	1.369	9.884	1.521	15
9 0	5.926	0.939	6.914	1.095	7.902	1.251	8.889	1.408	9.877	1.564	81 0
15	5.922	0.964	6.909	1.125	7.896	1.286	8.883	1.447	9.870	1.607	45
30	5.918	0.990	6.904	1.155	7.890	1.320	8.877	1.485	9.863	1.651	30
45	5.913	1.016	6.899	1.185	7.884	1.355	8.870	1.524	9.856	1.694	15
10 0	5.909	1.042	6.894	1.216	7.878	1.389	8.863	1.563	9.848	1.737	80 0
15	5.904	1.068	6.888	1.246	7.872	1.424	8.856	1.601	9.840	1.779	45
30	5.900	1.093	6.883	1.276	7.866	1.458	8.849	1.640	9.833	1.822	30
45	5.895	1.119	6.877	1.306	7.860	1.492	8.842	1.679	9.825	1.865	15
11 0	5.890	1.145	6.871	1.336	7.853	1.526	8.835	1.717	9.816	1.908	79 0
15	5.885	1.171	6.866	1.366	7.846	1.561	8.827	1.756	9.808	1.951	45
30	5.880	1.196	6.859	1.396	7.839	1.595	8.819	1.794	9.799	1.994	30
45	5.874	1.222	6.853	1.425	7.832	1.629	8.811	1.833	9.791	2.036	15
12 0	5.869	1.247	6.847	1.455	7.825	1.663	8.803	1.871	9.782	2.079	78 0
15	5.863	1.273	6.841	1.485	7.818	1.697	8.795	1.910	9.772	2.122	45
30	5.858	1.299	6.834	1.515	7.810	1.732	8.787	1.948	9.763	2.164	30
45	5.852	1.324	6.827	1.545	7.803	1.766	8.778	1.986	9.753	2.207	15
13 0	5.846	1.350	6.821	1.575	7.795	1.800	8.769	2.025	9.744	2.250	77 0
15	5.840	1.375	6.814	1.604	7.787	1.834	8.760	2.063	9.734	2.292	45
30	5.834	1.401	6.807	1.634	7.779	1.868	8.751	2.101	9.724	2.335	30
45	5.828	1.426	6.799	1.664	7.771	1.902	8.742	2.139	9.713	2.377	15
14 0	5.822	1.452	6.792	1.693	7.762	1.935	8.733	2.177	9.703	2.419	76 0
15	5.815	1.477	6.785	1.723	7.754	1.969	8.723	2.215	9.692	2.462	45
30	5.809	1.502	6.777	1.753	7.745	2.003	8.713	2.253	9.682	2.504	30
45	5.802	1.528	6.769	1.782	7.736	2.037	8.703	2.291	9.671	2.546	15
15 0	5.796	1.553	6.761	1.812	7.727	2.071	8.693	2.329	9.659	2.588	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
15 15	0.965	0.263	1.930	0.526	2.894	0.789	3.859	1.052	4.824	1.315	**74** 45
30	0.964	0.267	1.927	0.534	2.891	0.802	3.855	1.069	4.818	1.336	30
45	0.962	0.271	1.925	0.543	2.887	0.814	3.850	1.086	4.812	1.357	15
16 0	0.961	0.276	1.923	0.551	2.884	0.827	3.845	1.103	4.806	1.378	**74** 0
15	0.960	0.280	1.920	0.560	2.880	0.839	3.840	1.119	4.800	1.399	45
30	0.959	0.284	1.918	0.568	2.876	0.852	3.835	1.136	4.794	1.420	30
45	0.958	0.288	1.915	0.576	2.873	0.865	3.830	1.153	4.788	1.441	15
17 0	0.956	0.292	1.913	0.585	2.869	0.877	3.825	1.169	4.782	1.462	**73** 0
15	0.955	0.297	1.910	0.593	2.865	0.890	3.820	1.186	4.775	1.483	45
30	0.954	0.301	1.907	0.601	2.861	0.902	3.815	1.203	4.769	1.504	30
45	0.952	0.305	1.905	0.610	2.857	0.915	3.810	1.220	4.762	1.524	15
18 0	0.951	0.309	1.902	0.618	2.853	0.927	3.804	1.236	4.755	1.545	**72** 0
15	0.950	0.313	1.899	0.626	2.849	0.939	3.799	1.253	4.748	1.566	45
30	0.948	0.317	1.897	0.635	2.845	0.952	3.793	1.269	4.742	1.587	30
45	0.947	0.321	1.894	0.643	2.841	0.964	3.788	1.286	4.735	1.607	15
19 0	0.946	0.326	1.891	0.651	2.837	0.977	3.782	1.302	4.728	1.628	**71** 0
15	0.944	0.330	1.888	0.659	2.832	0.989	3.776	1.319	4.720	1.648	45
30	0.943	0.334	1.885	0.668	2.828	1.001	3.771	1.335	4.713	1.669	30
45	0.941	0.338	1.882	0.676	2.824	1.014	3.765	1.352	4.706	1.690	15
20 0	0.940	0.342	1.879	0.684	2.819	1.026	3.759	1.368	4.698	1.710	**70** 0
15	0.938	0.346	1.876	0.692	2.815	1.038	3.753	1.384	4.691	1.731	45
30	0.937	0.350	1.873	0.700	2.810	1.051	3.747	1.401	4.683	1.751	30
45	0.935	0.354	1.870	0.709	2.805	1.063	3.741	1.417	4.676	1.771	15
21 0	0.934	0.358	1.867	0.717	2.801	1.075	3.734	1.433	4.668	1.792	**69** 0
15	0.932	0.362	1.864	0.725	2.796	1.087	3.728	1.450	4.660	1.812	45
30	0.930	0.367	1.861	0.733	2.791	1.100	3.722	1.466	4.652	1.833	30
45	0.929	0.371	1.858	0.741	2.786	1.112	3.715	1.482	4.644	1.853	15
22 0	0.927	0.375	1.854	0.749	2.782	1.124	3.709	1.498	4.636	1.873	**68** 0
15	0.926	0.379	1.851	0.757	2.777	1.136	3.702	1.515	4.628	1.893	45
30	0.924	0.383	1.848	0.765	2.772	1.148	3.696	1.531	4.619	1.913	30
45	0.922	0.387	1.844	0.773	2.767	1.160	3.689	1.547	4.611	1.934	15
23 0	0.921	0.391	1.841	0.781	2.762	1.172	3.682	1.563	4.603	1.954	**67** 0
15	0.919	0.395	1.838	0.789	2.756	1.184	3.675	1.579	4.594	1.974	45
30	0.917	0.399	1.834	0.797	2.751	1.196	3.668	1.595	4.585	1.994	30
45	0.915	0.403	1.831	0.805	2.746	1.208	3.661	1.611	4.577	2.014	15
24 0	0.914	0.407	1.827	0.813	2.741	1.220	3.654	1.627	4.568	2.034	**66** 0
15	0.912	0.411	1.824	0.821	2.735	1.232	3.647	1.643	4.559	2.054	45
30	0.910	0.415	1.820	0.829	2.730	1.244	3.640	1.659	4.550	2.073	30
45	0.908	0.419	1.816	0.837	2.724	1.256	3.633	1.675	4.541	2.093	15
25 0	0.906	0.423	1.813	0.845	2.719	1.268	3.625	1.690	4.532	2.113	**65** 0
15	0.904	0.427	1.809	0.853	2.713	1.280	3.618	1.706	4.522	2.133	45
30	0.903	0.431	1.805	0.861	2.708	1.292	3.610	1.722	4.513	2.153	30
45	0.901	0.434	1.801	0.869	2.702	1.303	3.603	1.738	4.503	2.172	15
26 0	0.899	0.438	1.798	0.877	2.696	1.315	3.595	1.753	4.494	2.192	**64** 0
15	0.897	0.442	1.794	0.885	2.691	1.327	3.587	1.769	4.484	2.211	45
30	0.895	0.446	1.790	0.892	2.685	1.339	3.580	1.785	4.475	2.231	30
45	0.893	0.450	1.786	0.900	2.679	1.350	3.572	1.800	4.465	2.250	15
27 0	0.891	0.454	1.782	0.908	2.673	1.362	3.564	1.816	4.455	2.270	**63** 0
15	0.889	0.458	1.778	0.916	2.667	1.374	3.556	1.831	4.445	2.289	45
30	0.887	0.462	1.774	0.923	2.661	1.385	3.548	1.847	4.435	2.309	30
45	0.885	0.466	1.770	0.931	2.655	1.397	3.540	1.862	4.425	2.328	15
28 0	0.883	0.469	1.766	0.939	2.649	1.408	3.532	1.878	4.415	2.347	**62** 0
15	0.881	0.473	1.762	0.947	2.643	1.420	3.524	1.893	4.404	2.367	45
30	0.879	0.477	1.758	0.954	2.636	1.431	3.515	1.909	4.394	2.386	30
45	0.877	0.481	1.753	0.962	2.630	1.443	3.507	1.924	4.384	2.405	15
29 0	0.875	0.485	1.749	0.970	2.624	1.454	3.498	1.939	4.373	2.424	**61** 0
15	0.872	0.489	1.745	0.977	2.617	1.466	3.490	1.954	4.362	2.443	45
30	0.870	0.492	1.741	0.985	2.611	1.477	3.481	1.970	4.352	2.462	30
45	0.868	0.496	1.736	0.992	2.605	1.489	3.473	1.985	4.341	2.481	15
30 0	0.866	0.500	1.732	1.000	2.598	1.500	3.464	2.000	4.330	2.500	**60** 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.

Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
15 15	5.789	1.578	6.754	1.841	7.718	2.104	8.683	2.367	9.648	2.630	74 45
30	5.782	1.603	6.745	1.871	7.709	2.138	8.673	2.405	9.636	2.672	30
45	5.775	1.629	6.737	1.900	7.700	2.172	8.662	2.443	9.625	2.714	15
16 0	5.768	1.654	6.729	1.929	7.690	2.205	8.651	2.481	9.613	2.756	74 0
15	5.760	1.679	6.720	1.959	7.680	2.239	8.640	2.518	9.601	2.798	45
30	5.753	1.704	6.712	1.988	7.671	2.272	8.629	2.556	9.588	2.840	30
45	5.745	1.729	6.703	2.017	7.661	2.306	8.618	2.594	9.576	2.882	15
17 0	5.738	1.754	6.694	2.047	7.650	2.339	8.607	2.631	9.563	2.924	73 0
15	5.730	1.779	6.685	2.076	7.640	2.372	8.595	2.669	9.550	2.965	45
30	5.722	1.804	6.676	2.105	7.630	2.406	8.583	2.706	9.537	3.007	30
45	5.714	1.829	6.667	2.134	7.619	2.439	8.572	2.744	9.524	3.049	15
18 0	5.706	1.854	6.657	2.163	7.608	2.472	8.560	2.781	9.511	3.090	72 0
15	5.698	1.879	6.648	2.192	7.598	2.505	8.547	2.818	9.497	3.132	45
30	5.690	1.904	6.638	2.221	7.587	2.538	8.535	2.856	9.483	3.173	30
45	5.682	1.929	6.629	2.250	7.575	2.572	8.522	2.893	9.469	3.214	15
19 0	5.673	1.953	6.619	2.279	7.564	2.605	8.510	2.930	9.455	3.256	71 0
15	5.665	1.978	6.609	2.308	7.553	2.638	8.497	2.967	9.441	3.297	45
30	5.656	2.003	6.598	2.337	7.541	2.670	8.484	3.004	9.426	3.338	30
45	5.647	2.028	6.588	2.365	7.529	2.703	8.471	3.041	9.412	3.379	15
20 0	5.638	2.052	6.578	2.394	7.518	2.736	8.457	3.078	9.397	3.420	70 0
15	5.629	2.077	6.567	2.423	7.506	2.769	8.444	3.115	9.382	3.461	45
30	5.620	2.101	6.557	2.451	7.493	2.802	8.430	3.152	9.367	3.502	30
45	5.611	2.126	6.546	2.480	7.481	2.834	8.416	3.189	9.351	3.543	15
21 0	5.601	2.150	6.535	2.509	7.469	2.867	8.402	3.225	9.336	3.584	69 0
15	5.592	2.175	6.524	2.537	7.456	2.900	8.388	3.262	9.320	3.624	45
30	5.582	2.199	6.513	2.566	7.443	2.932	8.374	3.299	9.304	3.665	30
45	5.573	2.223	6.502	2.594	7.430	2.964	8.359	3.335	9.288	3.706	15
22 0	5.563	2.248	6.490	2.622	7.417	2.997	8.345	3.371	9.272	3.746	68 0
15	5.553	2.272	6.479	2.651	7.404	3.029	8.330	3.408	9.255	3.787	45
30	5.543	2.296	6.467	2.679	7.391	3.061	8.315	3.444	9.239	3.827	30
45	5.533	2.320	6.455	2.707	7.378	3.094	8.300	3.480	9.222	3.867	15
23 0	5.523	2.344	6.444	2.735	7.364	3.126	8.285	3.517	9.205	3.907	67 0
15	5.513	2.368	6.432	2.763	7.350	3.158	8.269	3.553	9.188	3.947	45
30	5.502	2.392	6.419	2.791	7.336	3.190	8.254	3.589	9.171	3.988	30
45	5.492	2.416	6.407	2.819	7.322	3.222	8.238	3.625	9.153	4.028	15
24 0	5.481	2.440	6.395	2.847	7.308	3.254	8.222	3.661	9.136	4.067	66 0
15	5.471	2.464	6.382	2.875	7.294	3.286	8.206	3.696	9.118	4.107	45
30	5.460	2.488	6.370	2.903	7.280	3.318	8.190	3.732	9.100	4.147	30
45	5.449	2.512	6.357	2.931	7.265	3.349	8.173	3.768	9.081	4.187	15
25 0	5.438	2.536	6.344	2.958	7.250	3.381	8.157	3.804	9.063	4.226	65 0
15	5.427	2.559	6.331	2.986	7.236	3.413	8.140	3.839	9.045	4.266	45
30	5.416	2.583	6.318	3.014	7.221	3.444	8.123	3.875	9.026	4.305	30
45	5.404	2.607	6.305	3.041	7.206	3.476	8.106	3.910	9.007	4.345	15
26 0	5.393	2.630	6.292	3.069	7.190	3.507	8.089	3.945	8.988	4.384	64 0
15	5.381	2.654	6.278	3.096	7.175	3.538	8.072	3.981	8.969	4.423	45
30	5.370	2.677	6.265	3.123	7.160	3.570	8.054	4.016	8.949	4.462	30
45	5.358	2.701	6.251	3.151	7.144	3.601	8.037	4.051	8.930	4.501	15
27 0	5.346	2.724	6.237	3.178	7.128	3.632	8.019	4.086	8.910	4.540	63 0
15	5.334	2.747	6.223	3.205	7.112	3.663	8.001	4.121	8.890	4.579	45
30	5.322	2.770	6.209	3.232	7.096	3.694	7.983	4.156	8.870	4.618	30
45	5.310	2.794	6.195	3.259	7.080	3.725	7.965	4.190	8.850	4.656	15
28 0	5.298	2.817	6.181	3.286	7.064	3.756	7.947	4.225	8.829	4.695	62 0
15	5.285	2.840	6.166	3.313	7.047	3.787	7.928	4.260	8.809	4.733	45
30	5.273	2.863	6.152	3.340	7.031	3.817	7.909	4.294	8.788	4.772	30
45	5.260	2.886	6.137	3.367	7.014	3.848	7.891	4.329	8.767	4.810	15
29 0	5.248	2.909	6.122	3.394	6.997	3.878	7.872	4.363	8.746	4.848	61 0
15	5.235	2.932	6.107	3.420	6.980	3.909	7.852	4.398	8.725	4.886	45
30	5.222	2.955	6.093	3.447	6.963	3.939	7.833	4.432	8.704	4.924	30
45	5.209	2.977	6.077	3.474	6.946	3.970	7.814	4.466	8.682	4.962	15
30 0	5.196	3.000	6.062	3.500	6.928	4.000	7.794	4.500	8.660	5.000	60 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.

30° — 45°

Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
30 15	0.864	0.504	1.728	1.008	2.592	1.511	3.455	2.015	4.319	2.519	**59** 45
30	0.862	0.508	1.723	1.015	2.585	1.523	3.447	2.030	4.308	2.538	30
45	0.859	0.511	1.719	1.023	2.578	1.534	3.438	2.045	4.297	2.556	15
31 0	0.857	0.515	1.714	1.030	2.572	1.545	3.429	2.060	4.286	2.575	**59** 0
15	0.855	0.519	1.710	1.038	2.565	1.556	3.420	2.075	4.275	2.594	45
30	0.853	0.522	1.705	1.045	2.558	1.567	3.411	2.090	4.263	2.612	30
45	0.850	0.526	1.701	1.052	2.551	1.579	3.401	2.105	4.252	2.631	15
32 0	0.848	0.530	1.696	1.060	2.544	1.590	3.392	2.120	4.240	2.650	**58** 0
15	0.846	0.534	1.691	1.067	2.537	1.601	3.383	2.134	4.229	2.668	45
30	0.843	0.537	1.687	1.075	2.530	1.612	3.374	2.149	4.217	2.686	30
45	0.841	0.541	1.682	1.082	2.523	1.623	3.364	2.164	4.205	2.705	15
33 0	0.839	0.545	1.677	1.089	2.516	1.634	3.355	2.179	4.193	2.723	**57** 0
15	0.836	0.548	1.673	1.097	2.509	1.645	3.345	2.193	4.181	2.741	45
30	0.834	0.552	1.668	1.104	2.502	1.656	3.336	2.208	4.169	2.760	30
45	0.831	0.556	1.663	1.111	2.494	1.667	3.326	2.222	4.157	2.778	15
34 0	0.829	0.559	1.658	1.118	2.487	1.678	3.316	2.237	4.145	2.796	**56** 0
15	0.827	0.563	1.653	1.126	2.480	1.688	3.306	2.251	4.133	2.814	45
30	0.824	0.566	1.648	1.133	2.472	1.699	3.297	2.266	4.121	2.832	30
45	0.822	0.570	1.643	1.140	2.465	1.710	3.287	2.280	4.108	2.850	15
35 0	0.819	0.574	1.638	1.147	2.457	1.721	3.277	2.294	4.096	2.868	**55** 0
15	0.817	0.577	1.633	1.154	2.450	1.731	3.267	2.309	4.083	2.886	45
30	0.814	0.581	1.628	1.161	2.442	1.742	3.257	2.323	4.071	2.904	30
45	0.812	0.584	1.623	1.168	2.435	1.753	3.246	2.337	4.058	2.921	15
36 0	0.809	0.588	1.618	1.176	2.427	1.763	3.236	2.351	4.045	2.939	**54** 0
15	0.806	0.591	1.613	1.183	2.419	1.774	3.226	2.365	4.032	2.957	45
30	0.804	0.595	1.608	1.190	2.412	1.784	3.215	2.379	4.019	2.974	30
45	0.801	0.598	1.603	1.197	2.404	1.795	3.205	2.393	4.006	2.992	15
37 0	0.799	0.602	1.597	1.204	2.396	1.805	3.195	2.407	3.993	3.009	**53** 0
15	0.796	0.605	1.592	1.211	2.388	1.816	3.184	2.421	3.980	3.026	45
30	0.793	0.609	1.587	1.218	2.380	1.826	3.173	2.435	3.967	3.044	30
45	0.791	0.612	1.581	1.224	2.372	1.837	3.163	2.449	3.953	3.061	15
38 0	0.788	0.616	1.576	1.231	2.364	1.847	3.152	2.463	3.940	3.078	**52** 0
15	0.785	0.619	1.571	1.238	2.356	1.857	3.141	2.476	3.927	3.095	45
30	0.783	0.623	1.565	1.245	2.348	1.868	3.130	2.490	3.913	3.113	30
45	0.780	0.626	1.560	1.252	2.340	1.878	3.120	2.504	3.899	3.130	15
39 0	0.777	0.629	1.554	1.259	2.331	1.888	3.109	2.517	3.886	3.147	**51** 0
15	0.774	0.633	1.549	1.265	2.323	1.898	3.098	2.531	3.872	3.164	45
30	0.772	0.636	1.543	1.272	2.315	1.908	3.086	2.544	3.858	3.180	30
45	0.769	0.639	1.538	1.279	2.307	1.918	3.075	2.558	3.844	3.197	15
40 0	0.766	0.643	1.532	1.286	2.298	1.928	3.064	2.571	3.830	3.214	**50** 0
15	0.763	0.646	1.526	1.292	2.290	1.938	3.053	2.584	3.816	3.231	45
30	0.760	0.649	1.521	1.299	2.281	1.948	3.042	2.598	3.802	3.247	30
45	0.758	0.653	1.515	1.306	2.273	1.958	3.030	2.611	3.788	3.264	15
41 0	0.755	0.656	1.509	1.312	2.264	1.968	3.019	2.624	3.774	3.280	**49** 0
15	0.752	0.659	1.504	1.319	2.256	1.978	3.007	2.637	3.759	3.297	45
30	0.749	0.663	1.498	1.325	2.247	1.988	2.996	2.650	3.745	3.313	30
45	0.746	0.666	1.492	1.332	2.238	1.998	2.984	2.664	3.730	3.329	15
42 0	0.743	0.669	1.486	1.338	2.229	2.007	2.973	2.677	3.716	3.346	**48** 0
15	0.740	0.672	1.480	1.345	2.221	2.017	2.961	2.689	3.701	3.362	45
30	0.737	0.676	1.475	1.351	2.212	2.027	2.949	2.702	3.686	3.378	30
45	0.734	0.679	1.469	1.358	2.203	2.036	2.937	2.715	3.672	3.394	15
43 0	0.731	0.682	1.463	1.364	2.194	2.046	2.925	2.728	3.657	3.410	**47** 0
15	0.728	0.685	1.457	1.370	2.185	2.056	2.913	2.741	3.642	3.426	45
30	0.725	0.688	1.451	1.377	2.176	2.065	2.901	2.753	3.627	3.442	30
45	0.722	0.692	1.445	1.383	2.167	2.075	2.889	2.766	3.612	3.458	15
44 0	0.719	0.695	1.439	1.389	2.158	2.084	2.877	2.779	3.597	3.473	**46** 0
15	0.716	0.698	1.433	1.396	2.149	2.093	2.865	2.791	3.582	3.489	45
30	0.713	0.701	1.427	1.402	2.140	2.103	2.853	2.804	3.566	3.505	30
45	0.710	0.704	1.420	1.408	2.131	2.112	2.841	2.816	3.551	3.520	15
45 0	0.707	0.707	1.414	1.414	2.121	2.121	2.828	2.828	3.536	3.536	**45** 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 1.		Distance 2.		Distance 3.		Distance 4.		Distance 5.		Bearing.

45° — 60°

Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
30 15	5.183	3.023	6.047	3.526	6.911	4.030	7.775	4.534	8.638	5.038	**59** 45
30	5.170	3.045	6.031	3.553	6.893	4.060	7.755	4.568	8.616	5.075	30
45	5.156	3.068	6.016	3.579	6.875	4.090	7.735	4.602	8.594	5.113	15
31 0	5.143	3.090	6.000	3.605	6.857	4.120	7.715	4.635	8.572	5.150	**59** 0
15	5.129	3.113	5.984	3.631	6.839	4.150	7.694	4.669	8.549	5.188	45
30	5.116	3.135	5.968	3.657	6.821	4.180	7.674	4.702	8.526	5.225	30
45	5.102	3.157	5.952	3.683	6.803	4.210	7.653	4.736	8.504	5.262	15
32 0	5.088	3.180	5.936	3.709	6.784	4.239	7.632	4.769	8.481	5.299	**58** 0
15	5.074	3.202	5.920	3.735	6.766	4.269	7.612	4.802	8.457	5.336	45
30	5.060	3.224	5.904	3.761	6.747	4.298	7.591	4.836	8.434	5.373	30
45	5.046	3.246	5.887	3.787	6.728	4.328	7.569	4.869	8.410	5.410	15
33 0	5.032	3.268	5.871	3.812	6.709	4.357	7.548	4.902	8.387	5.446	**57** 0
15	5.018	3.290	5.854	3.838	6.690	4.386	7.527	4.935	8.363	5.483	45
30	5.003	3.312	5.837	3.864	6.671	4.416	7.505	4.967	8.339	5.519	30
45	4.989	3.333	5.820	3.889	6.652	4.445	7.483	5.000	8.315	5.556	15
34 0	4.974	3.355	5.803	3.914	6.632	4.474	7.461	5.033	8.290	5.592	**56** 0
15	4.960	3.377	5.786	3.940	6.613	4.502	7.439	5.065	8.266	5.628	45
30	4.945	3.398	5.769	3.965	6.593	4.531	7.417	5.098	8.241	5.664	30
45	4.930	3.420	5.752	3.990	6.573	4.560	7.395	5.130	8.217	5.700	15
35 0	4.915	3.441	5.734	4.015	6.553	4.589	7.372	5.162	8.192	5.736	**55** 0
15	4.900	3.463	5.716	4.040	6.533	4.617	7.350	5.194	8.166	5.772	45
30	4.885	3.484	5.699	4.065	6.513	4.646	7.327	5.226	8.141	5.807	30
45	4.869	3.505	5.681	4.090	6.493	4.674	7.304	5.258	8.116	5.843	15
36 0	4.854	3.527	5.663	4.115	6.472	4.702	7.281	5.290	8.090	5.878	**54** 0
15	4.839	3.548	5.645	4.139	6.452	4.730	7.258	5.322	8.064	5.913	45
30	4.823	3.569	5.627	4.164	6.431	4.759	7.235	5.353	8.039	5.948	30
45	4.808	3.590	5.609	4.188	6.410	4.787	7.211	5.385	8.013	5.983	15
37 0	4.792	3.611	5.590	4.213	6.389	4.815	7.188	5.416	7.986	6.018	**53** 0
15	4.776	3.632	5.572	4.237	6.368	4.842	7.164	5.448	7.960	6.053	45
30	4.760	3.653	5.554	4.261	6.347	4.870	7.140	5.479	7.934	6.088	30
45	4.744	3.673	5.535	4.286	6.326	4.898	7.116	5.510	7.907	6.122	15
38 0	4.728	3.694	5.516	4.310	6.304	4.925	7.092	5.541	7.880	6.157	**52** 0
15	4.712	3.715	5.497	4.334	6.283	4.953	7.068	5.572	7.853	6.191	45
30	4.696	3.735	5.478	4.358	6.261	4.980	7.043	5.603	7.826	6.225	30
45	4.679	3.756	5.459	4.381	6.239	5.007	7.019	5.633	7.799	6.259	15
39 0	4.663	3.776	5.440	4.405	6.217	5.035	6.994	5.664	7.772	6.293	**51** 0
15	4.646	3.796	5.421	4.429	6.195	5.062	6.970	5.694	7.744	6.327	45
30	4.630	3.816	5.401	4.453	6.173	5.089	6.945	5.725	7.716	6.361	30
45	4.613	3.837	5.382	4.476	6.151	5.116	6.920	5.755	7.688	6.394	15
40 0	4.596	3.857	5.362	4.500	6.128	5.142	6.894	5.785	7.660	6.428	**50** 0
15	4.579	3.877	5.343	4.523	6.106	5.169	6.869	5.815	7.632	6.461	45
30	4.562	3.897	5.323	4.546	6.083	5.196	6.844	5.845	7.604	6.495	30
45	4.545	3.917	5.303	4.569	6.061	5.222	6.818	5.875	7.576	6.528	15
41 0	4.528	3.936	5.283	4.592	6.038	5.248	6.792	5.905	7.547	6.561	**49** 0
15	4.511	3.956	5.263	4.615	6.015	5.275	6.767	5.934	7.518	6.594	45
30	4.494	3.976	5.243	4.638	5.992	5.301	6.741	5.964	7.490	6.626	30
45	4.476	3.995	5.222	4.661	5.968	5.327	6.715	5.993	7.461	6.659	15
42 0	4.459	4.015	5.202	4.684	5.945	5.353	6.688	6.022	7.431	6.691	**48** 0
15	4.441	4.034	5.182	4.707	5.922	5.379	6.662	6.051	7.402	6.724	45
30	4.424	4.054	5.161	4.729	5.898	5.405	6.635	6.080	7.373	6.756	30
45	4.406	4.073	5.140	4.752	5.875	5.430	6.609	6.109	7.343	6.788	15
43 0	4.388	4.092	5.119	4.774	5.851	5.456	6.582	6.138	7.314	6.820	**47** 0
15	4.370	4.111	5.099	4.796	5.827	5.481	6.555	6.167	7.284	6.852	45
30	4.352	4.130	5.078	4.818	5.803	5.507	6.528	6.195	7.254	6.884	30
45	4.334	4.149	5.057	4.841	5.779	5.532	6.501	6.224	7.224	6.915	15
44 0	4.316	4.168	5.035	4.863	5.755	5.557	6.474	6.252	7.193	6.947	**46** 0
15	4.298	4.187	5.014	4.885	5.730	5.582	6.447	6.280	7.163	6.978	45
30	4.280	4.206	4.993	4.906	5.706	5.607	6.419	6.308	7.133	7.009	30
45	4.261	4.224	4.971	4.928	5.681	5.632	6.392	6.336	7.102	7.040	15
45 0	4.243	4.243	4.950	4.950	5.657	5.657	6.364	6.364	7.071	7.071	**45** 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
Bearing.	Distance 6.		Distance 7.		Distance 8.		Distance 9.		Distance 10.		Bearing.

′	0°		1°		2°		3°		4°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0000	1000	0175	9998	0349	9994	0523	9986	0698	9976	**60**
1	0003	1000	0177	9998	0352	9994	0526	9986	0700	9975	59
2	0006	1000	0180	9998	0355	9994	0529	9986	0703	9975	58
3	0009	1000	0183	9998	0358	9994	0532	9986	0706	9975	57
4	0012	1000	0186	9998	0361	9993	0535	9986	0709	9975	56
5	0015	1000	0189	9998	0364	9993	0538	9986	0712	9975	55
6	0017	1000	0192	9998	0366	9993	0541	9985	0715	9974	54
7	0020	1000	0195	9998	0369	9993	0544	9985	0718	9974	53
8	0023	1000	0198	9998	0372	9993	0547	9985	0721	9974	52
9	0026	1000	0201	9998	0375	9993	0550	9985	0724	9974	51
10	0029	1000	0204	9998	0378	9993	0552	9985	0727	9974	**50**
11	0032	1000	0207	9998	0381	9993	0555	9985	0729	9973	49
12	0035	1000	0209	9998	0384	9993	0558	9984	0732	9973	48
13	0038	1000	0212	9998	0387	9993	0561	9984	0735	9973	47
14	0041	1000	0215	9998	0390	9992	0564	9984	0738	9973	46
15	0044	1000	0218	9998	0393	9992	0567	9984	0741	9973	45
16	0047	1000	0221	9998	0396	9992	0570	9984	0744	9972	44
17	0049	1000	0224	9997	0398	9992	0573	9984	0747	9972	43
18	0052	1000	0227	9997	0401	9992	0576	9983	0750	9972	42
19	0055	1000	0230	9997	0404	9992	0579	9983	0753	9972	41
20	0058	1000	0233	9997	0407	9992	0581	9983	0756	9971	**40**
21	0061	1000	0236	9997	0410	9992	0584	9983	0758	9971	39
22	0064	1000	0239	9997	0413	9991	0587	9983	0761	9971	38
23	0067	1000	0241	9997	0416	9991	0590	9983	0764	9971	37
24	0070	1000	0244	9997	0419	9991	0593	9982	0767	9971	36
25	0073	1000	0247	9997	0422	9991	0596	9982	0770	9970	35
26	0076	1000	0250	9997	0425	9991	0599	9982	0773	9970	34
27	0079	1000	0253	9997	0427	9991	0602	9982	0776	9970	33
28	0081	1000	0256	9997	0430	9991	0605	9982	0779	9970	32
29	0084	1000	0259	9997	0433	9991	0608	9982	0782	9969	31
30	0087	1000	0262	9997	0436	9990	0610	9981	0785	9969	**30**
31	0090	1000	0265	9996	0439	9990	0613	9981	0787	9969	29
32	0093	1000	0268	9996	0442	9990	0616	9981	0790	9969	28
33	0096	1000	0270	9996	0445	9990	0619	9981	0793	9968	27
34	0099	1000	0273	9996	0448	9990	0622	9981	0796	9968	26
35	0102	9999	0276	9996	0451	9990	0625	9980	0799	9968	25
36	0105	9999	0279	9996	0454	9990	0628	9980	0802	9968	24
37	0108	9999	0282	9996	0457	9990	0631	9980	0805	9968	23
38	0111	9999	0285	9996	0459	9989	0634	9980	0808	9967	22
39	0113	9999	0288	9996	0462	9989	0637	9980	0811	9967	21
40	0116	9999	0291	9996	0465	9989	0640	9980	0814	9967	**20**
41	0119	9999	0294	9996	0468	9989	0642	9979	0816	9967	19
42	0122	9999	0297	9996	0471	9989	0645	9979	0819	9966	18
43	0125	9999	0300	9996	0474	9989	0648	9979	0822	9966	17
44	0128	9999	0302	9995	0477	9989	0651	9979	0825	9966	16
45	0131	9999	0305	9995	0480	9988	0654	9979	0828	9966	15
46	0134	9999	0308	9995	0483	9988	0657	9978	0831	9965	14
47	0137	9999	0311	9995	0486	9988	0660	9978	0834	9965	13
48	0140	9999	0314	9995	0488	9988	0663	9978	0837	9965	12
49	0143	9999	0317	9995	0491	9988	0666	9978	0840	9965	11
50	0145	9999	0320	9995	0494	9988	0669	9978	0843	9964	**10**
51	0148	9999	0323	9995	0497	9988	0671	9977	0845	9964	9
52	0151	9999	0326	9995	0500	9987	0674	9977	0848	9964	8
53	0154	9999	0329	9995	0503	9987	0677	9977	0851	9964	7
54	0157	9999	0332	9995	0506	9987	0680	9977	0854	9963	6
55	0160	9999	0334	9994	0509	9987	0683	9977	0857	9963	5
56	0163	9999	0337	9994	0512	9987	0686	9976	0860	9963	4
57	0166	9999	0340	9994	0515	9987	0689	9976	0863	9963	3
58	0169	9999	0343	9994	0518	9987	0692	9976	0866	9962	2
59	0172	9999	0346	9994	0520	9986	0695	9976	0869	9962	1
60	0175	9999	0349	9994	0523	9986	0698	9976	0872	9962	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0872	9962	1045	9945	1219	9925	1392	9903	1564	9877	**60**
1	0874	9962	1048	9945	1222	9925	1395	9902	1567	9876	59
2	0877	9461	1051	9945	1224	9925	1397	9902	1570	9876	58
3	0880	9961	1054	9944	1227	9924	1400	9901	1573	9876	57
4	0883	9961	1057	9944	1230	9924	1403	9901	1576	9875	56
5	0886	9961	1060	9944	1233	9924	1406	9901	1579	9875	55
6	0889	9960	1063	9943	1236	9923	1409	9900	1582	9874	54
7	0892	9960	1066	9943	1239	9923	1412	9900	1584	9874	53
8	0895	9960	1068	9943	1241	9923	1415	9899	1587	9873	52
9	0898	9960	1071	9942	1245	9922	1418	9899	1590	9873	51
10	0901	9959	1074	9942	1248	9922	1421	9899	1593	9872	**50**
11	0903	9959	1077	9942	1250	9922	1423	9898	1596	9872	49
12	0906	9959	1080	9942	1253	9921	1426	9898	1599	9871	48
13	0909	9959	1083	9941	1256	9921	1429	9897	1602	9871	47
14	0912	9958	1086	9941	1259	9920	1432	9897	1605	9870	46
15	0915	9958	1089	9941	1262	9920	1435	9897	1607	9870	45
16	0918	9958	1092	9940	1265	9920	1438	9896	1610	9869	44
17	0921	9958	1094	9940	1268	9919	1441	9896	1613	9869	43
18	0924	9957	1097	9940	1271	9919	1444	9895	1616	9869	42
19	0927	9957	1100	9939	1274	9919	1446	9895	1619	9868	41
20	0929	9957	1103	9939	1276	9918	1449	9894	1622	9868	**40**
21	0932	9956	1106	9939	1279	9918	1452	9894	1625	9867	39
22	0935	9956	1109	9938	1282	9917	1455	9894	1628	9867	38
23	0938	9956	1112	9938	1285	9917	1458	9893	1630	9866	37
24	0941	9956	1115	9938	1288	9917	1461	9893	1633	9866	36
25	0944	9955	1118	9937	1291	9916	1464	9892	1636	9865	35
26	0947	9955	1120	9937	1294	9916	1467	9892	1639	9865	34
27	0950	9955	1123	9937	1297	9916	1469	9891	1642	9864	33
28	0953	9955	1126	9936	1299	9915	1472	9891	1645	9864	32
29	0956	9954	1129	9936	1302	9915	1475	9891	1648	9863	31
30	0958	9954	1132	9936	1305	9914	1478	9890	1650	9863	**30**
31	0961	9954	1135	9935	1308	9914	1481	9890	1653	9862	29
32	9964	9953	1138	9935	1311	9914	1484	9889	1656	9862	28
33	0967	9953	1141	9935	1314	9913	1487	9889	1659	9861	27
34	0970	9953	1144	9934	1317	9913	1490	9888	1662	9861	26
35	0973	9553	1146	9934	1320	9913	1492	9888	1665	9860	25
36	0976	9952	1149	9934	1323	9912	1495	9888	1668	9860	24
37	0979	9952	1152	9933	1325	9912	1498	9887	1671	9859	23
38	0982	9952	1155	9933	1328	9911	1501	9887	1673	9859	22
39	0985	9951	1158	9933	1331	9911	1504	9886	1676	9859	21
40	0987	9951	1161	9932	1334	9911	1507	9886	1679	9858	**20**
41	0990	9951	1164	9932	1337	9910	1510	9885	1682	9858	19
42	0993	9951	1167	9932	1340	9910	1513	9885	1685	9857	18
43	0996	9950	1170	9931	1343	9909	1515	9884	1688	9857	17
44	0999	9950	1172	9931	1346	9909	1518	9884	1691	9856	16
45	1002	9950	1175	9931	1349	9909	1521	9884	1693	9856	15
46	1005	9949	1178	9930	1351	9908	1524	9883	1696	9855	14
47	1008	9949	1181	9930	1354	9908	1527	9883	1699	9855	13
48	1011	9949	1184	9930	1357	9907	1530	9882	1702	9854	12
49	1013	9949	1187	9929	1360	9907	1533	9882	1705	9854	11
50	1016	9948	1190	9929	1363	9907	1536	9881	1708	9853	**10**
51	1019	9948	1193	9929	1366	9906	1538	9881	1711	9853	9
52	1022	9948	1196	9928	1369	9906	1541	9880	1714	9852	8
53	1025	9947	1198	9928	1372	9905	1544	9880	1716	9852	7
54	1028	9947	1201	9928	1374	9905	1547	9880	1719	9851	6
55	1031	9947	1204	9927	1377	9905	1550	9879	1722	9851	5
56	1034	9946	1207	9927	1380	9904	1553	9879	1725	9850	4
57	1037	9946	1210	9927	1383	9904	1556	9878	1728	9850	3
58	1039	9946	1213	9926	1386	9903	1559	9878	1731	9849	2
59	1042	9946	1216	9926	1389	9903	1561	9877	1734	9849	1
60	1045	9945	1219	9925	1392	9903	1564	9877	1736	9848	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	1736	9848	1908	9816	2079	9781	2250	9744	2419	9703	**60**
1	1739	9848	1911	9816	2082	9781	2252	9743	2422	9702	59
2	1742	9847	1914	9815	2085	9780	2255	9742	2425	9702	58
3	1745	9847	1917	9815	2088	9780	2258	9742	2428	9701	57
4	1748	9846	1920	9814	2090	9779	2261	9741	2431	9700	56
5	1751	9846	1922	9813	2093	9778	2264	9740	2433	9699	55
6	1754	9845	1925	9813	2096	9778	2267	9740	2436	9699	54
7	1757	9845	1928	9812	2099	9777	2269	9739	2439	9698	53
8	1759	9844	1931	9812	2102	9777	2272	9738	2442	9697	52
9	1762	9843	1934	9811	2105	9776	2275	9738	2445	9697	51
10	1765	9843	1937	9811	2108	9775	2278	9737	2447	9696	**50**
11	1768	9842	1939	9810	2110	9775	2281	9736	2450	9695	49
12	1771	9842	1942	9810	2113	9774	2284	9736	2453	9694	48
13	1774	9841	1945	9809	2116	9774	2286	9735	2456	9694	47
14	1777	9841	1948	9808	2119	9773	2289	9734	2459	9693	46
15	1779	9840	1951	9808	2122	9772	2292	9734	2462	9692	45
16	1782	9840	1954	9807	2125	9772	2295	9733	2464	9692	44
17	1785	9839	1957	9807	2127	9771	2298	9732	2467	9691	43
18	1788	9839	1959	9806	2130	9770	2300	9732	2470	9690	42
19	1791	9838	1962	9806	2133	9770	2303	9731	2473	9689	41
20	1794	9838	1965	9805	2136	9769	2306	9730	2476	9689	**40**
21	1797	9837	1968	9804	2139	9769	2309	9730	2478	9688	39
22	1799	9837	1971	9804	2142	9768	2312	9729	2481	9687	38
23	1802	9836	1974	9803	2145	9767	2315	9728	2484	9687	37
24	1805	9836	1977	9803	2147	9767	2317	9728	2487	9686	36
25	1808	9835	1979	9802	2150	9766	2320	9727	2490	9685	35
26	1811	9835	1982	9802	2153	9765	2323	9726	2493	9684	34
27	1814	9834	1985	9801	2156	9765	2326	9726	2495	9684	33
28	1817	9834	1988	9800	2159	9764	2329	9725	2498	9683	32
29	1819	9833	1991	9800	2162	9764	2332	9724	2501	9682	31
30	1822	9833	1994	9799	2164	9763	2334	9724	2504	9681	**30**
31	1825	9832	1997	9799	2167	9762	2337	9723	2507	9681	29
32	1828	9831	1999	9798	2170	9762	2340	9722	2509	9680	28
33	1831	9831	2002	9798	2173	9761	3343	9722	2512	9679	27
34	1834	9830	2005	9797	2176	9760	2346	9721	2515	9679	26
35	1837	9830	2008	9796	2179	9760	2349	9720	2518	9678	25
36	1840	9829	2011	9796	2181	9759	2351	9720	2521	9677	24
37	1842	9829	2014	9795	2184	9759	2354	9719	2524	9676	23
38	1845	9828	2016	9795	2187	9758	2357	9718	2526	9676	22
39	1848	9828	2019	9794	2190	9757	2360	9718	2529	9675	21
40	1851	9827	2022	9793	2193	9757	2363	9717	2532	9674	**20**
41	1854	9827	2025	9793	2196	9756	2366	9716	2535	9673	19
42	1857	9826	2028	9792	2198	9755	2368	9715	2538	9673	18
43	1860	9826	2031	9792	2201	9755	2371	9715	2540	9672	17
44	1862	9825	2034	9791	2204	9754	2374	9714	2543	9671	16
45	1865	9825	2036	9790	2207	9753	2377	9713	2546	9670	15
46	1868	9824	2039	9790	2210	9753	2380	9713	2549	9670	14
47	1871	9823	2042	9789	2213	9752	2383	9712	2552	9669	13
48	1474	9823	2045	9789	2215	9751	2385	9711	2554	9668	12
49	1877	9822	2048	9788	2218	9751	2388	9711	2557	9667	11
50	1880	9822	2051	9787	2221	9750	2391	9710	2560	9667	**10**
51	1882	9821	2054	9787	2224	9750	2394	9709	2563	9666	9
52	1885	9821	2056	9786	2227	9749	2397	9709	2566	9665	8
53	1888	9820	2059	9786	2230	9748	2399	9708	2569	9665	7
54	1891	9820	2062	9785	2233	9748	2402	9707	2571	9664	6
55	1894	9819	2065	9784	2235	9747	2405	9706	2574	9663	5
56	1897	9818	2068	9784	2238	9746	2408	9706	2577	9662	4
57	1900	9818	2071	9783	2241	9746	2411	9705	2580	9662	3
58	1902	9817	2073	9783	2244	9745	2414	9704	2583	9661	2
59	1905	9817	2076	9782	2247	9744	2416	9704	2585	9660	1
60	1908	9816	2079	9781	2250	9744	2419	9703	2588	9659	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	2588	9659	2756	9613	2924	9563	3090	9511	3256	9455	**60**
1	2591	9659	2759	9612	2926	9562	3093	9510	3258	9454	59
2	2594	9658	2762	9611	2929	9561	3096	9509	3261	9453	58
3	2597	9657	2765	9610	2932	9560	3098	9508	3264	9452	57
4	2599	9656	2768	9609	2935	9560	3101	9507	3267	9451	56
5	2602	9655	2770	9609	2938	9559	3104	9506	3269	9450	55
6	2605	9655	2773	9608	2940	9558	3107	9505	3272	9449	54
7	2608	9654	2776	9607	2943	9557	3110	9504	3275	9449	53
8	2611	9653	2779	9606	2946	9556	3112	9503	3278	9448	52
9	2613	9652	2782	9605	2949	9555	3115	9502	3280	9447	51
10	2616	9652	2784	9605	2952	9555	3118	9502	3283	9446	**50**
11	2619	9651	2787	9604	2954	9554	3121	9501	3286	9445	49
12	2622	9650	2790	9603	2957	9553	3123	9500	3289	9444	48
13	2625	9649	2793	9602	2960	9552	3126	9499	3291	9443	47
14	2628	9649	2795	9601	2963	9551	3129	9498	3294	9442	46
15	2630	9648	2798	9600	2965	9550	3132	9497	3297	9441	45
16	2633	9647	2801	9600	2968	9549	3134	9496	3300	9440	44
17	2636	9646	2804	9599	2971	9548	3137	9495	3302	9439	43
18	2639	9646	2807	9598	2974	9548	3140	9494	3305	9438	42
19	2642	9645	2809	9597	2977	9547	3143	9493	3308	9437	41
20	2644	9644	2812	9596	2979	9546	3145	9492	3311	9436	**40**
21	2647	9643	2815	9596	2982	9545	3148	9492	3313	9435	39
22	2650	9642	2818	9595	2985	9544	3151	9491	3316	9434	38
23	2653	9642	2821	9594	2988	9543	3154	9490	3319	9433	37
24	2656	9641	2823	9593	2990	9542	3156	9489	3322	9432	36
25	2658	9640	2826	9592	2993	9542	3159	9488	3324	9431	35
26	2661	9639	2829	9591	2996	9541	3162	9487	3327	9430	34
27	2664	9639	2832	9591	2999	9540	3165	9486	3330	9429	33
28	2667	9638	2835	9590	3002	9539	3168	9485	3333	9428	32
29	2670	9637	2837	9589	3004	9538	3170	9484	3335	9427	31
30	2672	9636	2840	9588	3007	9537	3173	9483	3338	9426	**30**
31	2675	9636	2843	9587	3010	9536	3176	9482	3341	9425	29
32	2678	9635	2846	9587	3013	9535	3179	9481	3344	9424	28
33	2681	9634	2849	9586	3015	9535	3181	9480	3346	9423	27
34	2684	9633	2851	9585	3018	9534	3184	9480	3349	9423	26
35	2686	9632	2854	9584	3021	9533	3187	9479	3352	9422	25
36	2689	9632	2857	9583	3024	9532	3190	9478	3355	9421	24
37	2692	9631	2860	9582	3026	9531	3192	9477	3357	9420	23
38	2695	9630	2862	9582	3029	9530	3195	9476	3360	9419	22
39	2698	9629	2865	9581	3032	9529	3198	9475	3363	9418	21
40	2700	9628	2868	9580	3035	9528	3201	9474	3365	9417	**20**
41	2703	9628	2871	9579	3038	9527	3203	9473	3368	9416	19
42	2706	9627	2874	9578	3040	9527	3206	9472	3371	9415	18
43	2709	9626	2876	9577	3043	9526	3209	9471	3374	9414	17
44	2712	9625	2879	9577	3046	9525	3212	9470	3376	9413	16
45	2714	9625	2882	9576	3049	9524	3214	9469	3379	9412	15
46	2717	9624	2885	9575	3051	9523	3217	9468	3382	9411	14
47	2720	9623	2888	9574	3054	9522	3220	9467	3385	9410	13
48	2723	9622	2890	9573	3057	9521	3223	9466	3387	9409	12
49	2726	9621	2893	9572	3060	9520	3225	9466	3390	9408	11
50	2728	9621	2896	9572	3062	9520	3228	9465	3393	9407	**10**
51	2731	9620	2899	9571	3065	9519	3231	9464	3396	9406	9
52	2734	9619	2901	9570	3068	9518	3234	9463	3398	9405	8
53	2737	9618	2904	9569	3071	9517	3236	9462	3401	9404	7
54	2740	9617	2907	9568	3074	9516	3239	9461	3404	9403	6
55	2742	9617	2910	9567	3076	9515	3242	9460	3407	9402	5
56	2745	9616	2913	9566	3079	9514	3245	9459	3409	9401	4
57	2748	9615	2915	9566	3082	9513	3247	9458	3412	9400	3
58	2751	9614	2918	9565	3085	9512	3250	9457	3415	9399	2
59	2754	9613	2921	9564	3087	9511	3253	9456	3417	9398	1
60	2756	9613	2924	9563	3090	9511	3256	9455	3420	9397	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	3420	9397	3584	9336	3746	9272	3907	9205	4067	9135	**60**
1	3423	9396	3586	9335	3749	9271	3910	9204	4070	9134	59
2	3426	9395	3589	9334	3751	9270	3913	9203	4073	9133	58
3	3428	9394	3592	9333	3754	9269	3915	9202	4075	9132	57
4	3431	9393	3595	9332	3757	9267	3918	9200	4078	9131	56
5	3434	9392	3597	9331	3760	9266	3921	9199	4081	9130	55
6	3437	9391	3600	9330	3762	9265	3923	9198	4083	9128	54
7	3439	9390	3603	9328	3765	9264	3926	9197	4086	9127	53
8	3442	9389	3605	9327	3768	9263	3929	9196	4089	9126	52
9	3445	9388	3608	9326	3770	9262	3931	9195	4091	9125	51
10	3448	9387	3611	9325	3773	9261	3934	9194	4094	9124	**50**
11	3450	9386	3614	9324	3776	9260	3937	9192	4097	9122	49
12	3453	9385	3616	9323	3778	9259	3939	9191	4099	9121	48
13	3456	9384	3619	9322	3781	9258	3942	9190	4102	9120	47
14	3458	9383	3622	9321	3784	9257	3945	9189	4105	9119	46
15	3461	9382	3624	9320	3786	9255	3947	9188	4107	9118	45
16	3464	9381	3627	9319	3789	9254	3950	9187	4110	9116	44
17	3467	9380	3630	9318	3792	9253	3953	9186	4112	9115	43
18	3469	9379	3633	9317	3795	9252	3955	9184	4115	9114	42
19	3472	9378	3635	9316	3797	9251	3958	9183	4118	9113	41
20	3475	9377	3638	9315	3800	9250	3961	9182	4120	9112	**40**
21	3478	9376	3641	9314	3803	9249	3963	9181	4123	9110	39
22	3480	9375	3643	9313	3805	9248	3966	9180	4126	9109	38
23	3483	9374	3646	9312	3808	9247	3969	9179	4128	9108	37
24	3486	9373	3649	9311	3811	9245	3971	9178	4131	9107	36
25	3488	9372	3651	9309	3813	9244	3974	9176	4134	9106	35
26	3491	9371	3654	9308	3816	9243	3977	9175	4136	9104	34
27	3494	9370	3657	9307	3819	9242	3979	9174	4139	9103	33
28	3497	9369	3660	9306	3821	9241	3982	9173	4142	9102	32
29	3499	9368	3662	9305	3824	9240	3985	9172	4144	9101	31
30	3502	9367	3665	9304	3827	9239	3987	9171	4147	9100	**30**
31	3505	9366	3668	9303	3830	9238	3990	9169	4150	9098	29
32	3508	9365	3670	9302	3832	9237	3993	9168	4152	9097	28
33	3510	9364	3673	9301	3835	9235	3995	9167	4155	9096	27
34	3513	9363	3676	9300	3838	9234	3998	9166	4158	9095	26
35	3516	9362	3679	9299	3840	9233	4001	9165	4160	9094	25
36	3518	9361	3681	9298	3843	9232	4003	9164	4163	9092	24
37	3521	9360	3684	9297	3846	9231	4006	9162	4165	9091	23
38	3524	9359	3687	9296	3848	9230	4009	9161	4168	9090	22
39	3527	9358	3689	9295	3851	9229	4011	9160	4171	9088	21
40	3529	9356	3692	9293	3854	9228	4014	9159	4173	9088	**20**
41	3532	9355	3695	9292	3856	9227	4017	9158	4176	9086	19
42	3535	9354	3697	9291	3859	9225	4019	9157	4179	9085	18
43	3537	9353	3700	9290	3862	9224	4022	9155	4181	9084	17
44	3540	9352	3703	9289	3864	9223	4025	9154	4184	9083	16
45	3543	9351	3706	9288	3867	9222	4027	9153	4187	9081	15
46	3546	9350	3708	9287	3870	9221	4030	9152	4189	9080	14
47	3548	9349	3711	9286	3872	9220	4033	9151	4192	9079	13
48	3551	9348	3714	9285	3875	9219	4035	9150	4195	9078	12
49	3554	9347	3716	9284	3878	9218	4038	9148	4197	9077	11
50	3557	9346	3719	9283	3881	9216	4041	9147	4200	9075	**10**
51	3559	9345	3722	9282	3883	9215	4043	9146	4202	9074	9
52	3562	9344	3724	9281	3886	9214	4046	9145	4205	9073	8
53	3565	9343	3727	9279	3889	9213	4049	9144	4208	9072	7
54	3567	9342	3730	9278	3891	9212	4051	9143	4210	9070	6
55	3570	9341	3733	9277	3894	9211	4054	9141	4213	9069	5
56	3573	9340	3735	9276	3897	9210	4057	9140	4216	9068	4
57	3576	9339	3738	9275	3899	9208	4059	9139	4218	9067	3
58	3578	9338	3741	9274	3902	9207	4062	9138	4221	9066	2
59	3581	9337	3743	9273	3905	9206	4065	9137	4224	9064	1
60	3584	9336	3746	9272	3907	9205	4067	9135	4226	9063	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	69°		68°		67°		66°		65°		′

′	25° sin	25° cos	26° sin	26° cos	27° sin	27° cos	28° sin	28° cos	29° sin	29° cos	′
0	4226	9063	4384	8988	4540	8910	4695	8829	4848	8746	**60**
1	4229	9062	4386	8987	4542	8909	4697	8828	4851	8745	59
2	4231	9061	4389	8985	4545	8907	4700	8827	4853	8743	58
3	4234	9059	4392	8984	4548	8906	4702	8825	4856	8742	57
4	4237	9058	4394	8983	4550	8905	4705	8824	4858	8741	56
5	4239	9057	4397	8982	4553	8903	4708	8823	4861	8739	55
6	4242	9056	4399	8980	4555	8902	4710	8821	4863	8738	54
7	4245	9054	4402	8979	4558	8901	4713	8820	4866	8736	53
8	4247	9053	4405	8978	4561	8899	4715	8819	4868	8735	52
9	4250	9052	4407	8976	4563	8898	4718	8817	4871	8733	51
10	4253	9051	4410	8975	4566	8897	4720	8816	4874	8732	**50**
11	4255	9050	4412	8974	4568	8895	4723	8814	4876	8731	49
12	4258	9048	4415	8973	4571	8894	4726	8813	4879	8729	48
13	4260	9047	4418	8971	4574	8893	4728	8812	4881	8728	47
14	4263	9046	4420	8970	4576	8892	4731	8810	4884	8726	46
15	4266	9045	4423	8969	4579	8890	4733	8809	4886	8725	45
16	4268	9043	4425	8967	4581	8889	4736	8808	4889	8724	44
17	4271	9042	4428	8966	4584	8888	4738	8806	4891	8722	43
18	4274	9041	4431	8965	4586	8886	4741	8805	4894	8721	42
19	4276	9040	4433	8964	4589	8885	4743	8803	4896	8719	41
20	4279	9038	4436	8962	4592	8884	4746	8802	4899	8718	**40**
21	4281	9037	4439	8961	4594	8882	4749	8801	4901	8716	39
22	4284	9036	4441	8960	4597	8881	4751	8799	4904	8715	38
23	4287	9035	4444	8958	4599	8879	4754	8798	4907	8714	37
24	4289	9033	4446	8957	4602	8878	4756	8796	4909	8712	36
25	4292	9032	4449	8956	4605	8877	4759	8795	4912	8711	35
26	4295	9031	4452	8955	4607	8875	4761	8794	4914	8709	34
27	4297	9030	4454	8953	4610	8874	4764	8792	4917	8708	33
28	4300	9028	4457	8952	4612	8873	4766	8791	4919	8706	32
29	4302	9027	4459	8951	4615	8871	4769	8790	4922	8705	31
30	4305	9026	4462	8949	4617	8870	4772	8788	4924	8704	**30**
31	4308	9025	4465	8948	4620	8869	4774	8787	4927	8702	29
32	4310	9023	4467	8947	4623	8867	4777	8785	4929	8701	28
33	4313	9022	4470	8945	4625	8866	4779	8784	4932	8699	27
34	4316	9021	4472	8944	4628	8865	4782	8783	4934	8698	26
35	4318	9020	4475	8943	4630	8863	4784	8781	4937	8696	25
36	4321	9018	4478	8942	4633	8862	4787	8780	4939	8695	24
37	4323	9017	4480	8940	4636	8861	4789	8778	4942	8694	23
38	4326	9016	4483	8939	4638	8859	4792	8777	4944	8692	22
39	4329	9015	4485	8938	4641	8858	4795	8776	4947	8691	21
40	4331	9013	4488	8936	4643	8857	4797	8774	4950	8689	**20**
41	4334	9012	4491	8935	4646	8855	4800	8773	4952	8688	19
42	4337	9011	4493	8934	4648	8854	4802	8771	4955	8686	18
43	4339	9010	4496	8932	4651	8853	4805	8770	4957	8685	17
44	4342	9008	4498	8931	4654	8851	4807	8769	4960	8683	16
45	4344	9007	4501	8930	4656	8850	4810	8767	4962	8682	15
46	4347	9006	4504	8928	4659	8849	4812	8766	4965	8681	14
47	4350	9004	4506	8927	4661	8847	4815	8764	4967	8679	13
48	4352	9003	4509	8926	4664	8846	4818	8763	4970	8678	12
49	4355	9002	4511	8925	4666	8844	4820	8762	4972	8676	11
50	4358	9001	4514	8923	4669	8843	4823	8760	4975	8675	**10**
51	4360	8999	4517	8922	4672	8842	4825	8759	4977	8673	9
52	4363	8998	4519	8921	4674	8840	4828	8757	4980	8672	8
53	4365	8997	4522	8919	4677	8839	4830	8756	4982	8670	7
54	4368	8996	4524	8918	4679	8838	4833	8755	4985	8669	6
55	4371	8994	4527	8917	4682	8836	4835	8753	4987	8668	5
56	4373	8993	4530	8915	4684	8835	4838	8752	4990	8666	4
57	4376	8992	4532	8914	4687	8834	4840	8750	4992	8665	3
58	4378	8990	4535	8913	4690	8832	4843	8749	4995	8663	2
59	4381	8989	4537	8911	4692	8831	4846	8748	4997	8662	1
60	4384	8988	4540	8910	4695	8829	4848	8746	5000	8660	**0**
′	64° cos	64° sin	63° cos	63° sin	62° cos	62° sin	61° cos	61° sin	60° cos	60° sin	′

′	30° sin	30° cos	31° sin	31° cos	32° sin	32° cos	33° sin	33° cos	34° sin	34° cos	′
0	5000	8660	5150	8572	5299	8480	5446	8387	5592	8290	**60**
1	5003	8659	5153	8570	5302	8479	5449	8385	5594	8289	59
2	5005	8657	5155	8569	5304	8477	5451	8384	5597	8287	58
3	5008	8656	5158	8567	5307	8476	5454	8382	5599	8285	57
4	5010	8654	5160	8566	5309	8474	5456	8380	5602	8284	56
5	5013	8653	5163	8564	5312	8473	5459	8379	5604	8282	55
6	5015	8652	5165	8563	5314	8471	5461	8377	5606	8281	54
7	5018	8650	5168	8561	5316	8470	5463	8376	5609	8279	53
8	5020	8649	5170	8560	5319	8468	5466	8374	5611	8277	52
9	5023	8647	5173	8558	5321	8467	5468	8372	5614	8276	51
10	5025	8646	5175	8557	5324	8465	5471	8371	5616	8274	**50**
11	5028	8644	5178	8555	5326	8463	5473	8369	5618	8272	49
12	5030	8643	5180	8554	5329	8462	5476	8368	5621	8271	48
13	5033	8641	5183	8552	5331	8460	5478	8366	5623	8269	47
14	5035	8640	5185	8551	5334	8459	5480	8364	5626	8268	46
15	5038	8638	5188	8549	5336	8457	5483	8363	5628	8266	45
16	5040	8637	5190	8548	5339	8456	5485	8361	5630	8264	44
17	5043	8635	5193	8546	5341	8454	5488	8360	5633	8263	43
18	5045	8634	5195	8545	5344	8453	5490	8358	5635	8261	42
19	5048	8632	5198	8543	5346	8451	5493	8356	5638	8259	41
20	5050	8631	5200	8542	5348	8450	5495	8355	5640	8258	**40**
21	5053	8630	5203	8540	5351	8448	5498	8353	5642	8256	39
22	5055	8628	5205	8539	5353	8446	5500	8352	5645	8254	38
23	5058	8627	5208	8537	5356	8445	5502	8350	5647	8253	37
24	5060	8625	5210	8536	5358	8443	5505	8348	5650	8251	36
25	5063	8624	5213	8534	5361	8442	5507	8347	5652	8249	35
26	5065	8622	5215	8532	5363	8440	5510	8345	5654	8248	34
27	5068	8621	5218	8531	5366	8439	5512	8344	5657	8246	33
28	5070	8619	5220	8529	5368	8437	5515	8342	5659	8245	32
29	5073	8618	5223	8528	5371	8435	5517	8340	5662	8243	31
30	5075	8616	5225	8526	5373	8434	5519	8339	5664	8241	**30**
31	5078	8615	5227	8525	5375	8432	5522	8337	5666	8240	29
32	5080	8613	5230	8523	5378	8431	5524	8336	5669	8238	28
33	5083	8612	5232	8522	5380	8429	5527	8334	5671	8236	27
34	5085	8610	5235	8520	5383	8428	5529	8332	5674	8235	26
35	5088	8609	5237	8519	5385	8426	5531	8331	5676	8233	25
36	5090	8607	5240	8517	5388	8425	5534	8329	5678	8231	24
37	5093	8606	5242	8516	5390	8423	5536	8328	5681	8230	23
38	5095	8604	5245	8514	5393	8421	5539	8326	5683	8228	22
39	5098	8603	5247	8513	5395	8420	5541	8324	5686	8226	21
40	5100	8601	5250	8511	5398	8418	5544	8323	5688	8225	**20**
41	5103	8600	5252	8510	5400	8417	5546	8321	5690	8223	19
42	5105	8599	5255	8508	5402	8415	5548	8320	5693	8221	18
43	5108	8597	5257	8507	5405	8414	5551	8318	5695	8220	17
44	5110	8596	5260	8505	5407	8412	5553	8316	5698	8218	16
45	5113	8594	5262	8504	5410	8410	5556	8315	5700	8216	15
46	5115	8593	5265	8502	5412	8409	5558	8313	5702	8215	14
47	5118	8591	5267	8500	5415	8407	5561	8311	5705	8213	13
48	5120	8590	5270	8499	5417	8406	5563	8310	5707	8211	12
49	5123	8588	5272	8497	5420	8404	5565	8308	5710	8210	11
50	5125	8587	5275	8496	5422	8403	5568	8307	5712	8208	**10**
51	5128	8585	5277	8494	5424	8401	5570	8305	5714	8207	9
52	5130	8584	5279	8493	5427	8399	5573	8303	5717	8205	8
53	5133	8582	5282	8491	5429	8398	5575	8302	5719	8203	7
54	5135	8581	5284	8490	5432	8396	5577	8300	5721	8202	6
55	5138	8579	5287	8488	5434	8395	5580	8299	5724	8200	5
56	5140	8578	5289	8487	5437	8393	5582	8297	5726	8198	4
57	5143	8576	5292	8485	5439	8391	5585	8295	5729	8197	3
58	5145	8575	5294	8484	5442	8390	5587	8294	5731	8195	2
59	5148	8573	5297	8482	5444	8388	5590	8292	5733	8193	1
60	5150	8572	5299	8480	5446	8387	5592	8290	5736	8192	**0**
′	59° cos	59° sin	58° cos	58° sin	57° cos	57° sin	56° cos	56° sin	55° cos	55° sin	′

′	35°		36°		37°		38°		39°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	5736	8192	5878	8090	6018	7986	6157	7880	6293	7771	**60**
1	5738	8190	5880	8088	6020	7985	6159	7878	6295	7770	59
2	5741	8188	5883	8087	6023	7983	6161	7877	6298	7768	58
3	5743	8187	5885	8085	6025	7981	6163	7875	6300	7766	57
4	5745	8185	5887	8083	6027	7979	6166	7873	6302	7764	56
5	5748	8183	5890	8082	6030	7978	6168	7871	6305	7762	55
6	5750	8181	5892	8080	6032	7976	6170	7869	6307	7760	54
7	5752	8180	5894	8078	6034	7974	6173	7868	6309	7759	53
8	5755	8178	5897	8076	6037	7972	6175	7866	6311	7757	52
9	5757	8176	5899	8075	6039	7971	6177	7864	6314	7755	51
10	5760	8175	5901	8073	6041	7969	6180	7862	6316	7753	**50**
11	5762	8173	5904	8071	6044	7967	6182	7860	6318	7751	49
12	5764	8171	5906	8070	6046	7965	6184	7859	6320	7749	48
13	5767	8170	5908	8068	6048	7964	6186	7857	6323	7748	47
14	5769	8168	5911	8066	6051	7962	6189	7855	6325	7746	46
15	5771	8166	5913	8064	6053	7960	6191	7853	6327	7744	45
16	5774	8165	5915	8063	6055	7958	6193	7851	6329	7742	44
17	5776	8163	5918	8061	6058	7956	6196	7850	6332	7740	43
18	5779	8161	5920	8059	6060	7955	6198	7848	6334	7738	42
19	5781	8160	5922	8058	6062	7953	6200	7846	6336	7737	41
20	5783	8158	5925	8056	6065	7951	6202	7844	6338	7735	**40**
21	5786	8156	5927	8054	6067	7950	6205	7842	6341	7733	39
22	5788	8155	5930	8052	6069	7948	6207	7841	6343	7731	38
23	5790	8153	5932	8051	6071	7946	6209	7839	6345	7729	37
24	5793	8151	5934	8049	6074	7944	6211	7837	6347	7727	36
25	5795	8150	5937	8047	6076	7942	6214	7835	6350	7725	35
26	5798	8148	5939	8045	6078	7941	6216	7833	6352	7724	34
27	5800	8146	5941	8044	6081	7939	6218	7832	6354	7722	33
28	5802	8145	5944	8042	6083	7937	6221	7830	6356	7720	32
29	5805	8143	5946	8040	6085	7935	6223	7828	6359	7718	31
30	5807	8141	5948	8039	6088	7934	6225	7826	6361	7716	**30**
31	5809	8139	5951	8037	6090	7932	6227	7824	6363	7714	29
32	5812	8138	5953	8035	6092	7930	6230	7822	6365	7713	28
33	5814	8136	5955	8033	6095	7928	6232	7821	6368	7711	27
34	5816	8134	5958	8032	6097	7926	6234	7819	6370	7709	26
35	5819	8133	5960	8030	6099	7925	6237	7817	6372	7707	25
36	5821	8131	5962	8028	6101	7923	6239	7815	6374	7705	24
37	5824	8129	5965	8026	6104	7921	6241	7813	6376	7703	23
38	5826	8128	5967	8025	6106	7919	6243	7812	6379	7701	22
39	5828	8126	5969	8023	6108	7918	6246	7810	6381	7700	21
40	5831	8124	5972	8021	6111	7916	6248	7808	6383	7698	**20**
41	5833	8123	5974	8020	6113	7914	6250	7806	6385	7696	19
42	5835	8121	5976	8018	6115	7912	6252	7804	6388	7694	18
43	5838	8119	5979	8016	6118	7910	6255	7802	6390	7692	17
44	5840	8117	5981	8014	6120	7909	6257	7801	6392	7690	16
45	5842	8116	5983	8013	6122	7907	6259	7799	6394	7688	15
46	5845	8114	5986	8011	6124	7905	6262	7797	6397	7687	14
47	5847	8112	5988	8009	6127	7903	6264	7795	6399	7685	13
48	5850	8111	5990	8007	6129	7902	6266	7793	6401	7683	12
49	5852	8109	5993	8006	6131	7900	6268	7792	6403	7681	11
50	5854	8107	5995	8004	6134	7898	6271	7790	6406	7679	**10**
51	5857	8106	5997	8002	6136	7896	6273	7788	6408	7677	9
52	5859	8104	6000	8000	6138	7894	6275	7786	6410	7675	8
53	5861	8102	6002	7999	6141	7893	6277	7784	6412	7674	7
54	5864	8100	6004	7997	6143	7891	6280	7782	6414	7672	6
55	5866	8099	6007	7995	6145	7889	6282	7781	6417	7670	5
56	5868	8097	6009	7993	6147	7887	6284	7779	6419	7668	4
57	5871	8095	6011	7992	6150	7885	6286	7777	6421	7666	3
58	5873	8094	6014	7990	6152	7884	6289	7775	6423	7664	2
59	5875	8092	6016	7988	6154	7882	6291	7773	6426	7662	1
60	5878	8090	6018	7986	6157	7880	6293	7771	6428	7660	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	6428	7660	6561	7547	6691	7431	6820	7314	6947	7193	**60**
1	6430	7659	6563	7545	6693	7430	6822	7312	6949	7191	59
2	6432	7657	6565	7543	6696	7428	6824	7310	6951	7189	58
3	6435	7655	6567	7541	6698	7426	6826	7308	6953	7187	57
4	6437	7653	6569	7539	6700	7424	6828	7306	6955	7185	56
5	6439	7651	6572	7538	6702	7422	6831	7304	6957	7183	55
6	6441	7649	6574	7536	6704	7420	6833	7302	6959	7181	54
7	6443	7647	6576	7534	6706	7418	6835	7300	6961	7179	53
8	6446	7645	6578	7532	6709	7416	6837	7298	6963	7177	52
9	6448	7644	6580	7530	6711	7414	6839	7296	6965	7175	51
10	6450	7642	6583	7528	6713	7412	6841	7294	6967	7173	**50**
11	6452	7640	6585	7526	6715	7410	6843	7292	6970	7171	49
12	6455	7638	6587	7524	6717	7408	6845	7290	6972	7169	48
13	6457	7636	6589	7522	6719	7406	6848	7288	6974	7167	47
14	6459	7634	6591	7520	6722	7404	6850	7286	6976	7165	46
15	6461	7632	6593	7518	6724	7402	6852	7284	6978	7163	45
16	6463	7630	6596	7516	6726	7400	6854	7282	6980	7161	44
17	6466	7629	6598	7515	6728	7398	6856	7280	6982	7159	43
18	6468	7627	6600	7513	6730	7396	6858	7278	6984	7157	42
19	6470	7625	6602	7511	6732	7394	6860	7276	6986	7155	41
20	6472	7623	6604	7509	6734	7392	6862	7274	6988	7153	**40**
21	6475	7621	6607	7507	6737	7390	6865	7272	6990	7151	39
22	6477	7619	6609	7505	6739	7388	6867	7270	6992	7149	38
23	6479	7617	6611	7503	6741	7387	6869	7268	6995	7147	37
24	6481	7615	6613	7501	6743	7385	6871	7266	6997	7145	36
25	6483	7613	6615	7499	6745	7383	6873	7264	6999	7143	35
26	6486	7612	6617	7497	6747	7381	6875	7262	7001	7141	34
27	6488	7610	6620	7495	6749	7379	6877	7260	7003	7139	33
28	6490	7608	6622	7493	6752	7377	6879	7258	7005	7137	32
29	6492	7606	6624	7491	6754	7375	6881	7256	7007	7135	31
30	6494	7604	6626	7490	6756	7373	6884	7254	7009	7133	**30**
31	6497	7602	6628	7488	6758	7371	6886	7252	7011	7130	29
32	6499	7600	6631	7486	6760	7369	6888	7250	7013	7128	28
33	6501	7598	6633	7484	6762	7367	6890	7248	7015	7126	27
34	6503	7596	6635	7482	6764	7365	6892	7246	7017	7124	26
35	6506	7595	6637	7480	6767	7363	6894	7244	7019	7122	25
36	6508	7593	6639	7478	6769	7361	6896	7242	7022	7120	24
37	6510	7591	6641	7476	6771	7359	6898	7240	7024	7118	23
38	6512	7589	6644	7474	6773	7357	6900	7238	7026	7116	22
39	6514	7587	6646	7472	6775	7355	6903	7236	7028	7114	21
40	6517	7585	6648	7470	6777	7353	6905	7234	7030	7112	**20**
41	6519	7583	6650	7468	6779	7351	6907	7232	7032	7110	19
42	6521	7581	6652	7466	6782	7349	6909	7230	7034	7108	18
43	6523	7579	6654	7464	6784	7347	6911	7228	7036	7106	17
44	6525	7578	6657	7463	6786	7345	6913	7226	7038	7104	16
45	6528	7576	6659	7461	6788	7343	6915	7224	7040	7102	15
46	6530	7574	6661	7459	6790	7341	6917	7222	7042	7100	14
47	6532	7572	6663	7457	6792	7339	6919	7220	7044	7098	13
48	6534	7570	6665	7455	6794	7337	6921	7218	7046	7096	12
49	6536	7568	6667	7453	6797	7335	6924	7216	7048	7094	11
50	6539	7566	6670	7451	6799	7333	6926	7214	7050	7092	**10**
51	6541	7564	6672	7449	6801	7331	6928	7212	7053	7090	9
52	6543	7562	6674	7447	6803	7329	6930	7210	7055	7088	8
53	6545	7560	6676	7445	6805	7327	6932	7208	7057	7085	7
54	6547	7559	6678	7443	6807	7325	6934	7206	7059	7083	6
55	6550	7557	6680	7441	6809	7323	6936	7203	7061	7081	5
56	6552	7555	6683	7439	6811	7321	6938	7201	7063	7079	4
57	6554	7553	6685	7437	6814	7319	6940	7199	7065	7077	3
58	6556	7551	6687	7435	6816	7318	6942	7197	7067	7075	2
59	6558	7549	6689	7433	6818	7316	6944	7195	7069	7073	1
60	6561	7547	6691	7431	6820	7314	6947	7193	7071	7071	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	49°		48°		47°		46°		45°		′

'	0°		1°		2°		3°		4°		'
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0000	Infinite	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	**60**
1	0003	3437.75	0177	56.3506	0352	28.3994	0527	18.9755	0702	14.2411	59
2	0006	1718.87	0180	55.4415	0355	28.1664	0530	18.8711	0705	14.1821	58
3	0009	1145.92	0183	54.5613	0358	27.9372	0533	18.7678	0708	14.1235	57
4	0012	859.436	0186	53.7086	0361	27.7117	0536	18.6656	0711	14.0655	56
5	0015	687.549	0189	52.8821	0364	27.4899	0539	18.5645	0714	14.0079	55
6	0017	572.957	0192	52.0807	0367	27.2715	0542	18.4645	0717	13.9507	54
7	0020	491.106	0195	51.3032	0370	27.0566	0544	18.3655	0720	13.8940	53
8	0023	429.718	0198	50.5485	0373	26.8450	0547	18.2677	0723	13.8378	52
9	0026	381.971	0201	49.8157	0375	26.6367	0550	18.1708	0726	13.7821	51
10	0029	343.774	0204	49.1039	0378	26.4316	0553	18.0750	0729	13.7267	**50**
11	0032	312.521	0207	48.4121	0381	26.2296	0556	17.9802	0731	13.6719	49
12	0035	286.478	0209	47.7395	0384	26.0307	0559	17.8863	0734	13.6174	48
13	0038	264.441	0212	47.0853	0387	25.8348	0562	17.7934	0737	13.5634	47
14	0041	245.552	0215	46.4489	0390	25.6418	0565	17.7015	0740	13.5098	46
15	0044	229.182	0218	45.8294	0393	25.4517	0568	17.6106	0743	13.4566	45
16	0047	214.858	0221	45.2261	0396	25.2644	0571	17.5205	0746	13.4039	44
17	0049	202.219	0224	44.6386	0399	25.0798	0574	17.4314	0749	13.3515	43
18	0052	190.984	0227	44.0661	0402	24.8978	0577	17.3432	0752	13.2996	42
19	0055	180.932	0230	43.5081	0405	24.7185	0580	17.2558	0755	13.2480	41
20	0058	171.885	0233	42.9641	0407	24.5418	0582	17.1693	0758	13.1969	**40**
21	0061	163.700	0236	42.4335	0410	24.3675	0585	17.0837	0761	13.1461	39
22	0064	156.259	0239	41.9158	0413	24.1957	0588	16.9990	0764	13.0958	38
23	0067	149.465	0241	41.4106	0416	24.0263	0591	16.9150	0767	13.0458	37
24	0070	143.237	0244	40.9174	0419	23.8593	0594	16.8319	0769	12.9962	36
25	0073	137.507	0247	40.4358	0422	23.6945	0597	16.7496	0772	12.9469	35
26	0076	132.219	0250	39.9655	0425	23.5321	0600	16.6681	0775	12.8981	34
27	0079	127.321	0253	39.5059	0428	23.3718	0603	16.5874	0778	12.8496	33
28	0081	122.774	0256	39.0568	0431	23.2137	0606	16.5075	0781	12.8014	32
29	0084	118.540	0259	38.6177	0434	23.0577	0609	16.4283	0784	12.7536	31
30	0087	114.589	0262	38.1885	0437	22.9038	0612	16.3499	0787	12.7062	**30**
31	0090	110.892	0265	37.7686	0440	22.7519	0615	16.2722	0790	12.6591	29
32	0093	107.426	0268	37.3579	0442	22.6020	0617	16.1952	0793	12.6124	28
33	0096	104.171	0271	36.9560	0445	22.4541	0620	16.1190	0796	12.5660	27
34	0099	101.107	0274	36.5627	0448	22.3081	0623	16.0435	0799	12.5199	26
35	0102	98.2179	0276	36.1776	0451	22.1640	0626	15.9687	0802	12.4742	25
36	0105	95.4895	0279	35.8006	0454	22.0217	0629	15.8945	0805	12.4288	24
37	0108	92.9085	0282	35.4313	0457	21.8813	0632	15.8211	0808	12.3838	23
38	0111	90.4633	0285	35.0695	0460	21.7426	0635	15.7483	0810	12.3390	22
39	0113	88.1436	0288	34.7151	0463	21.6056	0638	15.6762	0813	12.2946	21
40	0116	85.9398	0291	34.3678	0466	21.4704	0641	15.6048	0816	12.2505	**20**
41	0119	83.8435	0294	34.0273	0469	21.3369	0644	15.5340	0819	12.2067	19
42	0122	81.8470	0297	33.6935	0472	21.2049	0647	15.4638	0822	12.1632	18
43	0125	79.9434	0300	33.3662	0475	21.0747	0650	15.3943	0825	12.1201	17
44	0128	78.1263	0303	33.0452	0477	20.9460	0653	15.3254	0828	12.0772	16
45	0131	76.3900	0306	32.7303	0480	20.8188	0655	15.2571	0831	12.0346	15
46	0134	74.7292	0308	32.4213	0483	20.6932	0658	15.1893	0834	11.9923	14
47	0137	73.1390	0311	32.1181	0486	20.5691	0661	15.1222	0837	11.9504	13
48	0140	71.6151	0314	31.8205	0489	20.4465	0664	15.0557	0840	11.9087	12
49	0143	70.1533	0317	31.5284	0492	20.3253	0667	14.9898	0843	11.8673	11
50	0146	68.7501	0320	31.2416	0495	20.2056	0670	14.9244	0846	11.8262	**10**
51	0148	67.4019	0323	30.9599	0498	20.0872	0673	14.8596	0849	11.7853	9
52	0151	66.1055	0326	30.6833	0501	19.9702	0676	14.7954	0851	11.7448	8
53	0154	64.8580	0329	30.4116	0504	19.8546	0679	14.7317	0854	11.7045	7
54	0157	63.6567	0332	30.1446	0507	19.7403	0682	14.6685	0857	11.6645	6
55	0160	62.4992	0335	29.8823	0509	19.6273	0685	14.6059	0860	11.6248	5
56	0163	61.3829	0338	29.6245	0512	19.5156	0688	14.5438	0863	11.5853	4
57	0166	60.3058	0340	29.3711	0515	19.4051	0690	14.4823	0866	11.5461	3
58	0169	59.2659	0343	29.1220	0518	19.2959	0693	14.4212	0869	11.5072	2
59	0172	58.2612	0346	28.8771	0521	19.1879	0696	14.3607	0872	11.4685	1
60	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	0875	11.4301	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
'	89°		88°		87°		86°		85°		'

′	5°		6°		7°		8°		9°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0875	11.4301	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	**60**
1	0878	11.3919	1054	9.4878	1231	8.1248	1408	7.1004	1587	6.3019	59
2	0881	11.3540	1057	9.4614	1234	8.1054	1411	7.0855	1590	6.2901	58
3	0884	11.3163	1060	9.4352	1237	8.0860	1414	7.0706	1593	6.2783	57
4	0887	11.2789	1063	9.4090	1240	8.0667	1417	7.0558	1596	6.2666	56
5	0890	11.2417	1066	9.3831	1243	8.0476	1420	7.0410	1599	6.2549	55
6	0892	11.2048	1069	9.3572	1246	8.0285	1423	7.0264	1602	6.2432	54
7	0895	11.1681	1072	9.3315	1249	8.0095	1426	7.0117	1605	6.2316	53
8	0898	11.1316	1075	9.3060	1251	7.9906	1429	6.9972	1608	6.2200	52
9	0901	11.0954	1078	9·2806	1254	7.9718	1432	6.9827	1611	6.2085	51
10	0904	11.0594	1080	9.2553	1257	7.9530	1435	6.9682	1614	6.1970	**50**
11	0907	11.0237	1083	9.2302	1260	7.9344	1438	6.9538	1617	6.1856	49
12	0910	10.9882	1086	9.2052	1263	7.9158	1441	6.9395	1620	6.1742	48
13	0913	10.9529	1089	9.1803	1266	7.8973	1444	6.9252	1623	6.1628	47
14	0916	10.9178	1092	9.1555	1269	7.8789	1447	6.9110	1626	6.1515	46
15	0919	10.8829	1095	9.1309	1272	7.8606	1450	6.8969	1629	6.1402	45
16	0922	10.8483	1098	9.1065	1275	7.8424	1453	6.8828	1632	6.1290	44
17	0925	10.8139	1101	9.0821	1278	7.8243	1456	6.8687	1635	6.1178	43
18	0928	10.7797	1104	9.0579	1281	7.8062	1459	6.8548	1638	6.1066	42
19	0931	10.7457	1107	9.0338	1284	7.7883	1462	6.8408	1641	6.0955	41
20	0934	10.7119	1110	9.0098	1287	7.7704	1465	6.8269	1644	6.0844	**40**
21	0936	10.6783	1113	8.9860	1290	7.7525	1468	6.8131	1647	6.0734	39
22	0939	10.6450	1116	8.9623	1293	7.7348	1471	6.7994	1650	6.0624	38
23	0942	10.6118	1119	8.9387	1296	7.7171	1474	6.7856	1653	6.0514	37
24	0945	10.5789	1122	8.9152	1299	7.6996	1477	6.7720	1655	6.0405	36
25	0948	10.5462	1125	8.8919	1302	7.6821	1480	6.7584	1658	6.0296	35
26	0951	10.5136	1128	8.8686	1305	7.6647	1483	6.7448	1661	6.0188	34
27	0954	10.4813	1131	8.8455	1308	7.6473	1486	6.7313	1664	6.0080	33
28	0957	10.4491	1134	8.8225	1311	7.6301	1489	6.7179	1667	5.9972	32
29	0960	10.4172	1136	8.7996	1314	7.6129	1492	6.7045	1670	5.9865	31
30	0963	10.3854	1139	8.7769	1317	7.5958	1495	6.6912	1673	5.9758	**30**
31	0966	10.3538	1142	8.7542	1319	7.5787	1497	6.6779	1676	5.9651	29
32	0969	10.3224	1145	8.7317	1322	7.5618	1500	6.6646	1679	5.9545	28
33	0972	10.2913	1148	8.7093	1325	7.5449	1503	6.6514	1682	5.9439	27
34	0975	10.2602	1151	8.6870	1328	7.5281	1506	6.6383	1685	5.9333	26
35	0978	10.2294	1154	8.6648	1331	7.5113	1509	6.6252	1688	5.9228	25
36	0981	10.1988	1157	8.6427	1334	7.4947	1512	6.6122	1691	5.9124	24
37	0983	10.1683	1160	8.6208	1337	7.4781	1515	6.5992	1694	5.9019	23
38	0986	10.1381	1163	8.5989	1340	7.4615	1518	6.5863	1697	5.8915	22
39	0989	10.1080	1166	8.5772	1343	7.4451	1521	6.5734	1700	5.8811	21
40	0992	10.0780	1169	8.5555	1346	7.4287	1524	6.5606	1703	5.8708	**20**
41	0995	10.0483	1172	8.5340	1349	7.4124	1527	6.5478	1706	5.8605	19
42	0998	10.0187	1175	8.5126	1352	7.3962	1530	6.5350	1709	5.8502	18
43	1001	9.9893	1178	8.4913	1355	7.3800	1533	6.5223	1712	5.8400	17
44	1004	9.9601	1181	8.4701	1358	7.3639	1536	6.5097	1715	5.8298	16
45	1007	9.9310	1184	8.4490	1361	7.3479	1539	6.4971	1718	5.8197	15
46	1010	9.9021	1187	8.4280	1364	7.3319	1542	6.4846	1721	5.8095	14
47	1013	9.8734	1189	8.4071	1367	7.3160	1545	6.4721	1724	5.7994	13
48	1016	9.8448	1192	8.3863	1370	7.3002	1548	6.4596	1727	5.7894	12
49	1019	9.8164	1195	8.3656	1373	7.2844	1551	6.4472	1730	5.7794	11
50	1022	9.7882	1198	8.3450	1376	7.2687	1554	6.4348	1733	5.7694	**10**
51	1025	9.7601	1201	8.3245	1379	7.2531	1557	6.4225	1736	5.7594	9
52	1028	9.7322	1204	8.3041	1382	7.2375	1560	6.4103	1739	5.7495	8
53	1030	9.7044	1207	8.2838	1385	7.2220	1563	6.3980	1742	5.7396	7
54	1033	9.6768	1210	8.2636	1388	7.2066	1566	6.3859	1745	5.7297	6
55	1036	9.6499	1213	8.2434	1391	7.1912	1569	6.3737	1748	5.7199	5
56	1039	9.6220	1216	8.2234	1394	7.1759	1572	6.3617	1751	5.7101	4
57	1042	9.5949	1219	8.2035	1397	7.1607	1575	6.3496	1754	5.7004	3
58	1045	9.5679	1222	8.1837	1399	7.1455	1578	6.3376	1757	5.6906	2
59	1048	9.5411	1225	8.1640	1402	7.1304	1581	6.3257	1760	5.6809	1
60	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	1763	5.6713	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	1763	5.6713	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	**60**
1	1766	5.6617	1947	5.1366	2129	4.6979	2312	4.3257	2496	4.0058	59
2	1769	5.6521	1950	5.1286	2132	4.6912	2315	4.3200	2499	4.0009	58
3	1772	5.6425	1953	5.1207	2135	4.6845	2318	4.3143	2503	3.9959	57
4	1775	5.6330	1956	5.1128	2138	4.6779	2321	4.3086	2506	3.9910	56
5	1778	5.6234	1959	5.1049	2141	4.6712	2324	4.3029	2509	3.9861	55
6	1781	5.6140	1962	5.0970	2144	4.6646	2327	4.2972	2512	3.9812	54
7	1784	5.6045	1965	5.0892	2147	4.6580	2330	4.2916	2515	3.9763	53
8	1787	5.5951	1968	5.0814	2150	4.6514	2333	4.2859	2518	3.9714	52
9	1790	5.5857	1971	5.0736	2153	4.6448	2336	4.2803	2521	3.9665	51
10	1793	5.5764	1974	5.0658	2156	4.6382	2339	4.2747	2524	3.9617	**50**
11	1796	5.5671	1977	5.0581	2159	4.6317	2342	4.2691	2527	3.9568	49
12	1799	5.5578	1980	5.0504	2162	4.6252	2345	4.2635	2530	3.9520	48
13	1802	5.5485	1983	5.0427	2165	4.6187	2349	4.2580	2533	3.9471	47
14	1805	5.5393	1986	5.0350	2168	4.6122	2352	4.2524	2537	3.9423	46
15	1808	5.5301	1989	5.0273	2171	4.6057	2355	4.2468	2540	3.9375	45
16	1811	5.5209	1992	5.0197	2174	4.5993	2358	4.2413	2543	3.9327	44
17	1814	5.5118	1995	5.0121	2177	4.5928	2361	4.2358	2546	3.9279	43
18	1817	5.5026	1998	5.0045	2180	4.5864	2364	4.2303	2549	3.9232	42
19	1820	5.4936	2001	4.9969	2183	4.5800	2367	4.2248	2552	3.9184	41
20	1823	5.4845	2004	4.9894	2186	4.5736	2370	4.2193	2555	3.9136	**40**
21	1826	5.4755	2007	4.9819	2189	4.5673	2373	4.2139	2558	3.9089	39
22	1829	5.4665	2010	4.9744	2193	4.5609	2376	4.2084	2561	3.9042	38
23	1832	5.4575	2013	4.9669	2196	4.5546	2379	4.2030	2564	3.8995	37
24	1835	5.4486	2016	4.9594	2199	4.5483	2382	4.1976	2568	3.8947	36
25	1838	5.4397	2019	4.9520	2202	4.5420	2385	4.1922	2571	3.8900	35
26	1841	5.4308	2022	4.9446	2205	4.5357	2388	4.1868	2574	3.8854	34
27	1844	5.4219	2025	4.9372	2208	4.5294	2392	4.1814	2577	3.8807	33
28	1847	5.4131	2028	4.9298	2211	4.5232	2395	4.1760	2580	3.8760	32
29	1850	5.4043	2031	4.9225	2214	4.5169	2398	4.1706	2583	3.8714	31
30	1853	5.3955	2035	4.9152	2217	4.5107	2401	4.1653	2586	3.8667	**30**
31	1856	5.3868	2038	4.9078	2220	4.5045	2404	4.1600	2589	3.8621	29
32	1859	5.3781	2041	4.9006	2223	4.4983	2407	4.1547	2592	3.8575	28
33	1862	5.3694	2044	4.8933	2226	4.4922	2410	4.1493	2595	3.8528	27
34	1865	5.3607	2047	4.8860	2229	4.4860	2413	4.1441	2599	3.8482	26
35	1868	5.3521	2050	4.8788	2232	4.4799	2416	4.1388	2602	3.8436	25
36	1871	5.3435	2053	4.8716	2235	4.4737	2419	4.1335	2605	3.8391	24
37	1874	5.3349	2056	4.8644	2238	4.4676	2422	4.1282	2608	3.8345	23
38	1877	5.3263	2059	4.8573	2241	4.4615	2425	4.1230	2611	3.8299	22
39	1880	5.3178	2062	4.8501	2244	4.4555	2428	4.1178	2614	3.8254	21
40	1883	5.3093	2065	4.8430	2247	4.4494	2432	4.1126	2617	3.8208	**20**
41	1887	5.3008	2068	4.8359	2251	4.4434	2435	4.1074	2620	3.8163	19
42	1890	5.2924	2071	4.8288	2254	4.4374	2438	4.1022	2623	3.8118	18
43	1893	5.2839	2074	4.8218	2257	4.4313	2441	4.0970	2627	3.8073	17
44	1896	5.2755	2077	4.8147	2260	4.4253	2444	4.0918	2630	3.8028	16
45	1899	5.2672	2080	4.8077	2263	4.4194	2447	4.0867	2633	3.7983	15
46	1902	5.2588	2083	4.8007	2266	4.4134	2450	4.0815	2636	3.7938	14
47	1905	5.2505	2086	4.7937	2269	4.4075	2453	4.0764	2639	3.7893	13
48	1908	5.2422	2089	4.7867	2272	4.4015	2456	4.0713	2642	3.7848	12
49	1911	5.2339	2092	4.7798	2275	4.3956	2459	4.0662	2645	3.7804	11
50	1914	5.2257	2095	4.7729	2278	4.3897	2462	4.0611	2648	3.7760	**10**
51	1917	5.2174	2098	4.7659	2281	4.3838	2465	4.0560	2651	3.7715	9
52	1920	5.2092	2101	4.7591	2284	4.3779	2469	4.0509	2655	3.7671	8
53	1923	5.2011	2104	4.7522	2287	4.3721	2472	4.0459	2658	3.7627	7
54	1926	5.1929	2107	4.7453	2290	4.3662	2475	4.0408	2661	3.7583	6
55	1929	5.1848	2110	4.7385	2293	4.3604	2478	4.0358	2664	3.7539	5
56	1932	5.1767	2113	4.7317	2296	4.3546	2481	4.0308	2667	3.7495	4
57	1935	5.1686	2116	4.7249	2299	4.3488	2484	4.0257	2670	3.7451	3
58	1938	5.1606	2119	4.7181	2303	4.3430	2487	4.0207	2673	3.7408	2
59	1941	5.1526	2123	4.7114	2306	4.3372	2490	4.0158	2676	3.7364	1
60	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	2679	3.7321	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	2679	3.7321	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	**60**
1	2683	3.7277	2871	3.4836	3060	3.2675	3252	3.0746	3447	2.9015	59
2	2686	3.7234	2874	3.4798	3064	3.2641	3256	3.0716	3450	2.8987	58
3	2689	3.7191	2877	3.4760	3067	3.2607	3259	3.0686	3453	2.8960	57
4	2692	3.7148	2880	3.4722	3070	3.2573	3262	3.0655	3456	2.8933	56
5	2695	3.7105	2883	3.4684	3073	3.2539	3265	3.0625	3460	2.8905	55
6	2698	3.7062	2886	3.4646	3076	3.2506	3269	3.0595	3463	2.8878	54
7	2701	3.7019	2890	3.4608	3080	3.2472	3272	3.0565	3466	2.8851	53
8	2704	3.6976	2893	3.4570	3083	3.2438	3275	3.0535	3469	2.8824	52
9	2708	3.6933	2896	3.4533	3086	3.2405	3278	3.0505	3473	2.8797	51
10	2711	3.6891	2899	3.4495	3089	3.2371	3281	3.0475	3476	2.8770	**50**
11	2714	3.6848	2902	3.4458	3092	3.2338	3285	3.0445	3479	2.8743	49
12	2717	3.6806	2905	3.4420	3096	3.2305	3288	3.0415	3482	2.8716	48
13	2720	3.6764	2908	3.4383	3099	3.2272	3291	3.0385	3486	2.8689	47
14	2723	3.6722	2912	3.4346	3102	3.2238	3294	3.0356	3489	2.8662	46
15	2726	3.6680	2915	3.4308	3105	3.2205	3298	3.0326	3492	2.8636	45
16	2729	3.6638	2918	3.4271	3108	3.2172	3301	3.0296	3495	2.8609	44
17	2733	3.6596	2921	3.4234	3111	3.2139	3304	3.0267	3499	2.8582	43
18	2736	3.6554	2924	3.4197	3115	3.2106	3307	3.0237	3502	2.8556	42
19	2739	3.6512	2927	3.4160	3118	3.2073	3310	3.0208	3505	2.8529	41
20	2742	3.6470	2931	3.4124	3121	3.2041	3314	3.0178	3508	2.8502	**40**
21	2745	3.6429	2934	3.4087	3124	3.2008	3317	3.0149	3512	2.8476	39
22	2748	3.6387	2937	3.4050	3127	3.1975	3320	3.0120	3515	2.8449	38
23	2751	3.6346	2940	3.4014	3131	3.1943	3323	3.0090	3518	2.8423	37
24	2754	3.6305	2943	3.3977	3134	3.1910	3327	3.0061	3522	2.8397	36
25	2758	3.6264	2946	3.3941	3137	3.1878	3330	3.0032	3525	2.8370	35
26	2761	3.6222	2949	3.3904	3140	3.1845	3333	3.0003	3528	2.8344	34
27	2764	3.6181	2953	3.3868	3143	3.1813	3336	2.9974	3531	2.8318	33
28	2767	3.6140	2956	3.3832	3147	3.1780	3339	2.9945	3535	2.8291	32
29	2770	3.6100	2959	3.3796	3150	3.1748	3343	2.9916	3538	2.8265	31
30	2773	3.6059	2962	3.3759	3153	3.1716	3346	2.9887	3541	2.8239	**30**
31	2776	3.6018	2965	3.3723	3156	3.1684	3349	2.9858	3544	2.8213	29
32	2780	3.5978	2968	3.3687	3159	3.1652	3352	2.9829	3548	2.8187	28
33	2783	3.5937	2972	3.3652	3163	3.1620	3356	2.9800	3551	2.8161	27
34	2786	3.5897	2975	3.3616	3166	3.1588	3359	2.9772	3554	2.8135	26
35	2789	3.5856	2978	3.3580	3169	3.1556	3362	2.9743	3558	2.8109	25
36	2792	3.5816	2981	3.3544	3172	3.1524	3365	2.9714	3561	2.8083	24
37	2795	3.5776	2984	3.3509	3175	3.1492	3369	2.9686	3564	2.8057	23
38	2798	3.5736	2987	3.3473	3179	3.1460	3372	2.9657	3567	2.8032	22
39	2801	3.5696	2991	3.3438	3182	3.1429	3375	2.9629	3571	2.8006	21
40	2805	3.5656	2994	3.3402	3185	3.1397	3378	2.9600	3574	2.7980	**20**
41	2808	3.5616	2997	3.3367	3188	3.1366	3382	2.9572	3577	2.7955	19
42	2811	3.5576	3000	3.3332	3191	3.1334	3385	2.9544	3581	2.7929	18
43	2814	3.5536	3003	3.3297	3195	3.1303	3388	2.9515	3584	2.7903	17
44	2817	3.5497	3006	3.3261	3198	3.1271	3391	2.9487	3587	2.7878	16
45	2820	3.5457	3010	3.3226	3201	3.1240	3395	2.9459	3590	2.7852	15
46	2823	3.5418	3013	3.3191	3204	3.1209	3398	2.9431	3594	2.7827	14
47	2827	3.5379	3016	3.3156	3207	3.1178	3401	2.9403	3597	2.7801	13
48	2830	3.5339	3019	3.3122	3211	3.1146	3404	2.9375	3600	2.7776	12
49	2833	3.5300	3022	3.3087	3214	3.1115	3408	2.9347	3604	2.7751	11
50	2836	3.5261	3026	3.3052	3217	3.1084	3411	2.9319	3607	2.7725	**10**
51	2839	3.5222	3029	3.3017	3220	3.1053	3414	2.9291	3610	2.7700	9
52	2842	3.5183	3032	3.2983	3223	3.1022	3417	2.9263	3613	2.7675	8
53	2845	3.5144	3035	3.2948	3227	3.0991	3421	2.9235	3617	2.7650	7
54	2849	3.5105	3038	3.2914	3230	3.0961	3424	2.9208	3620	2.7625	6
55	2852	3.5067	3041	3.2880	3233	3.0930	3427	2.9180	3623	2.7500	5
56	2855	3.5028	3045	3.2845	3236	3.0899	3430	2.9152	3627	2.7575	4
57	2858	3.4989	3048	3.2811	3240	3.0868	3434	2.9125	3630	2.7550	3
58	2861	3.4951	3051	3.2777	3243	3.0838	3437	2.9097	3633	2.7525	2
59	2864	3.4912	3054	3.2743	3246	3.0807	3440	2.9070	3636	2.7500	1
60	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	3640	2.7475	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	3640	2.7475	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	**60**
1	3643	2.7450	3842	2.6028	4044	2.4730	4248	2.3539	4456	2.2443	59
2	3646	2.7425	3845	2.6006	4047	2.4709	4252	2.3520	4459	2.2425	58
3	3650	2.7400	3849	2.5983	4050	2.4689	4255	2.3501	4463	2.2408	57
4	3653	2.7376	3852	2.5961	4054	2.4668	4258	2.3483	4466	2.2390	56
5	3656	2.7351	3855	2.5938	4057	2.4648	4262	2.3464	4470	2.2373	55
6	3659	2.7326	3859	2.5916	4061	2.4627	4265	2.3445	4473	2.2355	54
7	3663	2.7302	3862	2.5893	4064	2.4606	4269	2.3426	4477	2.2338	53
8	3666	2.7277	3865	2.5871	4067	2.4586	4272	2.3407	4480	2.2320	52
9	3669	2.7253	3869	2.5848	4071	2.4566	4276	2.3388	4484	2.2303	51
10	3673	2.7228	3872	2.5826	4074	2.4545	4279	2.3369	4487	2.2286	**50**
11	3676	2.7204	3875	2.5804	4078	2.4525	4283	2.3351	4491	2.2268	49
12	3679	2.7179	3879	2.5782	4081	2.4504	4286	2.3332	4494	2.2251	48
13	3683	2.7155	3882	2.5759	4084	2.4484	4289	2.3313	4498	2.2234	47
14	3686	2.7130	3885	2.5737	4088	2.4464	4293	2.3294	4501	2.2216	46
15	3689	2.7106	3889	2.5715	4091	2.4443	4296	2.3276	4505	2.2199	45
16	3693	2.7082	3892	2.5693	4095	2.4423	4300	2.3257	4508	2.2182	44
17	3696	2.7058	3895	2.5671	4098	2.4403	4303	2.3238	4512	2.2165	43
18	3699	2.7034	3899	2.5649	4101	2.4383	4307	2.3220	4515	2.2148	42
19	3702	2.7009	3902	2.5627	4105	2.4362	4310	2.3201	4519	2.2130	41
20	3706	2.6985	3906	2.5605	4108	2.4342	4314	2.3183	4522	2.2113	**40**
21	3709	2.6961	3909	2.5583	4111	2.4322	4317	2.3164	4526	2.2096	39
22	3712	2.6937	3912	2.5561	4115	2.4302	4320	2.3146	4529	2.2079	38
23	3716	2.6913	3916	2.5539	4118	2.4282	4324	2.3127	4533	2.2062	37
24	3719	2.6889	3919	2.5517	4122	2.4262	4327	2.3109	4536	2.2045	36
25	3722	2.6865	3922	2.5495	4125	2.4242	4331	2.3090	4540	2.2028	35
26	3726	2.6841	3926	2.5473	4129	2.4222	4334	2.3072	4543	2.2011	34
27	3729	2.6818	3929	2.5452	4132	2.4202	4338	2.3053	4547	2.1994	33
28	3732	2.6794	3932	2.5430	4135	2.4182	4341	2.3035	4550	2.1977	32
29	3736	2.6770	3936	2.5408	4139	2.4162	4345	2.3017	4554	2.1960	31
30	3739	2.6746	3939	2.5386	4142	2.4142	4348	2.2998	4557	2.1943	**30**
31	3742	2.6723	3942	2.5365	4146	2.4122	4352	2.2980	4561	2.1926	29
32	3745	2.6699	3946	2.5343	4149	2.4102	4355	2.2962	4564	2.1909	28
33	3749	2.6675	3949	2.5322	4152	2.4083	4359	2.2944	4568	2.1892	27
34	3752	2.6652	3953	2.5300	4156	2.4063	4362	2.2925	4571	2.1876	26
35	3755	2.6628	3956	2.5279	4159	2.4043	4365	2.2907	4575	2.1859	25
36	3759	2.6605	3959	2.5257	4163	2.4023	4369	2.2889	4578	2.1842	24
37	3762	2.6581	3963	2.5236	4166	2.4004	4372	2.2871	4582	2.1825	23
38	3765	2.6558	3966	2.5214	4169	2.3984	4376	2.2853	4585	2.1808	22
39	3769	2.6534	3969	2.5193	4173	2.3964	4379	2.2835	4589	2.1792	21
40	3772	2.6511	3973	2.5172	4176	2.3945	4383	2.2817	4592	2.1775	**20**
41	3775	2.6488	3976	2.5150	4180	2.3925	4386	2.2799	4596	2.1758	19
42	3779	2.6464	3979	2.5129	4183	2.3906	4390	2.2781	4599	2.1742	18
43	3782	2.6441	3983	2.5108	4187	2.3886	4393	2.2763	4603	2.1725	17
44	3785	2.6418	3986	2.5086	4190	2.3867	4397	2.2745	4607	2.1708	16
45	3789	2.6395	3990	2.5065	4193	2.3847	4400	2.2727	4610	2.1692	15
46	3792	2.6371	3993	2.5044	4197	2.3828	4404	2.2709	4614	2.1675	14
47	3795	2.6348	3996	2.5023	4200	2.3808	4407	2.2691	4617	2.1659	13
48	3799	2.6325	4000	2.5002	4204	2.3789	4411	2.2673	4621	2.1642	12
49	3802	2.6302	4003	2.4981	4207	2.3770	4414	2.2655	4624	2.1625	11
50	3805	2.6279	4006	2.4960	4210	2.3750	4417	2.2637	4628	2.1609	**10**
51	3809	2.6256	4010	2.4939	4214	2.3731	4421	2.2620	4631	2.1592	9
52	3812	2.6233	4013	2.4918	4217	2.3712	4424	2.2602	4635	2.1576	8
53	3815	2.6210	4017	2.4897	4221	2.3693	4428	2.2584	4638	2.1560	7
54	3819	2.6187	4020	2.4876	4224	2.3673	4431	2.2566	4642	2.1543	6
55	3822	2.6165	4023	2.4855	4228	2.3654	4435	2.2549	4645	2.1527	5
56	3825	2.6142	4027	2.4834	4231	2.3635	4438	2.2531	4649	2.1510	4
57	3829	2.6119	4030	2.4813	4234	2.3616	4442	2.2513	4652	2.1494	3
58	3832	2.6096	4033	2.4792	4238	2.3597	4445	2.2496	4656	2.1478	2
59	3835	2.6074	4037	2.4772	4241	2.3578	4449	2.2478	4660	2.1461	1
60	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	4663	2.1445	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	4663	2.1445	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	**60**
1	4667	2.1429	4881	2.0488	5099	1.9612	5321	1.8794	5547	1.8028	59
2	4670	2.1413	4885	2.0473	5103	1.9598	5325	1.8781	5551	1.8016	58
3	4674	2.1396	4888	2.0458	5106	1.9584	5328	1.8768	5555	1.8003	57
4	4677	2.1380	4892	2.0443	5110	1.9570	5332	1.8755	5558	1.7991	56
5	4681	2.1364	4895	2.0428	5114	1.9556	5336	1.8741	5562	1.7979	55
6	4684	2.1348	4899	2.0413	5117	1.9542	5340	1.8728	5566	1.7966	54
7	4688	2.1332	4903	2.0398	5121	1.9528	5343	1.8715	5570	1.7954	53
8	4691	2.1315	4906	2.0383	5125	1.9514	5347	1.8702	5574	1.7942	52
9	4695	2.1299	4910	2.0368	5128	1.9500	5351	1.8689	5577	1.7930	51
10	4699	2.1283	4913	2.0353	5132	1.9486	5354	1.8676	5581	1.7917	**50**
11	4702	2.1267	4917	2.0338	5136	1.9472	5358	1.8663	5585	1.7905	49
12	4706	2.1251	4921	2.0323	5139	1.9458	5362	1.8650	5589	1.7893	48
13	4709	2.1235	4924	2.0308	5143	1.9444	5366	1.8637	5593	1.7881	47
14	4713	2.1219	4928	2.0293	5147	1.9430	5369	1.8624	5596	1.7868	46
15	4716	2.1203	4931	2.0278	5150	1.9416	5373	1.8611	5600	1.7856	45
16	4720	2.1187	4935	2.0263	5154	1.9402	5377	1.8598	5604	1.7844	44
17	4723	2.1171	4939	2.0248	5158	1.9388	5381	1.8585	5608	1.7832	43
18	4727	2.1155	4942	2.0233	5161	1.9375	5384	1.8572	5612	1.7820	42
19	4731	2.1139	4946	2.0219	5165	1.9361	5388	1.8559	5616	1.7808	41
20	4734	2.1123	4950	2.0204	5169	1.9347	5392	1.8546	5619	1.7796	**40**
21	4738	2.1107	4953	2.0189	5172	1.9333	5396	1.8533	5623	1.7783	39
22	4741	2.1092	4957	2.0174	5176	1.9319	5399	1.8520	5627	1.7771	38
23	4745	2.1076	4960	2.0160	5180	1.9306	5403	1.8507	5631	1.7759	37
24	4748	2.1060	4964	2.0145	5184	1.9292	5407	1.8495	5635	1.7747	36
25	4752	2.1044	4968	2.0130	5187	1.9278	5411	1.8482	5639	1.7735	35
26	4755	2.1028	4971	2.0115	5191	1.9265	5415	1.8469	5642	1.7723	34
27	4759	2.1013	4975	2.0101	5195	1.9251	5418	1.8456	5646	1.7711	33
28	4763	2.0997	4979	2.0086	5198	1.9237	5422	1.8443	5650	1.7699	32
29	4766	2.0981	4982	2.0072	5202	1.9223	5426	1.8430	5654	1.7687	31
30	4770	2.0965	4986	2.0057	5206	1.9210	5430	1.8418	5658	1.7675	**30**
31	4773	2.0950	4989	2.0042	5209	1.9196	5433	1.8405	5662	1.7663	29
32	4777	2.0934	4993	2.0028	5213	1.9183	5437	1.8392	5665	1.7651	28
33	4780	2.0918	4997	2.0013	5217	1.9169	5441	1.8379	5669	1.7639	27
34	4784	2.0903	5000	1.9999	5220	1.9155	5445	1.8367	5673	1.7627	26
35	4788	2.0887	5004	1.9984	5224	1.9142	5448	1.8354	5677	1.7615	25
36	4791	2.0872	5008	1.9970	5228	1.9128	5452	1.8341	5681	1.7603	24
37	4795	2.0856	5011	1.9955	5232	1.9115	5456	1.8329	5685	1.7591	23
38	4798	2.0840	5015	1.9941	5235	1.9101	5460	1.8316	5688	1.7579	22
39	4802	2.0825	5019	1.9926	5239	1.9088	5464	1.8303	5692	1.7567	21
40	4806	2.0809	5022	1.9912	5243	1.9074	5467	1.8291	5696	1.7556	**20**
41	4809	2.0794	5026	1.9897	5246	1.9061	5471	1.8278	5700	1.7544	19
42	4813	2.0778	5029	1.9883	5250	1.9047	5475	1.8265	5704	1.7532	18
43	4816	2.0763	5033	1.9868	5254	1.9034	5479	1.8253	5708	1.7520	17
44	4820	2.0748	5037	1.9854	5258	1.9020	5482	1.8240	5712	1.7508	16
45	4823	2.0732	5040	1.9840	5261	1.9007	5486	1.8228	5715	1.7496	15
46	4827	2.0717	5044	1.9825	5265	1.8993	5490	1.8215	5719	1.7485	14
47	4831	2.0701	5048	1.9811	5269	1.8980	5494	1.8202	5723	1.7473	13
48	4834	2.0686	5051	1.9797	5272	1.8967	5498	1.8190	5727	1.7461	12
49	4838	2.0671	5055	1.9782	5276	1.8953	5501	1.8177	5731	1.7449	11
50	4841	2.0655	5059	1.9768	5280	1.8940	5505	1.8165	5735	1.7437	**10**
51	4845	2.0640	5062	1.9754	5284	1.8927	5509	1.8152	5739	1.7426	9
52	4849	2.0625	5066	1.9740	5287	1.8913	5513	1.8140	5743	1.7414	8
53	4852	2.0609	5070	1.9725	5291	1.8900	5517	1.8127	5746	1.7402	7
54	4856	2.0594	5073	1.9711	5295	1.8887	5520	1.8115	5750	1.7391	6
55	4859	2.0579	5077	1.9697	5298	1.8873	5524	1.8103	5754	1.7379	5
56	4863	2.0564	5081	1.9683	5302	1.8860	5528	1.8090	5758	1.7367	4
57	4867	2.0549	5084	1.9669	5306	1.8847	5532	1.8078	5762	1.7355	3
58	4870	2.0533	5088	1.9654	5310	1.8834	5535	1.8065	5766	1.7344	2
59	4874	2.0518	5092	1.9640	5313	1.8820	5539	1.8053	5770	1.7332	1
60	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	5774	1.7321	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	5774	1.7321	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	**60**
1	5777	1.7309	6013	1.6632	6253	1.5993	6498	1.5389	6749	1.4816	59
2	5781	1.7297	6017	1.6621	6257	1.5983	6502	1.5379	6754	1.4807	58
3	5785	1.7286	6020	1.6610	6261	1.5972	6506	1.5369	6758	1.4798	57
4	5789	1.7274	6024	1.6599	6265	1.5962	6511	1.5359	6762	1.4788	56
5	5793	1.7262	6028	1.6588	6269	1.5952	6515	1.5350	6766	1.4779	55
6	5797	1.7251	6032	1.6577	6273	1.5941	6519	1.5340	6771	1.4770	54
7	5801	1.7239	6036	1.6566	6277	1.5931	6523	1.5330	6775	1.4761	53
8	5805	1.7228	6040	1.6555	6281	1.5921	6527	1.5320	6779	1.4751	52
9	5808	1.7216	6044	1.6545	6285	1.5911	6531	1.5311	6783	1.4742	51
10	5812	1.7205	6048	1.6534	6289	1.5900	6536	1.5301	6787	1.4733	**50**
11	5816	1.7193	6052	1.6523	6293	1.5890	6540	1.5291	6792	1.4724	49
12	5820	1.7182	6056	1.6512	6297	1.5880	6544	1.5282	6796	1.4715	48
13	5824	1.7170	6060	1.6501	6301	1.5869	6548	1.5272	6800	1.4705	47
14	5828	1.7159	6064	1.6490	6305	1.5859	6552	1.5262	6805	1.4696	46
15	5832	1.7147	6068	1.6479	6310	1.5849	6556	1.5253	6809	1.4687	45
16	5836	1.7136	6072	1.6469	6314	1.5839	6560	1.5243	6813	1.4678	44
17	5840	1.7124	6076	1.6458	6318	1.5829	6565	1.5233	6817	1.4669	43
18	5844	1.7113	6080	1.6447	6322	1.5818	6569	1.5224	6822	1.4659	42
19	5847	1.7102	6084	1.6436	6326	1.5808	6573	1.5214	6826	1.4650	41
20	5851	1.7090	6088	1.6426	6330	1.5798	6577	1.5204	6830	1.4641	**40**
21	5855	1.7079	6092	1.6415	6334	1.5788	6581	1.5195	6834	1.4632	39
22	5859	1.7067	6096	1.6404	6338	1.5778	6585	1.5185	6839	1.4623	38
23	5863	1.7056	6100	1.6393	6342	1.5768	6590	1.5175	6843	1.4614	37
24	5867	1.7045	6104	1.6383	6346	1.5757	6594	1.5166	6847	1.4605	36
25	5871	1.7033	6108	1.6372	6350	1.5747	6598	1.5156	6851	1.4596	35
26	5875	1.7022	6112	1.6361	6354	1.5737	6602	1.5147	6856	1.4586	34
27	5879	1.7011	6116	1.6351	6358	1.5727	6606	1.5137	6860	1.4577	33
28	5883	1.6999	6120	1.6340	6363	1.5717	6610	1.5127	6864	1.4568	32
29	5887	1.6988	6124	1.6329	6367	1.5707	6615	1.5118	6869	1.4559	31
30	5890	1.6977	6128	1.6319	6371	1.5697	6619	1.5108	6873	1.4550	**30**
31	5894	1.6965	6132	1.6308	6375	1.5687	6623	1.5099	6877	1.4541	29
32	5898	1.6954	6136	1.6297	6379	1.5677	6627	1.5089	6881	1.4532	28
33	5902	1.6943	6140	1.6287	6383	1.5667	6631	1.5080	6886	1.4523	27
34	5906	1.6932	6144	1.6276	6387	1.5657	6636	1.5070	6890	1.4514	26
35	5910	1.6920	6148	1.6265	6391	1.5647	6640	1.5061	6894	1.4505	25
36	5914	1.6909	6152	1.6255	6395	1.5637	6644	1.5051	6899	1.4496	24
37	5918	1.6898	6156	1.6244	6399	1.5627	6648	1.5042	6903	1.4487	23
38	5922	1.6887	6160	1.6234	6403	1.5617	6652	1.5032	6907	1.4478	22
39	5926	1.6875	6164	1.6223	6408	1.5607	6657	1.5023	6911	1.4469	21
40	5930	1.6864	6168	1.6212	6412	1.5597	6661	1.5013	6916	1.4460	**20**
41	5934	1.6853	6172	1.6202	6416	1.5587	6665	1.5004	6920	1.4451	19
42	5938	1.6842	6176	1.6191	6420	1.5577	6669	1.4994	6924	1.4442	18
43	5942	1.6831	6180	1.6181	6424	1.5567	6673	1.4985	6929	1.4433	17
44	5945	1.6820	6184	1.6170	6428	1.5557	6678	1.4975	6933	1.4424	16
45	5949	1.6808	6188	1.6160	6432	1.5547	6682	1.4966	6937	1.4415	15
46	5953	1.6797	6192	1.6149	6436	1.5537	6686	1.4957	6942	1.4406	14
47	5957	1.6786	6196	1.6139	6440	1.5527	6690	1.4947	6946	1.4397	13
48	5961	1.6775	6200	1.6128	6445	1.5517	6694	1.4938	6950	1.4388	12
49	5965	1.6764	6204	1.6118	6449	1.5507	6699	1.4928	6954	1.4379	11
50	5969	1.6753	6208	1.6107	6453	1.5497	6703	1.4919	6959	1.4370	**10**
51	5973	1.6742	6212	1.6097	6457	1.5487	6707	1.4910	6963	1.4361	9
52	5977	1.6731	6216	1.6087	6461	1.5477	6711	1.4900	6967	1.4352	8
53	5981	1.6720	6220	1.6076	6465	1.5468	6716	1.4891	6972	1.4344	7
54	5985	1.6709	6224	1.6066	6469	1.5458	6720	1.4882	6976	1.4335	6
55	5989	1.6698	6228	1.6055	6473	1.5448	6724	1.4872	6980	1.4326	5
56	5993	1.6687	6233	1.6045	6478	1.5438	6728	1.4863	6985	1.4317	4
57	5997	1.6676	6237	1.6034	6482	1.5428	6732	1.4854	6989	1.4308	3
58	6001	1.6665	6241	1.6024	6486	1.5418	6737	1.4844	6993	1.4299	2
59	6005	1.6654	6245	1.6014	6490	1.5408	6741	1.4835	6998	1.4290	1
60	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	7002	1.4281	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	7002	1.4281	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	**60**
1	7006	1.4273	7270	1.3755	7540	1.3262	7818	1.2792	8103	1.2342	59
2	7011	1.4264	7274	1.3747	7545	1.3254	7822	1.2784	8107	1.2334	58
3	7015	1.4255	7279	1.3739	7549	1.3246	7827	1.2776	8112	1.2327	57
4	7019	1.4246	7283	1.3730	7554	1.3238	7832	1.2769	8117	1.2320	56
5	7024	1.4237	7288	1.3722	7558	1.3230	7836	1.2761	8122	1.2312	55
6	7028	1.4229	7292	1.3713	7563	1.3222	7841	1.2753	8127	1.2305	54
7	7032	1.4220	7297	1.3705	7568	1.3214	7846	1.2746	8132	1.2298	53
8	7037	1.4211	7301	1.3697	7572	1.3206	7850	1.2738	8136	1.2290	52
9	7041	1.4202	7306	1.3688	7577	1.3198	7855	1.2731	8141	1.2283	51
10	7046	1.4193	7310	1.3680	7581	1.3190	7860	1.2723	8146	1.2276	**50**
11	7050	1.4185	7314	1.3672	7586	1.3182	7865	1.2715	8151	1.2268	49
12	7054	1.4176	7319	1.3663	7590	1.3175	7869	1.2708	8156	1.2261	48
13	7059	1.4167	7323	1.3655	7595	1.3167	7874	1.2700	8161	1.2254	47
14	7063	1.4158	7328	1.3647	7600	1.3159	7879	1.2693	8165	1.2247	46
15	7067	1.4150	7332	1.3638	7604	1.3151	7883	1.2685	8170	1.2239	45
16	7072	1.4141	7337	1.3630	7609	1.3143	7888	1.2677	8175	1.2232	44
17	7076	1.4132	7341	1.3622	7613	1.3135	7893	1.2670	8180	1.2225	43
18	7080	1.4124	7346	1.3613	7618	1.3127	7898	1.2662	8185	1.2218	42
19	7085	1.4115	7350	1.3605	7623	1.3119	7902	1.2655	8190	1.2210	41
20	7089	1.4106	7355	1.3597	7627	1.3111	7907	1.2647	8195	1.2203	**40**
21	7094	1.4097	7359	1.3588	7632	1.3103	7912	1.2640	8199	1.2196	39
22	7098	1.4089	7364	1.3580	7636	1.3095	7916	1.2632	8204	1.2189	38
23	7102	1.4080	7368	1.3572	7641	1.3087	7921	1.2624	8209	1.2181	37
24	7107	1.4071	7373	1.3564	7646	1.3079	7926	1.2617	8214	1.2174	36
25	7111	1.4063	7377	1.3555	7650	1.3072	7931	1.2609	8219	1.2167	35
26	7115	1.4054	7382	1.3547	7655	1.3064	7935	1.2602	8224	1.2160	34
27	7120	1.4045	7386	1.3539	7659	1.3056	7940	1.2594	8229	1.2153	33
28	7124	1.4037	7391	1.3531	7664	1.3048	7945	1.2587	8234	1.2145	32
29	7129	1.4028	7395	1.3522	7669	1.3040	7950	1.2579	8238	1.2138	31
30	7133	1.4019	7400	1.3514	7673	1.3032	7954	1.2572	8243	1.2131	**30**
31	7137	1.4011	7404	1.3506	7678	1.3024	7959	1.2564	8248	1.2124	29
32	7142	1.4002	7409	1.3498	7683	1.3017	7964	1.2557	8253	1.2117	28
33	7146	1.3994	7413	1.3490	7687	1.3009	7969	1.2549	8258	1.2109	27
34	7151	1.3985	7418	1.3481	7692	1.3001	7973	1.2542	8263	1.2102	26
35	7155	1.3976	7422	1.3473	7696	1.2993	7978	1.2534	8268	1.2095	25
36	7159	1.3968	7427	1.3465	7701	1.2985	7983	1.2527	8273	1.2088	24
37	7164	1.3959	7431	1.3457	7706	1.2977	7988	1.2519	8278	1.2081	23
38	7168	1.3951	7436	1.3449	7710	1.2970	7992	1.2512	8283	1.2074	22
39	7173	1.3942	7440	1.3440	7715	1.2962	7997	1.2504	8287	1.2066	21
40	7177	1.3934	7445	1.3432	7720	1.2954	8002	1.2497	8292	1.2059	**20**
41	7181	1.3925	7449	1.3424	7724	1.2946	8007	1.2489	8297	1.2052	19
42	7186	1.3916	7454	1.3416	7729	1.2938	8012	1.2482	8302	1.2045	18
43	7190	1.3908	7458	1.3408	7734	1.2931	8016	1.2475	8307	1.2038	17
44	7195	1.3899	7463	1.3400	7738	1.2923	8021	1.2467	8312	1.2031	16
45	7199	1.3891	7467	1.3392	7743	1.2915	8026	1.2460	8317	1.2024	15
46	7203	1.3882	7472	1.3384	7747	1.2907	8031	1.2452	8322	1.2017	14
47	7208	1.3874	7476	1.3375	7752	1.2900	8035	1.2445	8327	1.2009	13
48	7212	1.3865	7481	1.3367	7757	1.2892	8040	1.2437	8332	1.2002	12
49	7217	1.3857	7485	1.3359	7761	1.2884	8045	1.2430	8337	1.1995	11
50	7221	1.3848	7490	1.3351	7766	1.2876	8050	1.2423	8342	1.1988	**10**
51	7226	1.3840	7495	1.3343	7771	1.2869	8055	1.2415	8346	1.1981	9
52	7230	1.3831	7499	1.3335	7775	1.2861	8059	1.2408	8351	1.1974	8
53	7234	1.3823	7504	1.3327	7780	1.2853	8064	1.2401	8356	1.1967	7
54	7239	1.3814	7508	1.3319	7785	1.2846	8069	1.2393	8361	1.1960	6
55	7243	1.3806	7513	1.3311	7789	1.2838	8074	1.2386	8366	1.1953	5
56	7248	1.3798	7517	1.3303	7794	1.2830	8079	1.2378	8371	1.1946	4
57	7252	1.3789	7522	1.3295	7799	1.2822	8083	1.2371	8376	1.1939	3
58	7257	1.3781	7526	1.3287	7803	1.2815	8088	1.2364	8381	1.1932	2
59	7261	1.3772	7531	1.3278	7808	1.2807	8093	1.2356	8386	1.1925	1
60	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	8391	1.1918	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	54°		53°		52°		51°		50°		′

40° tan	40° cot	41° tan	41° cot	42° tan	42° cot	43° tan	43° cot	44° tan	44° cot	'
8391	1.1918	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	60
8396	1.1910	8698	1.1497	9009	1.1100	9331	1.0717	9663	1.0349	59
8401	1.1903	8703	1.1490	9015	1.1093	9336	1.0711	9668	1.0343	58
8406	1.1896	8708	1.1483	9020	1.1087	9341	1.0705	9674	1.0337	57
8411	1.1889	8713	1.1477	9025	1.1080	9347	1.0699	9679	1.0331	56
8416	1.1882	8718	1.1470	9030	1.1074	9352	1.0692	9685	1.0325	55
8421	1.1875	8724	1.1463	9036	1.1067	9358	1.0686	9691	1.0319	54
8426	1.1868	8729	1.1456	9041	1.1061	9363	1.0680	9696	1.0313	53
8431	1.1861	8734	1.1450	9046	1.1054	9369	1.0674	9702	1.0307	52
8436	1.1854	8739	1.1443	9052	1.1048	9374	1.0668	9708	1.0301	51
8441	1.1847	8744	1.1436	9057	1.1041	9380	1.0661	9713	1.0295	50
8446	1.1840	8749	1.1430	9062	1.1035	9385	1.0655	9719	1.0289	49
8451	1.1833	8754	1.1423	9067	1.1028	9391	1.0649	9725	1.0283	48
8456	1.1826	8759	1.1416	9073	1.1022	9396	1.0643	9730	1.0277	47
8461	1.1819	8765	1.1410	9078	1.1016	9402	1.0637	9736	1.0271	46
8466	1.1812	8770	1.1403	9083	1.1009	9407	1.0630	9742	1.0265	45
8471	1.1806	8775	1.1396	9089	1.1003	9413	1.0624	9747	1.0259	44
8476	1.1799	8780	1.1389	9094	1.0996	9418	1.0618	9753	1.0253	43
8481	1.1792	8785	1.1383	9099	1.0990	9424	1.0612	9759	1.0247	42
8486	1.1785	8790	1.1376	9105	1.0983	9429	1.0606	9764	1.0241	41
8491	1.1778	8796	1.1369	9110	1.0977	9435	1.0599	9770	1.0235	40
8496	1.1771	8801	1.1363	9115	1.0971	9440	1.0593	9776	1.0230	39
8501	1.1764	8806	1.1356	9121	1.0964	9446	1.0587	9781	1.0224	38
8506	1.1757	8811	1.1349	9126	1.0958	9451	1.0581	9787	1.0218	37
8511	1.1750	8816	1.1343	9131	1.0951	9457	1.0575	9793	1.0212	36
8516	1.1743	8821	1.1336	9137	1.0945	9462	1.0569	9798	1.0206	35
8521	1.1736	8827	1.1329	9142	1.0939	9468	1.0562	9804	1.0200	34
8526	1.1729	8832	1.1323	9147	1.0932	9473	1.0556	9810	1.0194	33
8531	1.1722	8837	1.1316	9153	1.0926	9479	1.0550	9816	1.0188	32
8536	1.1715	8842	1.1310	9158	1.0919	9484	1.0544	9821	1.0182	31
8541	1.1708	8847	1.1303	9163	1.0913	9490	1.0538	9827	1.0176	30
8546	1.1702	8852	1.1296	9169	1.0907	9495	1.0532	9833	1.0170	29
8551	1.1695	8858	1.1290	9174	1.0900	9501	1.0526	9838	1.0164	28
8556	1.1688	8863	1.1283	9179	1.0894	9506	1.0519	9844	1.0158	27
8561	1.1681	8868	1.1276	9185	1.0888	9512	1.0513	9850	1.0152	26
8566	1.1674	8873	1.1270	9190	1.0881	9517	1.0507	9856	1.0147	25
8571	1.1667	8878	1.1263	9195	1.0875	9523	1.0501	9861	1.0141	24
8576	1.1660	8884	1.1257	9201	1.0869	9528	1.0495	9867	1.0135	23
8581	1.1653	8889	1.1250	9206	1.0862	9534	1.0489	9873	1.0129	22
8586	1.1647	8894	1.1243	9212	1.0856	9540	1.0483	9879	1.0123	21
8591	1.1640	8899	1.1237	9217	1.0850	9545	1.0477	9884	1.0117	20
8596	1.1633	8904	1.1230	9222	1.0843	9551	1.0470	9890	1.0111	19
8601	1.1626	8910	1.1224	9228	1.0837	9556	1.0464	9896	1.0105	18
8606	1.1619	8915	1.1217	9233	1.0831	9562	1.0458	9902	1.0099	17
8611	1.1612	8920	1.1211	9239	1.0824	9567	1.0452	9907	1.0094	16
8617	1.1606	8925	1.1204	9244	1.0818	9573	1.0446	9913	1.0088	15
8622	1.1599	8931	1.1197	9249	1.0812	9578	1.0440	9919	1.0082	14
8627	1.1592	8936	1.1191	9255	1.0805	9584	1.0434	9925	1.0076	13
8632	1.1585	8941	1.1184	9260	1.0799	9590	1.0428	9930	1.0070	12
8637	1.1578	8946	1.1178	9266	1.0793	9595	1.0422	9936	1.0064	11
8642	1.1571	8952	1.1171	9271	1.0786	9601	1.0416	9942	1.0058	10
8647	1.1565	8957	1.1165	9276	1.0780	9606	1.0410	9948	1.0052	9
8652	1.1558	8962	1.1158	9282	1.0774	9612	1.0404	9954	1.0047	8
8657	1.1551	8967	1.1152	9287	1.0768	9618	1.0398	9959	1.0041	7
8662	1.1544	8972	1.1145	9293	1.0761	9623	1.0392	9965	1.0035	6
8667	1.1538	8978	1.1139	9298	1.0755	9629	1.0385	9971	1.0029	5
8672	1.1531	8983	1.1132	9303	1.0749	9634	1.0379	9977	1.0023	4
8678	1.1524	8988	1.1126	9309	1.0742	9640	1.0373	9983	1.0017	3
8683	1.1517	8994	1.1119	9314	1.0736	9646	1.0367	9988	1.0012	2
8688	1.1510	8999	1.1113	9320	1.0730	9651	1.0361	9994	1.0006	1
8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	1000	1.0000	0
49° cot	49° tan	48° cot	48° tan	47° cot	47° tan	46° cot	46° tan	45° cot	45° tan	'

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